Progress and plans for coupling continuum kinetics to M3D-C1

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[1]General Atomics

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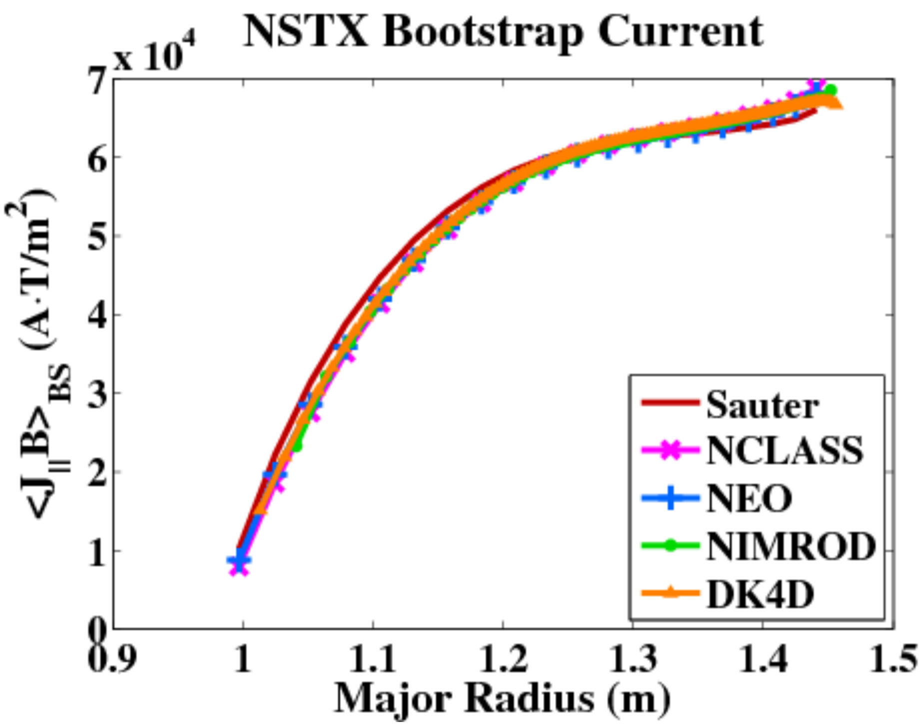
DK4D[1] is a time-dependent, axisymmetric drift-kinetic equation (DKE) solver

- **DK4D developed for purpose of extended-MHD coupling**
  - Solves non-Maxwellian part of distribution function
  - Linearized Fokker-Planck-Landau collision operators
  - Chapman-Enskog-like formulation
- **Encouraging initial results**
  - Benchmarked to Sauter model, along with NIMROD, NCLASS, and NEO codes[2]
  - Coupling to reduced MHD code produced self-consistent simulations of dynamic bootstrap current formation[3]



[1]B.C. Lyons, S.C. Jardin, & J.J. Ramos. Phys. Plasmas 22, 056103 (2015).
[2]E.D. Held et al. Phys. Plasmas 22, 032511 (2015).

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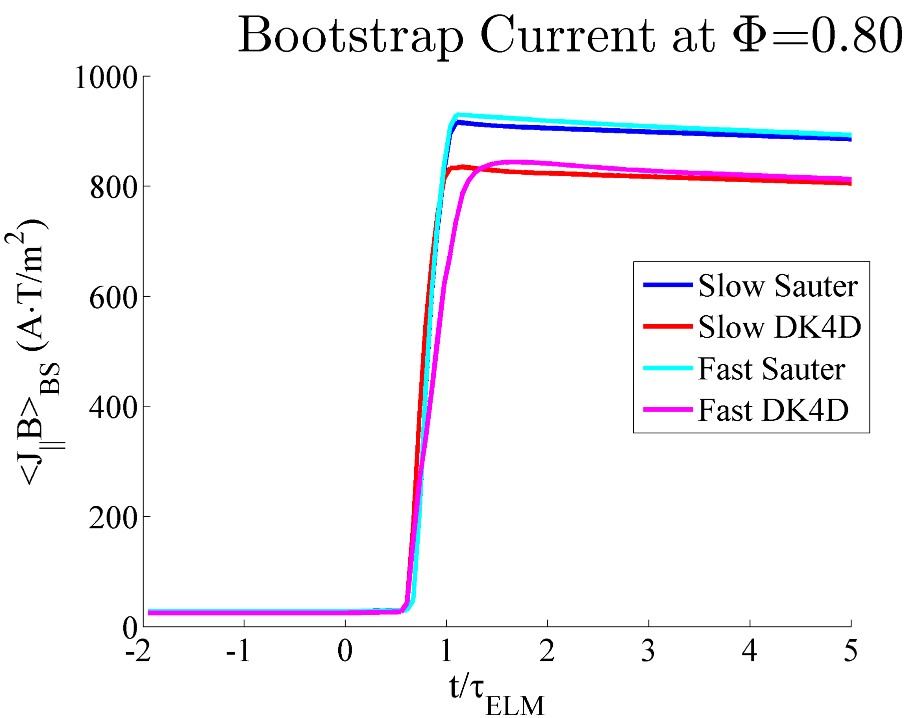
Bootstrap current evolution during slow & fast ELM-like pressure collapses



[3]B.C. Lyons. Doctoral dissertation. Princeton University 2014.

Extending DK4D will permit coupling to all M3D-C1 versions

- **Complex, linear 2D**
  - Must solve new DKE driven by linear MHD perturbations
  - Provides drift-kinetic corrections to linear stability (e.g., RWM)
- **Real, nonlinear 2D**
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GENERAL ATOMICS

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GENERAL ATOMICS

Modifications to DK4D would allow linear stability analysis of resistive wall modes

- **New coupled drift-kinetic MHD formulation derived by Ramos**
  - Chapman-Enskog-like for self-consistency
  - Fokker-Planck-Landau collisions
  - Rotation of order the sound speed
  - Two-fluid effects
  - Zero-Larmor-radius limit

As in DK4D

New to model

- **Linearized about**
  - Axisymmetric, single-fluid, collisionless equilibrium
  - Maxwellian distribution function
- **Linear solution evolves time-dependently**
  - n=0 perturbation which corrects assumed equilibrium
  - Single toroidal harmonic for stability analysis

GENERAL ATOMICS

New DKE has similar structure to the one already implemented in DK4D

DK4D

$$\frac{\partial \bar{f}_{NMs}}{\partial t} + wy\mathbf{b}\cdot\nabla \bar{f}_{NMs} - \frac{1}{2}w\left(1-y^2\right)\mathbf{b}\cdot\nabla\ln B\frac{\partial \bar{f}_{NMs}}{\partial y} = \langle C_{ss} + C_{ss'}\rangle_\alpha$$

$$+\left\{\frac{wy}{nT_s}\left[\frac{2}{3}\mathbf{b}\cdot\nabla\left(p_{s\parallel}-p_{s\perp}\right)-\left(p_{s\parallel}-p_{s\perp}\right)\mathbf{b}\cdot\nabla\ln B-\mathbf{b}\cdot\mathbf{F}_s^{coll}\right]\right.$$

$$+P_2(y)\frac{w^2}{3v_{ths}^2}\left(\nabla\cdot\mathbf{u}_s-3\mathbf{b}\cdot[\mathbf{b}\cdot\nabla\mathbf{u}_s]\right)+\frac{1}{3nT_s}\left(\frac{w^2}{v_{ths}^2}-3\right)\nabla\cdot\left(q_{s\parallel}\mathbf{b}\right)$$

$$\left.-\frac{\varsigma\left(e_s\right)I}{3m_s\Omega_s}\left[\frac{1}{2}P_2(y)\frac{w^2}{v_{ths}^2}\left(\frac{w^2}{v_{ths}^2}-5\right)+\frac{w^4}{v_{ths}^4}-10\frac{w^2}{v_{ths}^2}+15\right]\mathbf{b}\cdot\nabla\ln B\frac{dT_s}{d\psi}\right\}f_{Ms}$$

New

$$\frac{\partial \bar{f}_{s1}}{\partial t} + \left(\mathbf{u}_0 + w_\parallel\ \mathbf{b}_0\right)\cdot\frac{\partial \bar{f}_{s1}}{\partial \mathbf{x}} + \frac{\mathbf{b}_0}{m_s}\cdot\left(T_{s0}\nabla\ln n_0 + e_s\eta_{cl0}\ \mathbf{j}_0\right)\frac{\partial \bar{f}_{s1}}{\partial w_\parallel} + \frac{w_\perp}{2}\left(\mathbf{b}_0\cdot\nabla\ln B_0\right)\left(w_\parallel\frac{\partial \bar{f}_{s1}}{\partial w_\perp} - w_\perp\frac{\partial \bar{f}_{s1}}{\partial w_\parallel}\right) =$$

$$= \left\{-\left[\mathbf{u}_{s1} + \frac{n_1}{n_0}\ w_\parallel\ \mathbf{b}_0\right]\cdot\nabla\ln n_0\ + \left[\left(\frac{3}{2} - \frac{m_s w^2}{2T_{s0}}\right)\mathbf{u}_{s1} + \left(\frac{5}{2} - \frac{m_s w^2}{2T_{s0}}\right)\ \frac{w_\parallel}{B_0}\ \mathbf{B}_1\right]\cdot\nabla\ln T_{s0} + \right.$$

$$+\frac{w_\parallel}{n_0 T_{s0}}\ \mathbf{b}_0\cdot\left[\nabla p_{s\parallel 1} - \left(p_{s\parallel 1} - p_{s\perp 1}\right)\nabla\ln B_0 + e_s(n_0 - n_1)\eta_{cl0}\ \mathbf{j}_0 - \mathbf{F}_{s1}^{coll}\right]\ + \frac{e_s\eta_{cl0}w_\parallel}{B_0 T_{s0}}\left(\mathbf{B}_1 - B_1\mathbf{b}_0\right)\cdot\mathbf{j}_0 -$$

$$\left.-\frac{m_s}{2T_{s0}}\left[w_\perp^2\nabla\cdot\mathbf{u}_{s1} + \left(2w_\parallel^2 - w_\perp^2\right)\left(\mathbf{b}_0\mathbf{b}_0\right):\left(\nabla\mathbf{u}_{s1}\right) + \left(\frac{2w_\parallel^2 - w_\perp^2}{B_0}\right)\left(\mathbf{b}_0\mathbf{B}_1 + \mathbf{B}_1\mathbf{b}_0\right):\left(\nabla\mathbf{u}_{s0}\right)\right]\right\} f_{Ms0} +$$

$$+\sum_{s'}\left(C_{ss'}[f_{Ms0}, f_{Ms'0}] + C_{ss'}[f_{Ms0}, \bar{f}_{s'1}] + C_{ss'}[\bar{f}_{s1}, f_{Ms'0}]\right)$$

Development of DK4D & M3D-C1 required

- **DK4D**
  - Expand new DKE in DK4D variables and representations
  - Change to complex variables for linear stability analysis ($e^{in\varphi}$)
  - **Ax**=**B** system
    - **A** will require some modification, but much can be reused
    - **B** will be redone completely for new source terms
- **M3D-C1**
  - Add new closure terms based on moments of drift-kinetic solution
  - Calculate equilibria in flux coordinates
  - Fast evolution of instability must be treated carefully
- **Codes will likely need to be compiled together and run simultaneously**
  - Both codes would be linear
  - Reuse matrices and LU decompositions for maximum efficiency

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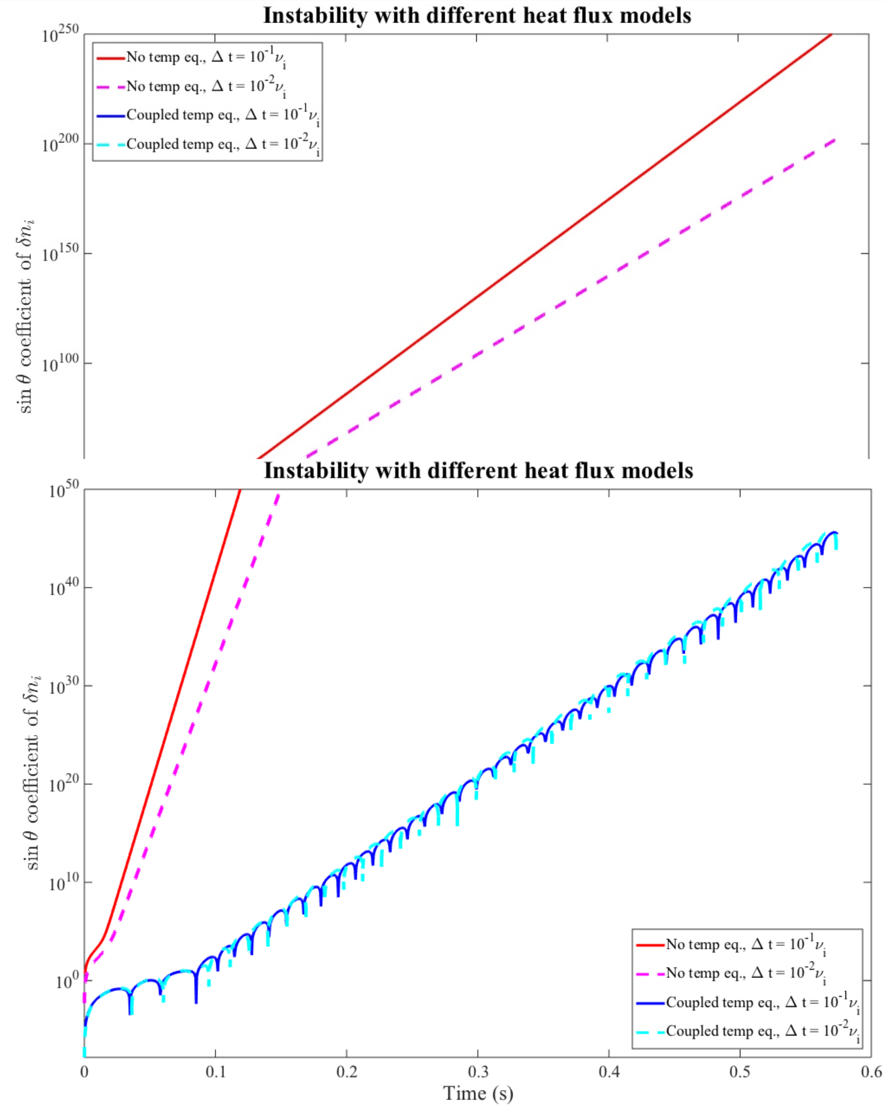
Implicit heat flux causes instability in DK4D

- **$\mathbf{B}\cdot\nabla T$ should develop if $q_{//}$ does not have the Pfrisch-Schulter-like component**
- **Should require tight coupling to temperature equation**
  - Without: exponential growth
  - With: oscillatory, but exponential
- **Unclear if the coupled system is still missing physics or, more likely, there's a numerical instability**



GENERAL ATOMICS

Coupling M3D-C1 & axisymmetric DK4D would permit realistic transport-timescale simulations

- **M3D-C1 can calculate steady-state, axisymmetric equilibria**
  - Sources included in two-fluid MHD equations (e.g., particle source)
  - Transport model assumed (Spitzer resistivity, Braginskii pressure tensor)
  - Evolve to steady-state to predict axisymmetric equilibria (e.g. NSTX[1])
- **DK4D would provide neoclassical transport model**
  - Friction force and pressure anisotropy close momentum equations
  - Parallel heat flux closes temperature equation (if needed)
  - Self-consistent neoclassical resistivity and bootstrap current would develop
- **Code development has some similarity to linear stability work**

[1]Ferraro, N.M. and S.C. Jardin. *J. Comput. Phys.* 228 (2009) 7742–7770.

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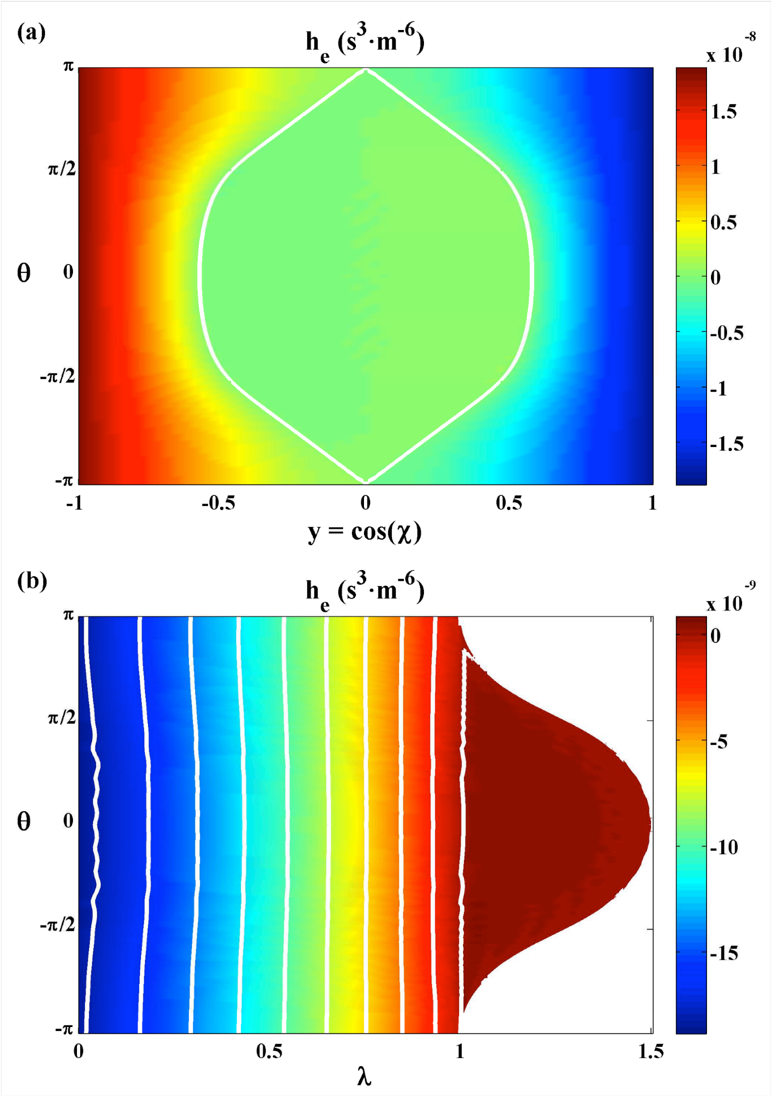
Full finite-element representation may be superior to DK4D's mixed representation

- **DK4D uses Legendre polynomials in $y = \cos\chi$ and Fourier modes in θ**
- **Finite elements could improve resolution of trapped-passing boundary**
  - Triangular mesh in y and θ
  - Adaptive packing along TPB
  - Used in M3D-C[1] to resolve resistive layers at rational surfaces
- **Could be some difficulties**
  - Requires unstructured and periodic mesh
  - Denser matrices, particularly with implicit moments



GENERAL ATOMICS

Full, non-axisymmetric geometries would require substantial upgrades to DK4D

- **Can no longer assume good, nested flux surfaces**
  - Important effects in and around magnetic islands
  - Flux tubes could replace flux surfaces, allowing for extensive parallelization
- **Must determine best representation in toroidal angle**
  - Fourier modes may be simplest to implement
  - Maybe finite elements if also implemented for y and θ?
- **Close collaboration with ASCR colleagues likely required**

GENERAL ATOMICS