Kinetic calculations in NIMROD
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E. Held   J.-Y. Ji   Andy Spencer   NIMROD Team

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DKE in NIMROD

- Recast form of Hazeltine's DKE implemented in NIMROD:

$$\frac{\partial f}{\partial t} + (\mathbf{v}_\parallel + \mathbf{v}_D)\cdot\nabla f - s\frac{\partial f}{\partial s}\left[\frac{\partial}{\partial t} + (\mathbf{v}_\parallel + \mathbf{v}_D)\cdot\nabla\right]\ln v_0$$

$$-\xi(1-\xi^2)\left[\frac{\mu_0}{2B^2}\mathbf{J}_\parallel\cdot\mathbf{E} + \frac{T_0 s^2}{q}\mathbf{b}\cdot\nabla\left(\frac{\mu_0 J_\parallel}{B^2}\right)\right]\frac{\partial f}{\partial \xi}$$

$$+\frac{1-\xi^2}{2\xi}\left[-\xi^2\frac{\partial \ln B}{\partial t} + (\mathbf{v}_\parallel + \mathbf{v}_c^*)\cdot\left(\frac{q}{T_0 s^2}\mathbf{E} - \nabla\ln B\right) + \xi^2\mathbf{v}_{\mathbf{E}\times\mathbf{B}}\cdot\nabla\ln B\right]\frac{\partial f}{\partial \xi}$$

$$+\frac{s}{2}\left[(1-\xi^2)\frac{\partial \ln B}{\partial t} + (\mathbf{v}_\parallel + \mathbf{v}_c)\cdot\frac{q}{T_0 s^2}\mathbf{E} + (1+\xi^2)\mathbf{v}_{\mathbf{E}\times\mathbf{B}}\cdot\nabla\ln B\right]\frac{\partial f}{\partial s} = \mathbf{C}(f)$$

- Typically solve for δf beyond a fixed f_0:

  -for energetic ions: $f_0 = f_{\mathrm{slow}} + f_{\mathrm{tail}} = A/(1+s^3) + f_{\mathrm{tail}}$

RWMs with energetic ions.

- Advance energetic particle δf and couple to MHD through closure for anisotropic pressure tensor.
- Plans for improvements:
  -optimize loops in integrand routines,
  -finish implementation of nonlinear terms in DKE,
  -apply static condensation in pitch-angle,
  -develop infastructure for 3D FEs in NIMROD; DKE implementation using (R, Z, ξ)-elements (TECH-X).

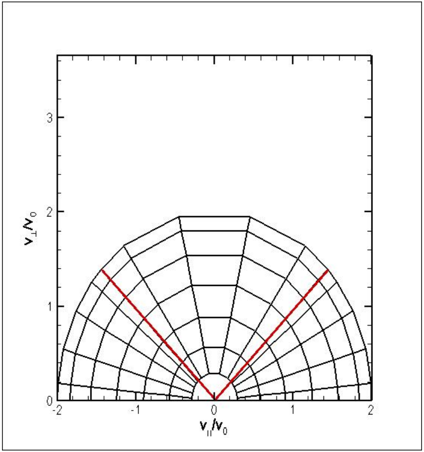


Figure: 2D velocity grid

Chapman-Enskog-like (CEL) DKE in NIMROD

- CEL-DKE in the fluid frame (Ramos, *Phys Plasmas* **17**, 082502 (2010)) implemented in NIMROD:

$$
\begin{aligned}
&\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial t} + v_\parallel' \mathbf{b}\cdot\nabla \bar{f}_{\mathrm{NM}} - \frac{1-\xi^2}{2\xi} v_\parallel' \mathbf{b}\cdot\nabla \ln B \frac{\partial \bar{f}_{\mathrm{NM}}}{\partial \xi} \\
&+\frac{v_0}{2}(\mathbf{b}\cdot\nabla\ln n)[\xi\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial s} + \frac{1-\xi^2}{s}\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial \xi}] - s[\xi\mathbf{b}\cdot\nabla + \frac{\partial}{\partial t}]\ln v_0 \frac{\partial \bar{f}_{\mathrm{NM}}}{\partial s} = \langle C(f)\rangle \\
&+\Bigg[(\frac{5}{2}-s^2)v_\parallel'\mathbf{b}\cdot\nabla\ln T + \frac{v_\parallel'}{nT}\mathbf{b}\cdot[\frac{2}{3}\nabla\pi_\parallel - \pi_\parallel\nabla\ln B - \mathbf{F}^{\mathrm{coll}}] \\
&+2s^2(\frac{3}{2}\xi^2-\frac{1}{2})[\frac{1}{3}\nabla\cdot\mathbf{u} - \mathbf{bb}\cdot\nabla\mathbf{u}] + \frac{2}{3nT}(s^2-\frac{5}{2})[\mathbf{b}\cdot\nabla q_\parallel - q_\parallel\mathbf{b}\cdot\nabla\ln B - G^{\mathrm{coll}}] \\
&+\frac{2}{3eB}s^2(\frac{3}{2}\xi^2-\frac{1}{2})[(\frac{5}{2}-s^2)(\nabla\ln B - 2\kappa) + \nabla\ln n]\cdot\nabla T\times\mathbf{b} \\
&+\frac{4}{3eB}(\frac{s^4}{2}-\frac{5}{2}s^2+\frac{15}{8})(\nabla\ln B+\kappa)\cdot\nabla T\times\mathbf{b}\Bigg] f_M
\end{aligned}
$$

Anisotropic thermal transport using CEL-DKE

- Test temperature flattening across magnetic islands by evolving

$$\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial t} + v_\parallel' \mathbf{b} \cdot \nabla \bar{f}_{\mathrm{NM}} - \frac{1-\xi^2}{2\xi} v_\parallel' \mathbf{b} \cdot \nabla \ln B \frac{\partial \bar{f}_{\mathrm{NM}}}{\partial \xi}$$

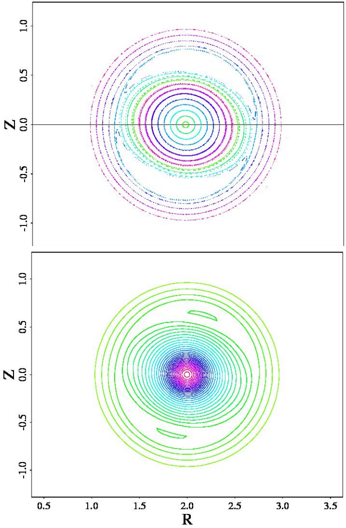
$$-s[\xi \mathbf{b} \cdot \nabla + \frac{\partial}{\partial t}] \ln v_0 \frac{\partial \bar{f}_{\mathrm{NM}}}{\partial s} = \langle C(f) \rangle +$$

$$\left[(\frac{5}{2} - s^2) v_\parallel' \mathbf{b} \cdot \nabla \ln T + \frac{2}{3nT} (s^2 - \frac{5}{2}) \nabla \cdot q_\parallel \mathbf{b} \right] f_M$$

$$\frac{3n}{2} \frac{\partial T}{\partial t} + \nabla \cdot q_\parallel \mathbf{b} = 0$$

to steady state.

- Simple prelude to NTMs to test
  -velocity-space resolution requirements,
  -reduction of $q_\parallel$ due to particle trapping,
  -efficiency of preconditioneed, matrix-free DKE solves.



Aspects of CEL-DKE formulation

- Allows for a tight coupling, *i.e.*, hybrid fluid/kinetic capability that is rigorous and consistent: 1, $v'_{\|}$ and v'^2 moments of $\bar{f}_{\mathrm{NM}}$ vanish.

- DKEs written in moving frame of fluid makes taking moments easy.

- Numerical considerations:
  -time centering of fluid and kinetic variables,
  -enforcing the requirement that fluid moments of $\bar{f}_{\mathrm{NM}}$ vanish,
  -ability to evolve linearized system that expands about an axisymmetric $\bar{f}_{\mathrm{NM}}$ and its closure moments.