Update on NTM Modeling Using Heuristic Closures

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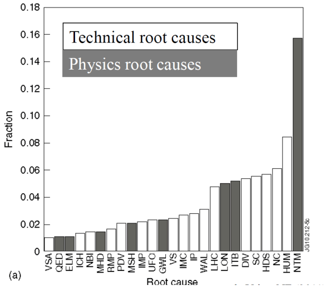
Outline

NTMs are a Concern for ITER and Beyond

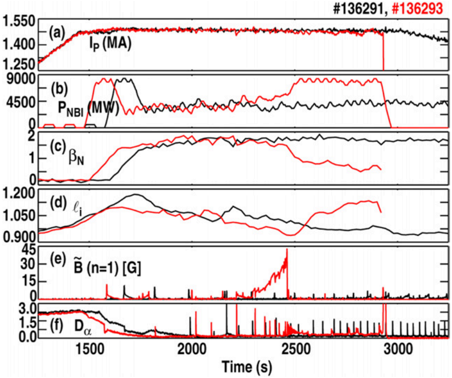
- Ideal MHD limits are relatively well-understood: $\beta_N \lesssim 4l_i$
- Rotating tearing modes result in a soft β limit: $\beta_N \lesssim 2l_i$
- Locked modes are a leading cause of disruptions



P. de Vries, NF 2011

NTM Modeling is Computationally Challenging due to their Slow Evolution and Kinetic Nature

- Evolution spans multiple ELM and sawtooth cycles
  - τ_{ntm}: 100 – 1000ms
  - τ_{elm}: 5 – 20ms
  - τ_{saw}: 100ms
- Neoclassical effects govern island evolution
  - Self-consistent modeling requires solving 5D DKE



F. Turco NF 2010

Our Numerical Simulations do not Model the Experimental Control Systems

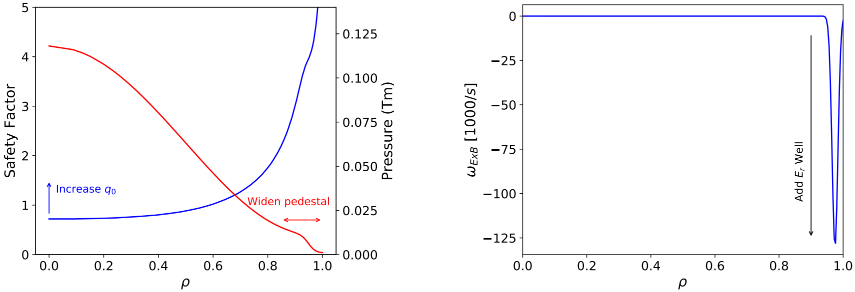
- NTM control is routine on large experiments
  - ECCD, entrainment and spin-up
  - Disruption threshold depends on MHD and control system
- Experiments are preemptively terminated when large amplitude locked modes are observed
  - Decrease I_p, turn off heating
  - Changes to actuators alters dynamics
  - Muddies validation

Our Approach to Studying NTM Disruptions

- Construct experimentally relevant model equilibria
  - Start from approximate DIII-D ITER baseline discharge
  - Modify equilibria to remove extraneous dynamics
- Use heuristic closures to model neoclassical stresses
  - Heuristic closures model dominant neoclassical effects
  - Use model parameters to scale island width
- Investigate how locked NTM cause disruption

Generation of Model Equilibria

Model Equilibria are Created to Focus on NTMs



- Raise $q_0 > 1$ to avoid sawteeth
- Widen pedestal and add E_r well to stabilize edge modes
- Increase λ locally to increase Δ'

Proportional-Derivative Feedback System Implemented for Shape Control

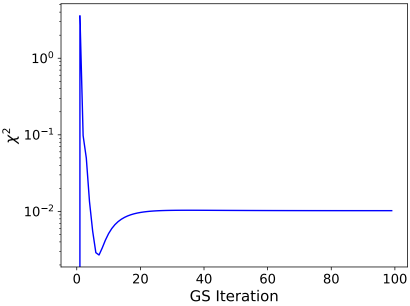
- External currents are updated using a PD controller:

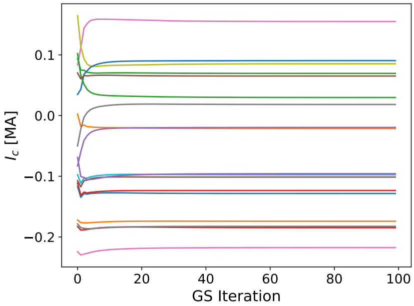
$$\delta I_j^n = \alpha_p J_{ji}^{(k)\sim 1} e_i^n + \alpha_d J_{ji}^{(k)\sim 1} \left(e_i^n - e_j^{n-1} \right)$$

Shape control points:	$\{r_i\}$
Flux on control surface:	ψ_0
Square error:	$e_i^2 = w_i \left(\psi\left(r_i\right) - \psi_0\right)^2$
Jacobian:	$J_{ij} = \frac{\partial e_i}{\partial I_j} \equiv U \Sigma V^T$
SVD pseudo-inverse:[2]	$J^{(k)\sim 1} \equiv V \Sigma^k U^T$

[2] J. D. Hanson NF 2009

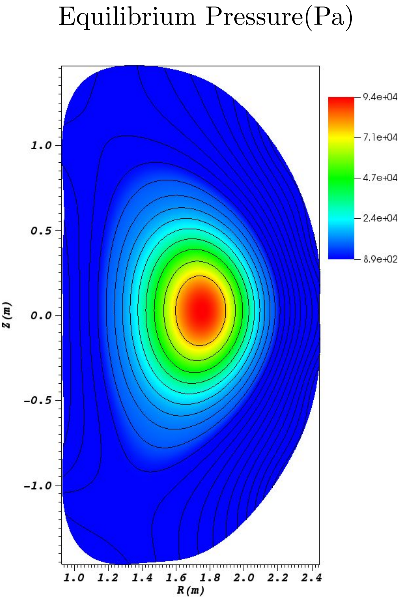
Example Case Illustrates the Performance of the Feedback System

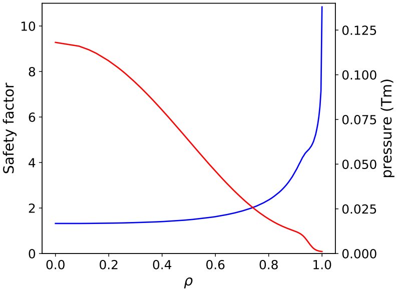




- 100 control points specify the $\hat{\psi} = 0.95$ surface
- Provides vertical and radial stability
- Performance is sensitive to the number of singular values
  - A cutoff tolerance of 10^{-4} works well

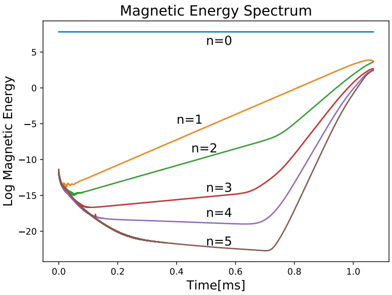
Modifications Enable Creation of ITER-Like Equilibria



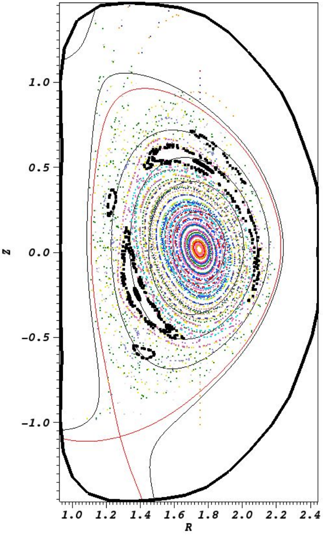


I_p	0.958 MA
ϵ	0.33
κ	1.83
q_0	1.32
q_{95}	5.2
β_N	1.85
β_N/l_i	2.31

Equilibria is $n = 1$ Tearing Unstable



S	10^5
Pr_m	10
$\nu_{visc}/\chi_\perp$	1
$k_\parallel/k_\perp$	10^4



Numerical Implementation and Testing of Heuristic Closures

Neoclassical Effects Modify MHD Through the Stresses (and Heat Fluxes)

Continuity: $\frac{\partial n}{\partial t} + \vec{v} \cdot \nabla n = n \nabla \cdot \vec{v}$

Momentum: $\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = \vec{J} \times \vec{B} - \nabla p - \nabla \cdot \vec{\vec{\Pi}}_\nu - \nabla \cdot \vec{\vec{\Pi}}_i$

Temperature: $\frac{n}{\Gamma - 1} \left(\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T \right) = -nT \nabla \cdot \vec{v} - \nabla \cdot \vec{q} + Q$

Faraday's Law: $\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$

Ohm's Law: $E = -\vec{v} \times \vec{B} + \eta J - \frac{1}{ne} \nabla \cdot \vec{\vec{\Pi}}_e$

Heuristic Closures Model the Neoclassical Stresses

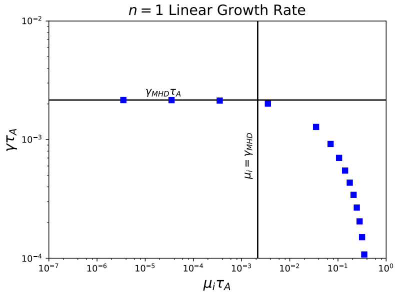
$$\nabla \cdot \vec{\vec{\Pi}}_i = n m_i \mu_i \left\langle B^2 \right\rangle \frac{\vec{v} \cdot \vec{e}_\Theta}{\left(\vec{B} \cdot \vec{e}_\Theta\right)^2} \vec{e}_\Theta$$

$$\nabla \cdot \vec{\vec{\Pi}}_e = -\frac{n m_e \mu_e}{ne} \left\langle B^2 \right\rangle \frac{\vec{J} \cdot \vec{e}_\Theta}{\left(\vec{B} \cdot \vec{e}_\Theta\right)^2} \vec{e}_\Theta$$

- Heuristic closures model dominant neoclassical effects[3]
  - Poloidal ion flow damping
  - Enhancement of polarization current
  - Bootstrap current

[3] Gianakon, POP 9, 2002

Neoclassical Enhancement of the Polarization Current is Used to Test the Ion Stress



S	10^5
Pr_m	1
$\nu_{visc}/\chi_\perp$	1
$k_\parallel/k_\perp$	10^8
μ_e	0

- The ion stress damps poloidal flows associated with tearing modes
- Damping increases inertia by a factor of $\left(1 + C\frac{\mu_i}{\gamma}\right)$
- Growth rate is reduced when $\mu_i \gtrsim \gamma_{MHD}$

Conclusions/Future Work

- Modifications enable the generation of experimentally relevant equilibria
  - New feedback system enables shape control
  - Equilibria designed to focus on NTM physics
- Heuristic closures are used to model neoclassical stresses
  - Tests of the closures are ongoing
- Modifications will be used to investigate the mechanisms by which NTMs cause disruptions