Center for Tokamak Transients Simulation

VDE and RWM Status and Plans

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CTTS Meeting

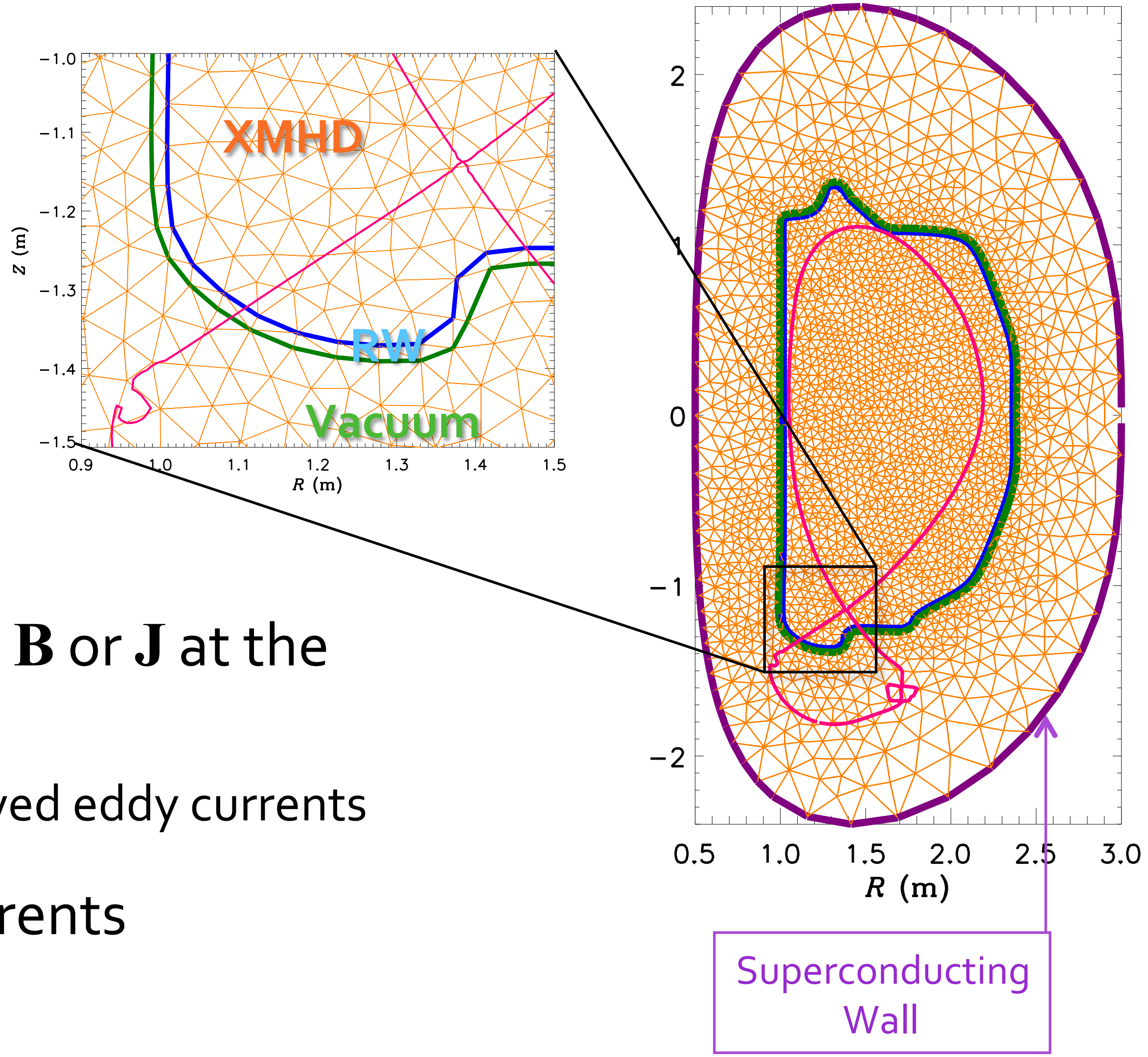
Milwaukee, WI

October 22, 2017

M3D-C1

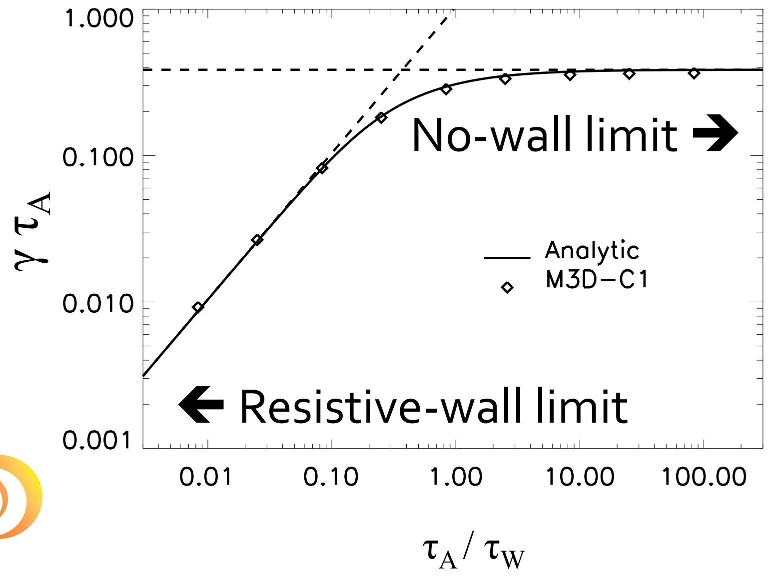
M3D-C1 Uses Multi-Region Mesh to Model Resistive Wall

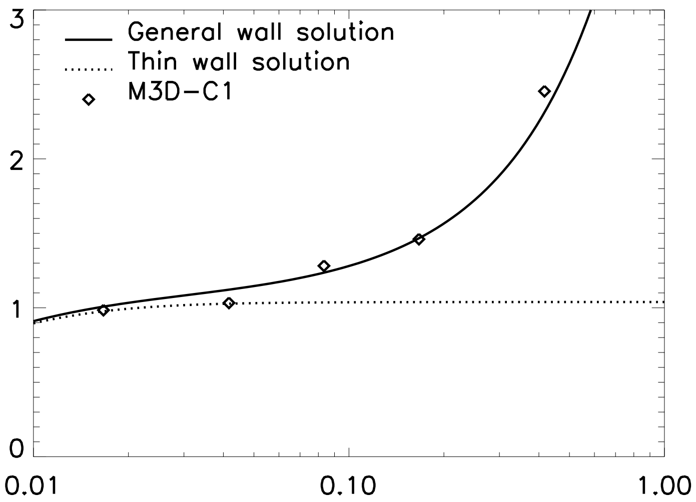
- Three mesh regions:
  - XMHD (extended-MHD)
  - Resistive Wall ($\mathbf{E} = \eta\ \mathbf{J}$)
  - Vacuum ($\mathbf{J} = 0$)
- Resistive wall is region of arbitrary thickness
- No boundary or jump conditions on $\mathbf{B}$ or $\mathbf{J}$ at the resistive wall
  - Allows Halo currents and spatially-resolved eddy currents
- Fully implicit treatment of eddy currents

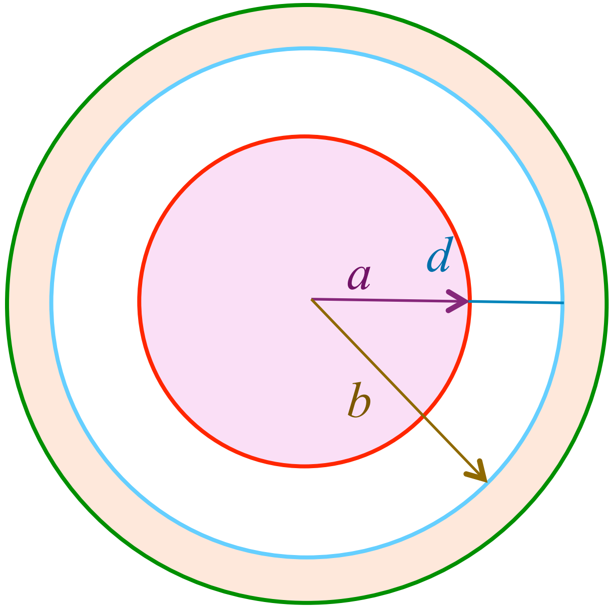


Linear M3D-C1 Simulations of RWMs Successfully Validated

- Linear growth rates compared to analytic model using reduced MHD
- Agreement spans resistive-wall to inertial regimes
- Agreement for walls of arbitrary thickness

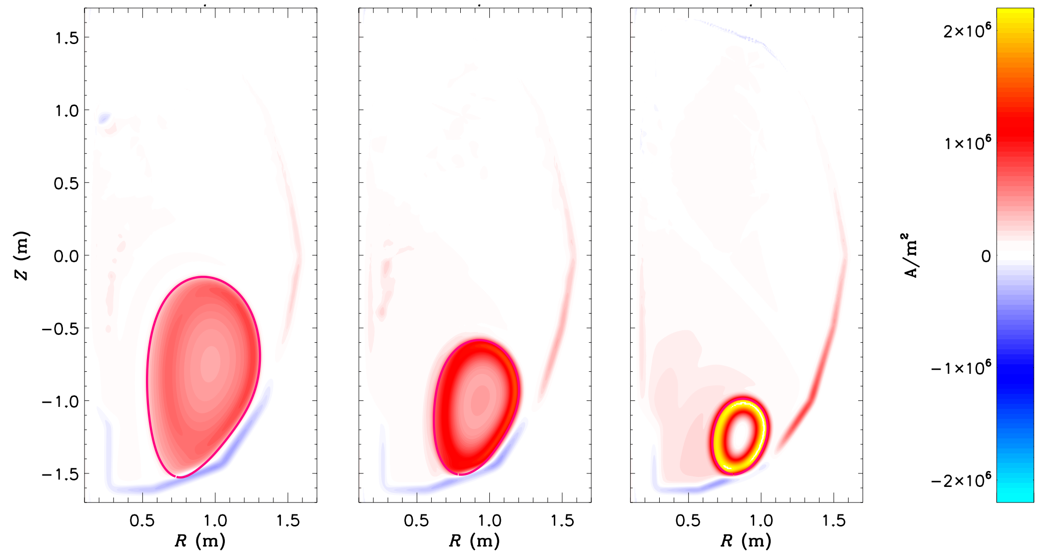


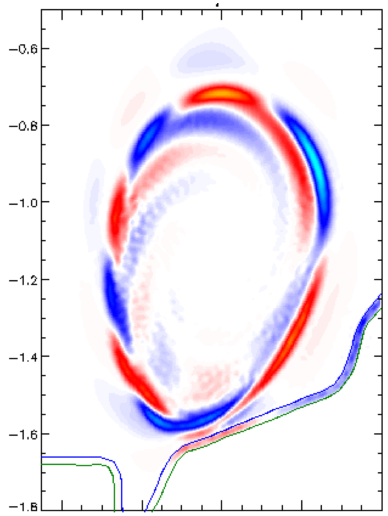


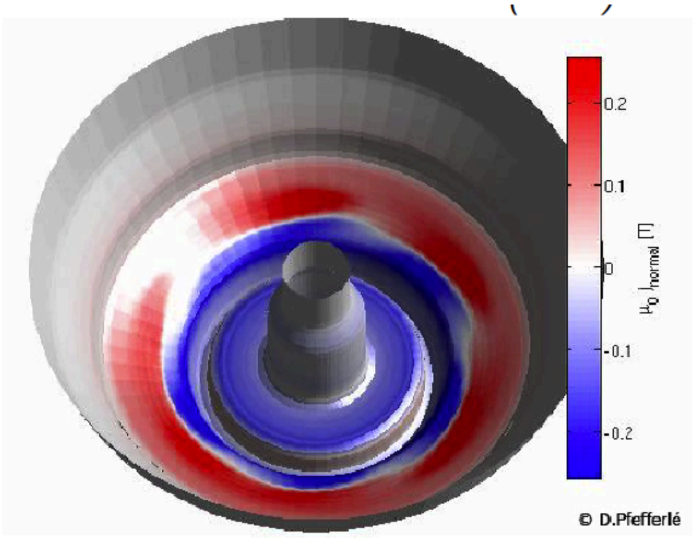


M3D-C1 Being Used for 3D Nonlinear Simulations of VDEs

- Simulations initialized with vertically unstable Grad-Shafranov Equilibrium
- Axisymmetric model is used until q_a drops and plasma becomes unstable to $n > 0$ modes
- Fully 3D model is then used to follow non-axisymmetric evolution of plasma and Halo currents
- **See talk by D. Pfefferlé, Wednesday at 9:30 am (NI3.00001)**



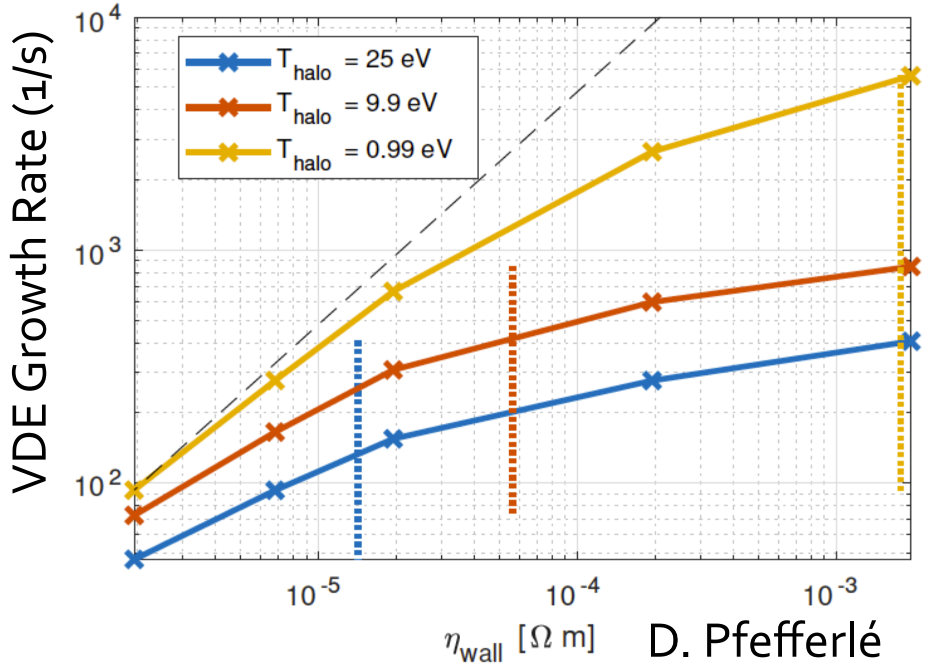




Primary Challenge for VDE Simulations is Resolving Gradients Near Wall

- Hot plasma touching cold wall leads to huge gradients
- Several strategies are used to mitigate this:
  - Aggressive mesh packing
  - The Schnack Protocol (*i.e.* increase viscosity when the code crashes)
  - Make wall (and SOL) artificially hot
- Artificially hot SOL makes resistivity too small
  - Stabilizing to VDE and secondary non-axisymmetric instabilities
  - We correct this by subtracting T_e offset in resistivity calculation

$$\eta = \eta_0 \, (T_e - T_{off})^{-3/2}$$

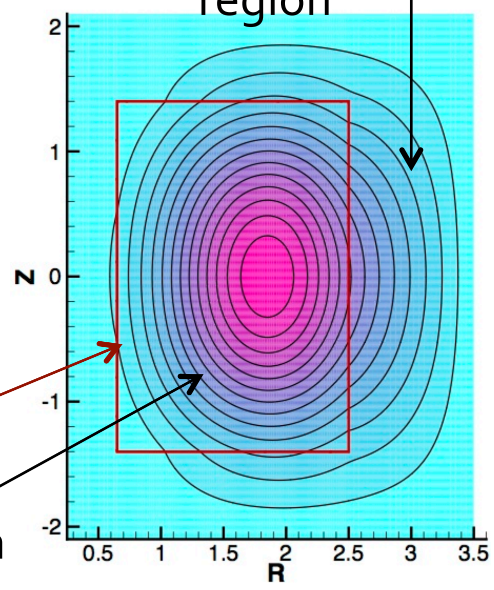


D. Pfefferlé

NIMROD

Two Resistive-Wall Formulations Have Been Implemented in NIMROD

- Both link the inner (plasma) region to an outer region via the thin resistive-wall model.
- One uses a Green's function/boundary element method that couples a NIMROD inner region to the GRIN code (Pletzer).
- The other uses a meshed outer region that solves a magnetic diffusion equation.
    - Large η_{outer} approximates vacuum.
    - Meshed layers of moderate η_{outer} can be used to represent a thick wall.



With the Meshed Outer Region, A Weak Form of the Thin Wall Equation is Used

- In NIMROD's weak form, the PDE from Faraday's law for the interior of each region is

$$\int_R \mathbf{A}^* \cdot \frac{\partial \mathbf{B}}{\partial t} dVol = -\int_R \mathbf{E} \cdot \nabla \times \mathbf{A}^* dVol + \oint_{\partial R} \mathbf{A}^* \times \mathbf{E} \cdot \hat{\mathbf{n}} dS$$

for all vector test functions

$$\mathbf{A}_{k,n,\nu}(R,Z,\phi) = \alpha_k\left[\xi(R,Z),\eta(R,Z)\right] e^{in\phi} \hat{\mathbf{e}}_\nu(\phi)$$

used in the expansion for the magnetic field. Here, $\alpha_k(\xi,\eta)$ is a 2D nodal spectral element, $\hat{\mathbf{e}}_1 = \hat{\mathbf{R}}(\phi), \hat{\mathbf{e}}_2 = \mathbf{Z}, \hat{\mathbf{e}}_3 = \hat{\boldsymbol{\phi}}(\phi)$, and

$$\mathbf{B}(R,Z,\phi,t) = \sum_{k,n,\nu} B_{k,n,\nu}(t) \mathbf{A}_{k,n,\nu}(R,Z,\phi)$$

The resistive-wall $\mathbf{E} = v_w \hat{\mathbf{n}} \times \delta \mathbf{B}$ is used in the surface integral.

$$v_w \equiv \frac{\eta_w}{\mu_0 \delta x}$$

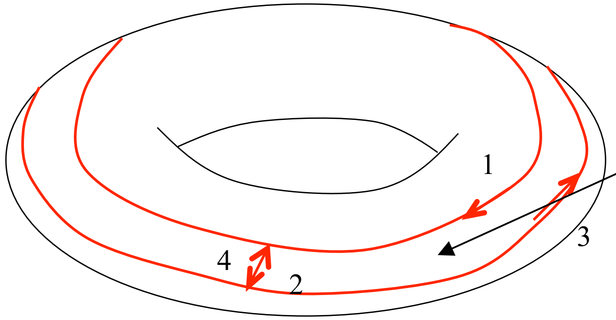
Evolution of the Normal Component is Imposed as an Integral Constraint with the Same Test Functions

- Applying Faraday's law along an interface between regions,

$$\int_{\partial R} \mathbf{A}^* \cdot \hat{\mathbf{n}}\hat{\mathbf{n}} \cdot \frac{\partial \mathbf{B}}{\partial t} dS = -\int_{\partial R} \mathbf{A}^* \cdot \hat{\mathbf{n}}\hat{\mathbf{n}} \cdot \nabla \times \mathbf{E}\, dS$$

$$= \int_{\partial R} \hat{\mathbf{n}} \cdot \nabla\left(\mathbf{A}^* \cdot \hat{\mathbf{n}}\right) \times \mathbf{E}\, dS - \sum_j \oint_{\partial R_j} \mathbf{A}^* \cdot \hat{\mathbf{n}}\mathbf{E} \cdot d\mathbf{l}$$

with path integrals defined by curves swept by the corners of the 2D elements.

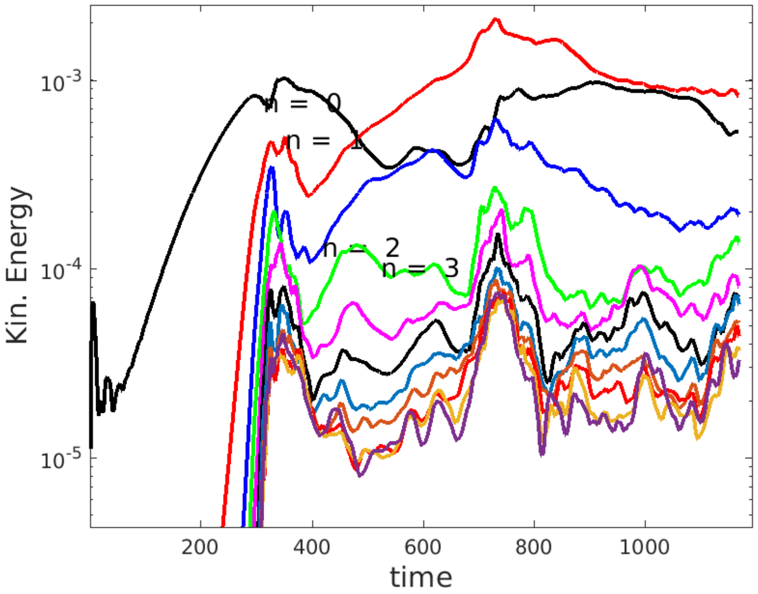


- Adding the constraint equation with a Lagrange multiplier λ provides an unsplit relation, and E has implicit $v_w \hat{\mathbf{n}} \times \Delta\delta\mathbf{B}$ terms. $\left[\Delta f \equiv f(t+\Delta t) - f(t)\right]$

$$\int_R \mathbf{A}^* \cdot \Delta\mathbf{B}\, dVol + \Delta t \int_R \mathbf{E} \cdot \nabla \times \mathbf{A}^*\, dVol - \Delta t \oint_{\partial R} \mathbf{A}^* \times \mathbf{E} \cdot \hat{\mathbf{n}}\, dS$$

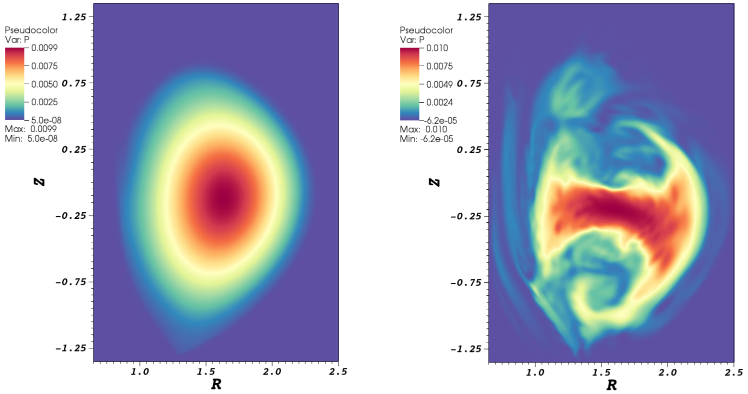
$$+\lambda\left[\int_{\partial R} \mathbf{A}^* \cdot \hat{\mathbf{n}}\hat{\mathbf{n}} \cdot \Delta\mathbf{B}\, dS - \Delta t \int_{\partial R} \hat{\mathbf{n}} \cdot \nabla\left(\mathbf{A}^* \cdot \hat{\mathbf{n}}\right) \times \mathbf{E}\, dS + \Delta t \sum_j \oint_{\partial R_j} \mathbf{A}^* \cdot \hat{\mathbf{n}}\mathbf{E} \cdot d\mathbf{l}\right] = 0$$

Recent Forced-VDE Computations with Asymmetric Instability use the Coupled-Region Implementation

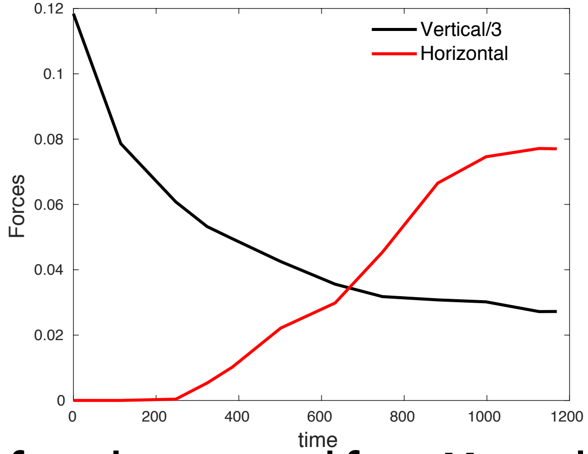


Evolution of kinetic energy fluctuations ($0 \leq n \leq 10$) with $\chi_{||}$=0.75.

- See posters CP11.00104 and CP11.00105, Monday PM, for more information



Plasma pressure at t=250 τ_A (left) and t=500 τ_A (right).



Net force is computed from Maxwell stress over outside of resistive wall.

Plans

Three Areas of Development Are Needed for VDE Studies

1. Sheath boundary conditions
   - Results are sensitive to conditions for *T* along wall.
   - Loizu, *et. al.* (PoP **19**, 122307) formulated bc's for reduced turbulence modeling.
   - We are adapting these conditions and will implement.
2. Wall geometry
   - Many NIMROD tokamak computations use conformal meshes outside the separatrix.
   - Special cross-section shapes have been meshed by hand.
   - Development of spectral triangles or blocks of unstructured quads will facilitate application to experiments.
3. Implicit advances
   - 3D simulation over long times (CQ) encounter many dynamics.
   - *A*-stable methods may facilitate computations by suppressing unimportant fluctuations.

For RWM Studies, We Proposed to Incorporate Kinetic Effects

- The wall modeling described earlier is suitable for RWM studies.
- Meshing developments would also benefit RWM applications.
- Kinetic effects are considered important for RWM stability properties [Berkery, *et. al.* PoP **17**, 082504].
  - We proposed to apply NIMROD's drift-kinetic modeling.
  - To date, there have been no attempts to invoke both RW and DKE in a computation.
  - The Green's function approach may be the easier starting point.

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M3D-C1 Development Will Include Radiation Models and Wall Asymmetries

- Implementation of KPRAD module in M3D-C1 is underway
  - Will track charge states and radiation from one impurity species
- Will implement non-axisymmetric wall structures by using a non-axisymmetric wall resistivity
  - Mesh will remain axisymmetric
  - Presently have capability for rectangular (in (R, φ, Z) coordinates) ports / breaks
- Particle source as a function of halo current may be necessary to capture halo current rotation

VDE Benchmark Should Test Primary Physical Prediction Targets

- Linear growth rate of VDE
- Halo currents as a function of time and space
- Plasma current and q-profile evolution during CQ
- Sideways ($n = 1$) forces, and rotation of these forces
- Is comparison of unstable VDE calculations a viable near term goal?
  - We have many DIII-D and NSTX geqdsks available ($n = 0$ so results shouldn't depend strongly on getting perfect equilibrium match)
  - Start with axisymmetric model
  - Can use an analytic (Miller-like) wall shape

RWM Benchmark Could Focus on Braking

- RWM benchmark doesn't need to be a resistive wall mode
  - Could be resistive wall tearing mode, or other mode that penetrates wall
  - Tearing mode rotates with plasma; RWM rotates at resistive wall frequency

- Focusing on braking would make benchmark interesting
  - Mode causes plasma to slow down by interaction of response fields with eddy currents
  - Not the same as ongoing error field penetration benchmark, which does not involve a resistive wall
  - Could start with case like Matt Beidler's (straight cylinder, circular cross-section), but unstable and without RMPs.