# Force-free motion of a cold plasma during the current quench

D. I. Kiramov and B. N. Breizman, Phys. Plasmas 24 , 100702 (2017).
D. I. Kiramov and B. N. Breizman, Phys. Plasmas 25 , 092501 (2018).

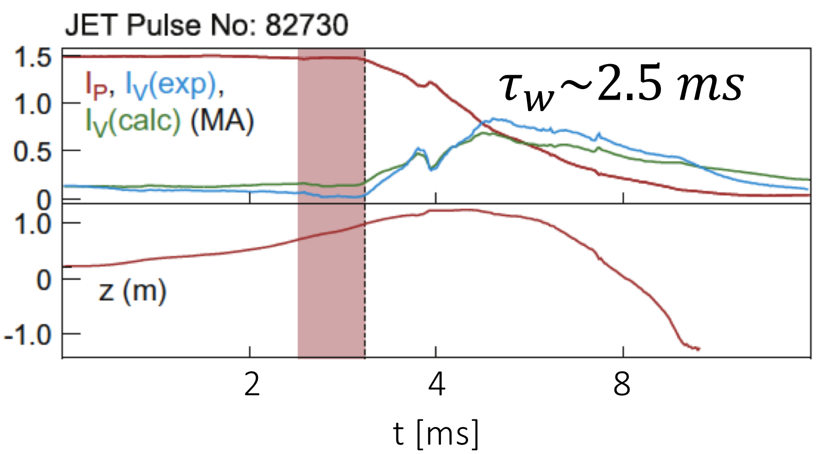
# Outline

- Motivation
- Three-wire problem
- 2D treatment
- Conclusions

# Motivation

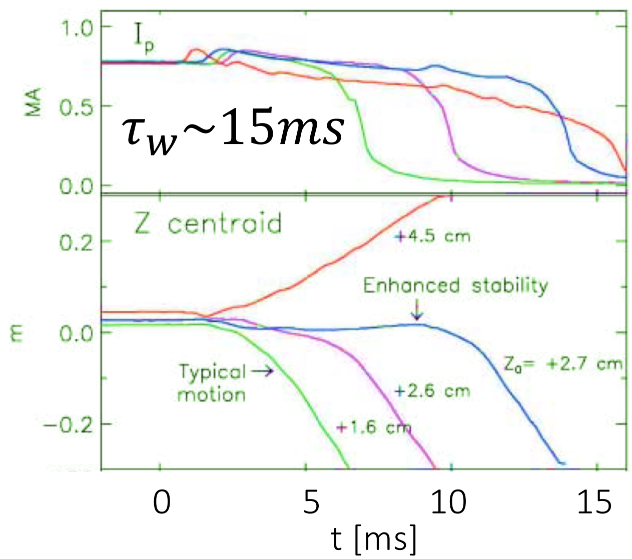
Cold disruptive plasma tends to move during the current quench. Its motion is essentially ***force-free*** since the current quench timescale is resistive rather than Alfvénic. In contrast with the hot vertical displacement events, ***the frozen-in condition is violated*** in the cold plasma case, and the plasma motion is not governed by magnetic flux conservation but rather by its dissipation.

JET




M. Lehnen et. al, J. NUCL. MATER. 438 , S102 (2013).

ALCATOR C-MOD




J. Irby, et. al, Fusion Science and Technology 51 , 460 (2007).

$$\boxed{\tau_{VDE} \gg \tau_A}$$

$$\boxed{\tau_w \gg \tau_{CQ}}$$

$$\tau_{w\ ITER} \sim 500\ ms$$

$$\tau_{CQ\ ITER} \sim 50\ ms$$

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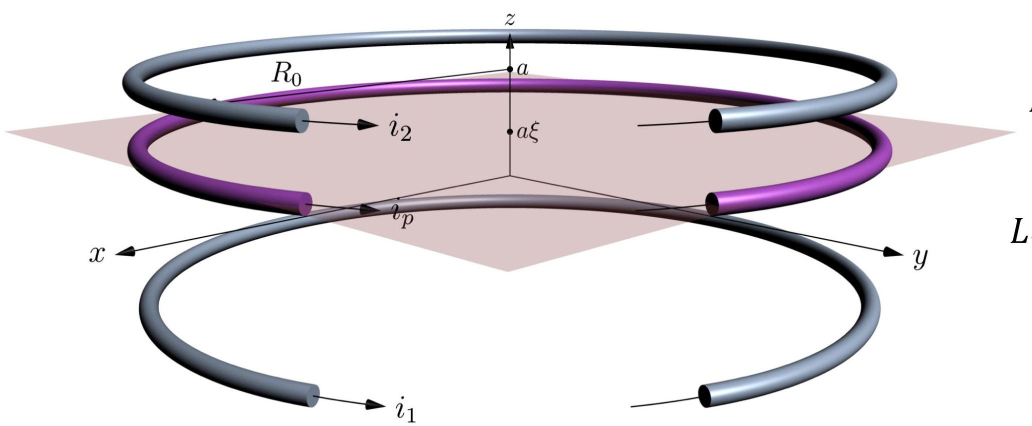
# Three-wire problem




D. I. Kiramov and B. N. Breizman, Phys. Plasmas 24 , 100702 (2017).
D. Pfefferlé and A. Bhattacharjee, Phys. Plasmas 25 , 022516 (2018).

The three-wire model reduces the problem to a set of circuit equations

$$L_w \frac{d}{dt} i_1 + L_{12} \frac{d}{dt} + L_{wp} \frac{d}{dt} [1 - \beta \ln(1+\xi)] i_p = -R_w i_1$$

$$L_{12} \frac{d}{dt} i_1 + L_w \frac{d}{dt} i_2 + L_{wp} \frac{d}{dt} [1 - \beta \ln(1-\xi)] i_p = -R_w i_2$$

$$L_{wp} \frac{d}{dt} [1 - \beta \ln(1+\xi)](i_1 + i_e) + L_{wp} \frac{d}{dt} [1 - \beta \ln(1-\xi)](i_2 + i_e) + L_p \frac{d i_p}{dt} = -R_p i_p$$

with a constraint that the net magnetic force vanishes at the plasma position

$$\boxed{\xi = \frac{i_1 - i_2}{i_1 + i_2 + 2 i_e}}$$

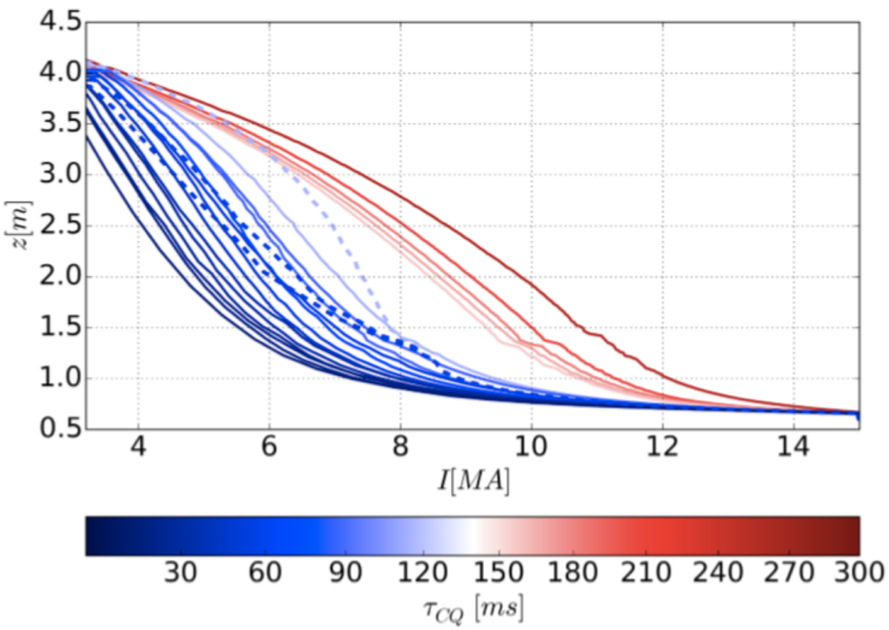
# Three-wire problem

$$\boxed{\frac{L_{pl}}{R_{pl}} \ll \frac{L_w}{R_w}} \quad \Rightarrow \quad \boxed{\begin{aligned}\frac{i_p}{2} = -\frac{i_e(L_w - L_{12})[\xi(t) - \xi(0)]}{L_{wp}\big(l_1(\xi) - L\xi l_2(\xi)\big)} - \\ \frac{i_p(0)L_{wp}[\beta\xi(0) + L\xi(t)]}{L_{wp}\big(l_1(\xi) - L\xi l_2(\xi)\big)}\end{aligned}}$$

- Valid if $t \ll \tau_w = L_w/R_w$, $|i_1(0)|, |i_2(0)| \ll \left|i_p(0)\right|, |\xi(0)| \ll 1$;
- Presents a monotonic relation between the plasma current and plasma displacement for $\xi \in [-1,1]$;
- The absolute value of $\xi$ should increase in step with the current decay during current quench of mitigated disruptions;
- Implies that the plasma ***remains stable*** with respect ***to ideal MHD perturbations.***

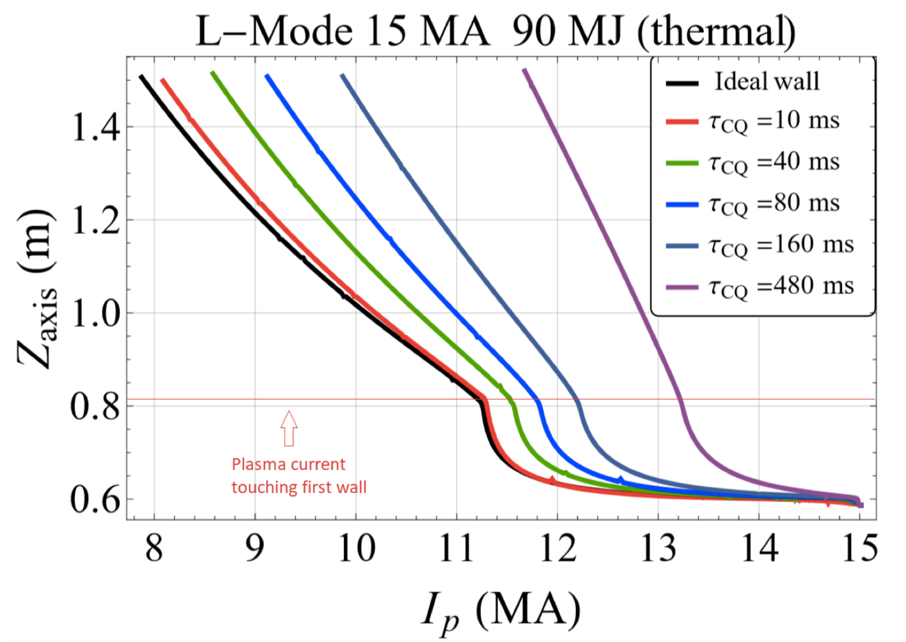
# Full-scale numerical simulations

DINA results




- Temperature profile is not fixed;
- Includes cases with runaway electrons (dashed curves);

JOREK results




- Temperature profile is fixed;
- Ohmic cases only;

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# 2D treatment

Magnetic field representation:

$$\boldsymbol{B} = \nabla\psi \times \nabla\zeta + \frac{2I}{c}\nabla\zeta$$

Flux diffusion/advection:

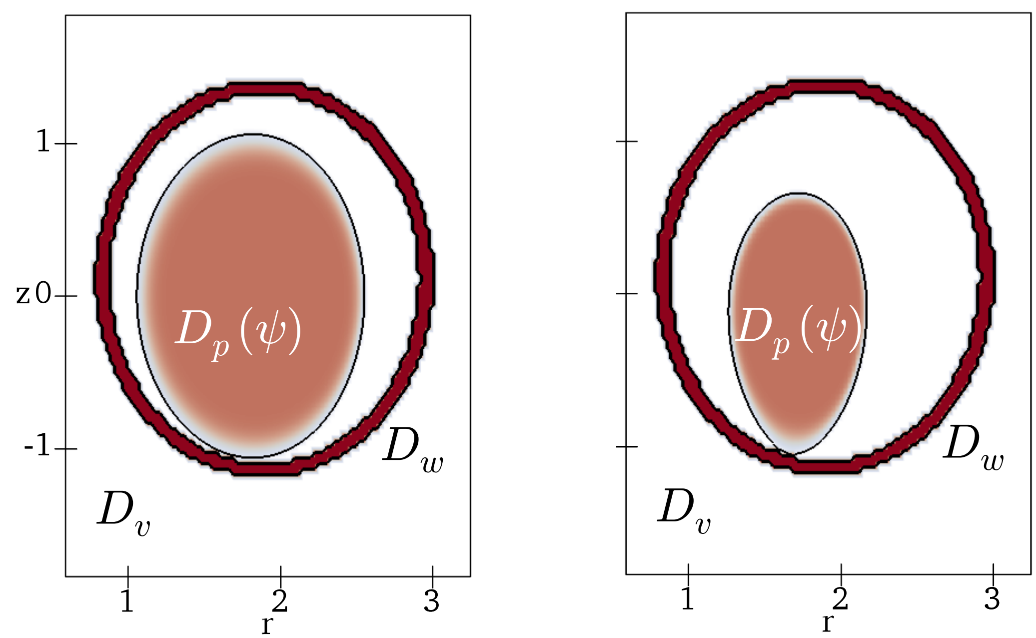
$$\frac{\partial\psi}{\partial t} = Dr^2\nabla\cdot\left(\frac{\nabla\psi}{r^2}\right) - \boldsymbol{V}\cdot\nabla\psi$$
$$\frac{\partial I}{\partial t} = r^2\nabla\cdot\left(\frac{D\nabla I - \boldsymbol{V}I}{r^2}\right)$$

Force-free constraint:

$$r^2\nabla\cdot\left(\frac{\nabla\psi}{r^2}\right) = -I\frac{dI}{d\psi}$$

D. I. Kiramov and B. N. Breizman, Phys. Plasmas 25 , 092501 (2018).

Snapshots of the plasma shape




Ideal wall limit: $D_v \gg D_p(\psi) \gg D_w$

# 2D treatment

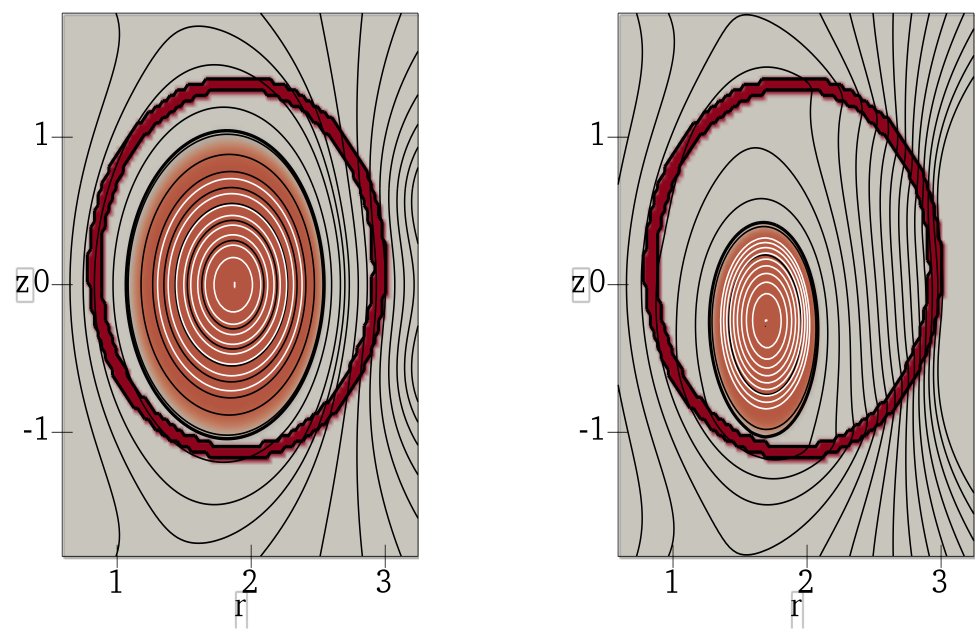
We do not assume:

$\boldsymbol{V} \cdot \boldsymbol{n}_{wall} = 0$

We assume:

1) A perfectly absorbing wall;
2) $\frac{l_{connect}}{V_{\parallel}} \ll \tau_{VDE}$ (once the plasma enters the region of open magnetic flux surfaces, it instantly flows into the wall).

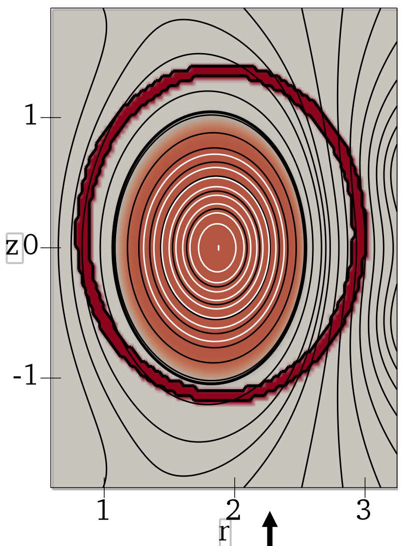
Snapshots of the poloidal flux




Magnetic field diffusion coefficient ***changes in time*** since during the plasma current dissipation, ***the last closed flux surface evolves*** thus ***changing the region of definition*** of the piecewise-defined function $D$.

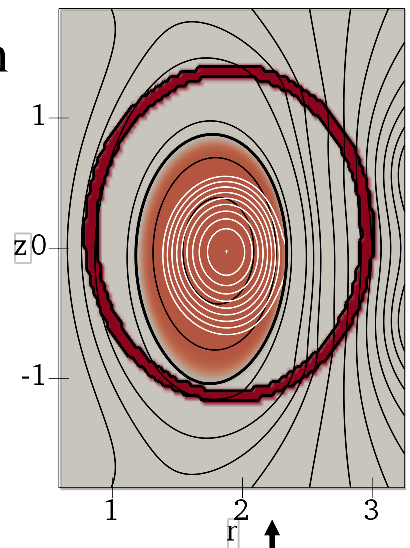
# Numerical procedure

Initial equilibrium:




$$r^2\nabla\cdot\left(\frac{\nabla\psi}{r^2}\right) = -I\frac{dI}{d\psi}$$

Diffusion step:




$$r^2\nabla\cdot\left(\frac{\nabla\psi}{r^2}\right) \neq -I\frac{dI}{d\psi}$$

Advance the velocity field to keep the plasma force-free after the diffusion time step:

***Iterations:***

$$\boldsymbol{V}_{i+1} = \boldsymbol{V}_i + \delta\boldsymbol{V}_i$$

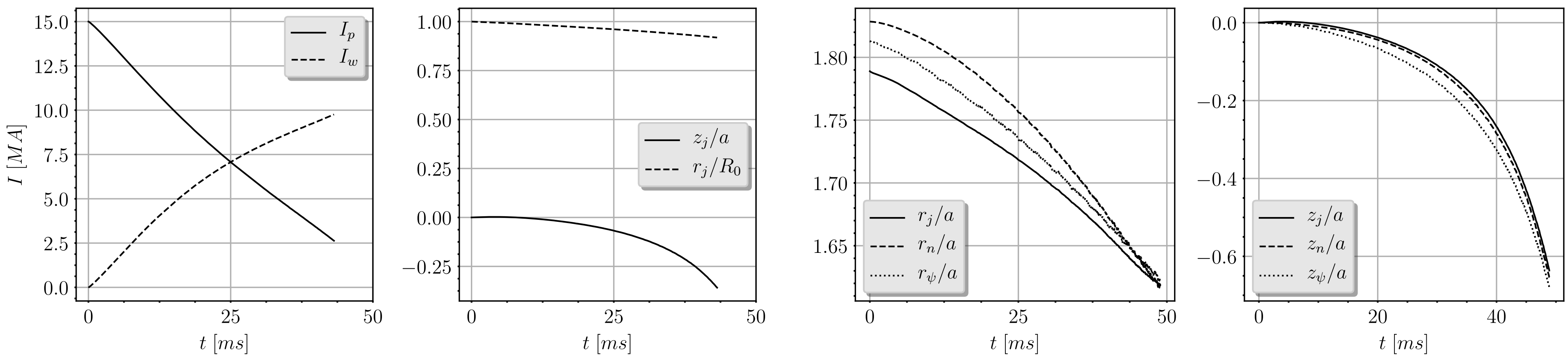
$$\boxed{\gamma\delta\boldsymbol{V}_i} = \boxed{\delta\boldsymbol{f}_i(\delta\psi_i, \delta I_i, \psi_0, I_0, V_i)}$$

artificial friction

perturbed magnetic force

L. E. Zakharov and X. Li, Phys. Plasmas 22 , 062511 (2015).

# Numerical results



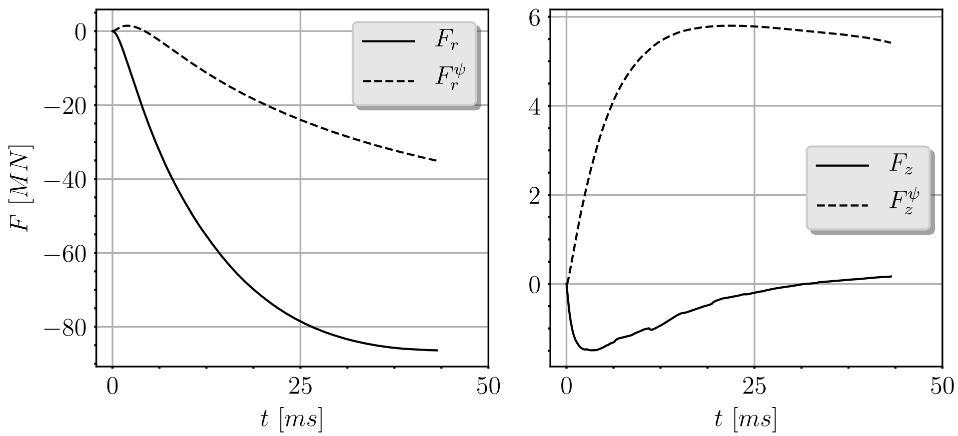

As in 1D case the absolute value of plasma vertical displacement increases in step with the plasma current decay

$z_j$ - current centroid $r$ position

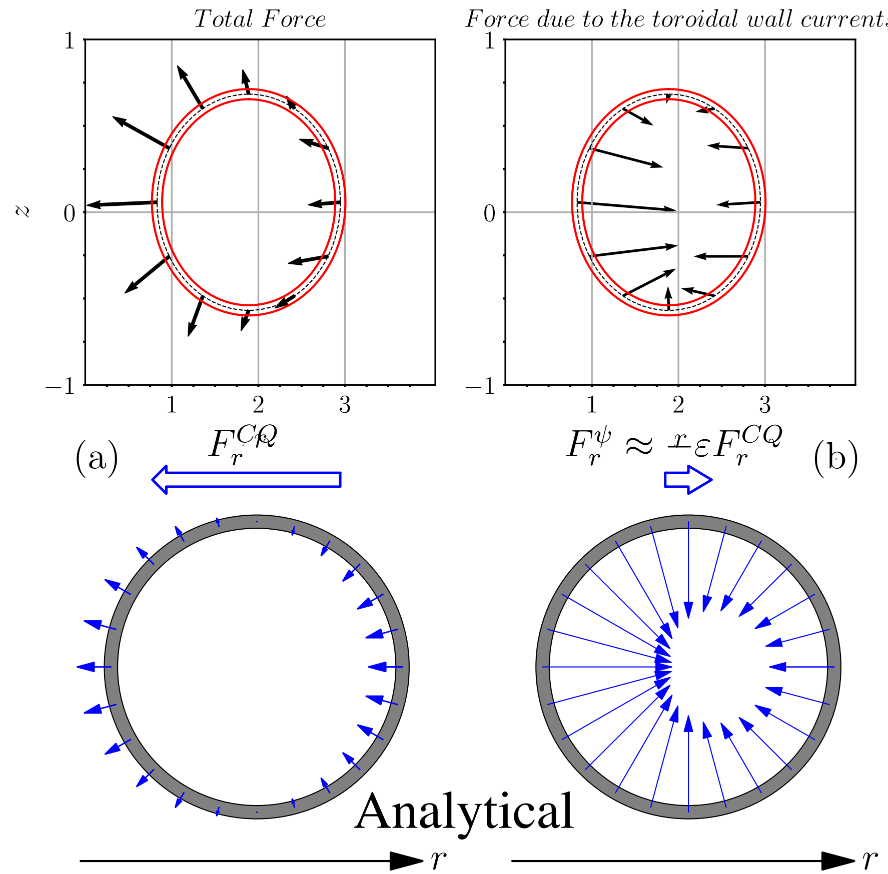
$r_j$ - current centroid $r$ position

The plasma positions calculated with $j_{tor}$, $n$, and $\psi$ do not match. Describing the vertical position of the plasma during the cold VDE, one has to distinguish the current, density, and poloidal magnetic flux centroids from each other.

# Wall forces




Numerical




$F_{r,z}$ - integral wall force

$F_{r,z}^{\psi}$ - force due to the toroidal wall currents

The poloidal wall currents make the same order of magnitude contribution to the total wall force as the toroidal ones.

V. D. Pustovitov and D. I. Kiramov, Plasma Phys. Controlled Fusion 60, 045011 (2018).

# Conclusions

- The force-free nature of the plasma motion leads to an adiabatic rather than exponential growth of the vertical displacement;
- Induced poloidal and toroidal currents produce comparable mechanical forces during the CQ;
- Displacement of the current profile differs from the plasma displacement during the cold CQ.