Collision Energy Exchange in the CEL-DKE
CTTS Meeting at APS-DPP, Portland, OR

E. Held A. Spencer J. Ji NIMROD Team

November 4, 2018

Electron Chapman-Enskog-like (CEL)-DKE in NIMROD

- Assume $f = f_{Me} + F_e$ with $F_e = O(\delta^2 f_{Me})$
- Write CEL-DKE in the fluid frame (Ramos, *Phys Plasmas* **17**, 082502 (2010)) using $s = v/v_{te}$ and $\xi = v_\parallel / v$ variables:

$$
\begin{aligned}
&\frac{\partial F_e}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla F_e - \frac{1-\xi^2}{2\xi} v_\parallel \mathbf{b}\cdot\nabla \ln B \frac{\partial F_e}{\partial \xi} \\
&+\frac{v_0}{2}(\mathbf{b}\cdot\nabla\ln n)[\xi\frac{\partial F_e}{\partial s} + \frac{1-\xi^2}{s}\frac{\partial F_e}{\partial \xi}] - s[\xi\mathbf{b}\cdot\nabla + \frac{\partial}{\partial t}]\ln v_0 \frac{\partial F_e}{\partial s} = \langle C(f)\rangle \\
&+\Bigg[(\frac{5}{2} - s^2)v_\parallel \mathbf{b}\cdot\nabla\ln T + \frac{v_\parallel}{nT}\mathbf{b}\cdot[\frac{2}{3}\nabla\pi_\parallel - \pi_\parallel\nabla\ln B - \mathbf{F}_{ei}] \\
&+2s^2(\frac{3}{2}\xi^2 - \frac{1}{2})[\frac{1}{3}\nabla\cdot\mathbf{u} - \mathbf{bb}\cdot\nabla\mathbf{u}] + \frac{2}{3nT}(s^2 - \frac{3}{2})[\mathbf{b}\cdot\nabla q_\parallel - q_\parallel\mathbf{b}\cdot\nabla\ln B - G_{ei}] \\
&+\frac{2}{3eB}s^2(\frac{3}{2}\xi^2 - \frac{1}{2})[(\frac{5}{2} - s^2)(\nabla\ln B - 2\kappa) + \nabla\ln n]\cdot\nabla T\times\mathbf{b} \\
&+\frac{4}{3eB}(\frac{s^4}{2} - \frac{5}{2}s^2 + \frac{15}{8})(\nabla\ln B + \kappa)\cdot\nabla T\times\mathbf{b}\Bigg] f_{Me}
\end{aligned}
$$

Spitzer thermalization problem using CEL approach.

- Test accuracy of electron/ion collisional energy exchange.

$$\frac{\partial F_e}{\partial t} - \frac{s}{2}\frac{\partial \ln T}{\partial t}\frac{\partial F_e}{\partial s} = C(F_e, f_{Me}) + C(f_{Me}, F_e)$$
$$+ C(F_e, f_{Mi}) + C(f_{Me}, f_i) + \frac{2}{3nT}(\frac{3}{2} - s^2) G_{ei} f_{Me}$$

Here accurate evaluation of e/i collision operator and its moment, G_{ei}, are needed:

$$G_{ei} = \frac{m_e}{2} \int d\mathbf{v} v^2 \{C(F_e, f_{Mi}) + C(f_{Me}, f_i)\}$$

and

$$\frac{3n_e}{2}\frac{\partial T_e}{\partial t} = G_{ei}, \qquad \frac{3n_i}{2}\frac{\partial T_i}{\partial t} = G_{ie}$$

Drives in CEL-DKE can have complicated s-dependence.

- Collocation approach in speed:
  - solve DKE at set of quadrature points in s,
  - derivatives done using $F = \sum_{i=1}^{ns} F_i L_i(s) w(s)$ where $\int dsw(s) L_i L_j = \delta_{ij}$

- Most "drives" in CEL-DKE have simple form: polynomial-in-s * $exp(-s^2)$:

  suggests expansion $F = \sum_{i=1}^{ns} F_i L_i(s) \exp(-s^2)$.

- But, s-dependence in $C(f_{Me}, f_i)$ has large response at low s:

  suggests expansion $F = \sum_{i=1}^{ns1} F_i L_i'(s) De0(s)$
  on domain $s \in [0, root(De0)]$.

Collisional energy and momentum exchange

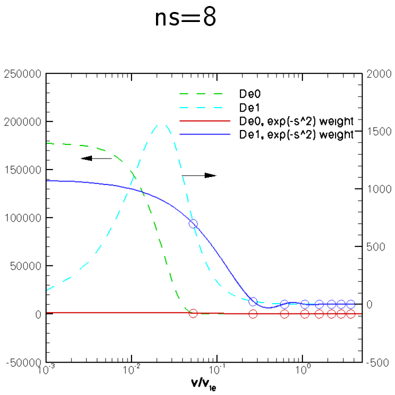
- Large response in $C(f_{Me}, f_i)$ at low electron speed present in functions $De0(s)$ and $De1(s)$.

$$C(f_{Me}, f_i) = \frac{3}{4\pi}\frac{e^2 n_e^2}{m_i v_{te}^3}\eta_\perp(1-\frac{T_i}{T_e})\underbrace{\left(\frac{4}{\sqrt{\pi}}\left[\frac{v_{te}}{v_{ti}}\right]^3 e^{-s_i^2} - \frac{2}{s_e}\left(E(s_i) - s_i E'(s_i)\right)\right) e^{-s_e^2}}_{De0}$$

$$+\frac{3}{2\pi}\frac{e n_e}{m_e}\frac{\eta_\perp}{v_{te}^4}\xi j_\parallel \underbrace{\frac{1}{s_e^2}\left(E(s_i) - s_i E'(s_i)\right) e^{-s_e^2}}_{De1}.$$

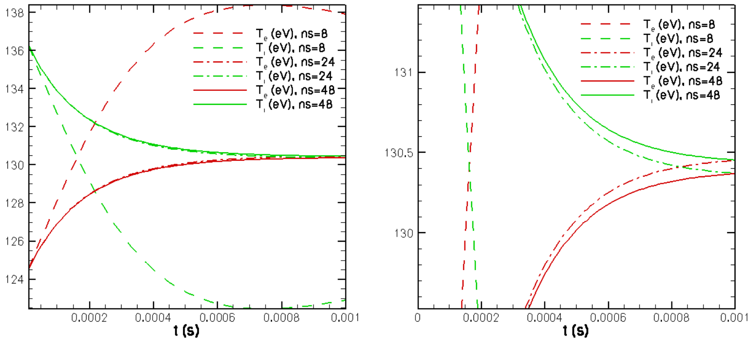
Here $E(s_i)$ and $E'(s_i)$ are the error function and its derivative and $De0$ and $De1$ control collisional energy and momentum exchange responses, respectively.

De0 and De1 behavior with $w = \exp(-s^2)$ on $s \in [0, \infty]$.



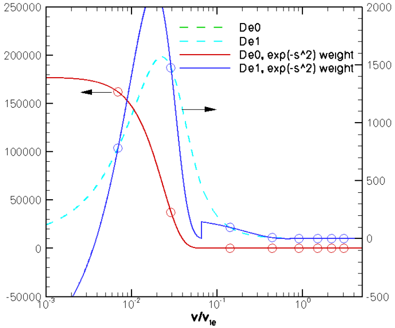
Spitzer thermalization recovered for $\exp(-s^2)$ weighting.

- Accurate thermalization obtained for $ns = 48$.

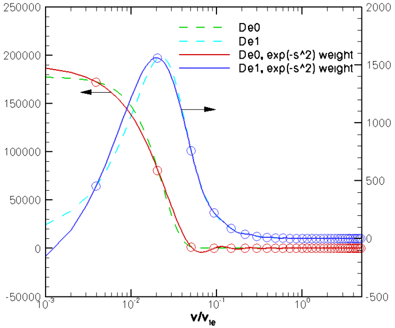


De0 and De1 behavior with $w = De0$ on $s \in [0, root(De0)]$.

ns1=2 $(w(s) = De0)$, **ns2=6** $(w(s) = \exp(-s^2))$

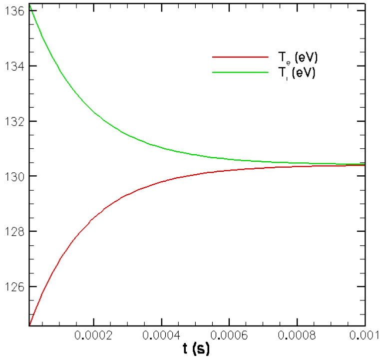


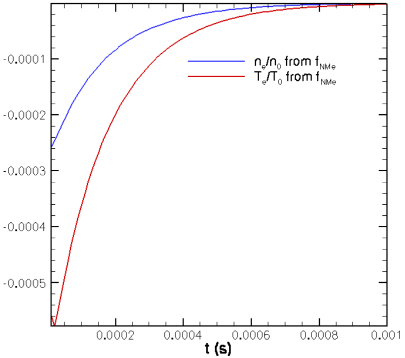
ns=48, $w = \exp(-s^2)$



Spitzer thermalization recovered for low-s, $De0(s)$ weighting.

- 8 speed collocation points:
  2 at low s ($w = De0$), 6 at high-s ($w = \exp(-s^2)$).





Conclusions

- Quadrature schemes in s with $De0(s)$ weight at low s and $\exp(-s^2)$ at high s helpful for accurate thermalization.
- Refinement of this approach needed for simultaneously capturing collisional momentum exchange response at low s?