
VDE simulations with M3D-C¹

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Outline

- **Features of the M3D-C¹ code**
- **Benchmark Studies with NIMROD, JOREK, CarMa0NL**
- **ITER VDE Studies**

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3D Extended MHD Equations in M3D-C¹

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = \nabla \cdot D_n \nabla n + S_n$$

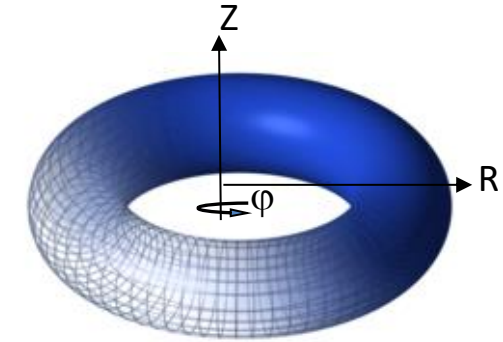
$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla \Phi, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla_{\perp} \cdot \frac{1}{R^2} \nabla \Phi = -\nabla_{\perp} \cdot \frac{1}{R^2} \mathbf{E}$$

$$nM_i \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{\Pi}_i + \mathbf{S}_m$$

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne} \left(\mathbf{R}_c + \mathbf{J} \times \mathbf{B} - \nabla p_e - \nabla \cdot \mathbf{\Pi}_e \right) - \frac{m_e}{e} \left(\frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \cdot \nabla \mathbf{V}_e \right) + \mathbf{S}_{CD}$$

$$\frac{3}{2} \left[\frac{\partial p_e}{\partial t} + \nabla \cdot (p_e \mathbf{V}) \right] = -p_e \nabla \cdot \mathbf{V} + \frac{\mathbf{J}}{ne} \cdot \left[\frac{3}{2} \nabla p_e - \frac{5}{2} \frac{p_e}{n} \nabla n + \mathbf{R}_c \right] + \nabla \cdot \left(\frac{\mathbf{J}}{ne} \right) : \mathbf{\Pi}_e - \nabla \cdot \mathbf{q}_e + Q_{\Delta} + S_{eE}$$

$$\frac{3}{2} \left[\frac{\partial p_i}{\partial t} + \nabla \cdot (p_i \mathbf{V}) \right] = -p_i \nabla \cdot \mathbf{V} - \mathbf{\Pi}_i : \nabla \mathbf{V} - \nabla \cdot \mathbf{q}_i - Q_{\Delta} + S_{iE}$$



$$\mathbf{V}_e = \mathbf{V}_i - \mathbf{J} / ne$$

$$\mathbf{R}_c = \eta ne \mathbf{J}, \quad \mathbf{\Pi}_i = -\mu \left[\nabla \mathbf{V} + \nabla \mathbf{V}^{\dagger} \right] - 2(\mu_c - \mu)(\nabla \cdot \mathbf{V}) \mathbf{I} + \mathbf{\Pi}_i^{GV}$$

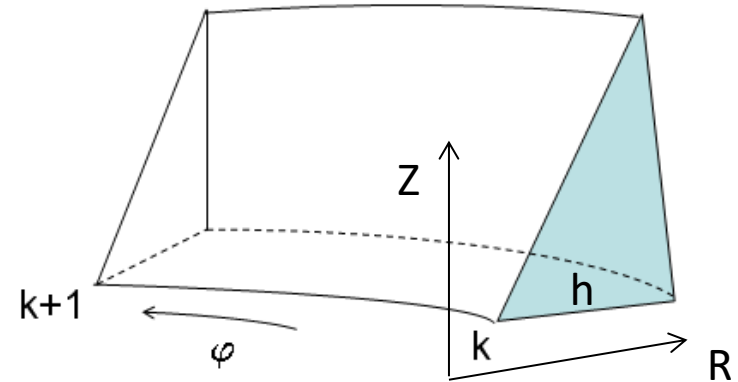
$$\mathbf{q}_{e,i} = -\kappa_{e,i} \nabla T_{e,i} - \kappa_{\parallel} \nabla_{\parallel} T_{e,i}$$

$$\mathbf{\Pi}_e = (\mathbf{B} / B^2) \nabla \cdot \left[\lambda_h \nabla (\mathbf{J} \cdot \mathbf{B} / B^2) \right], \quad Q_{\Delta} = 3m_e (p_i - p_e) / (M_i \tau_e)$$

Blue terms are 2-fluid terms. Also, now have impurity and pellet models for disruption mitigation. NOT reduced MHD.

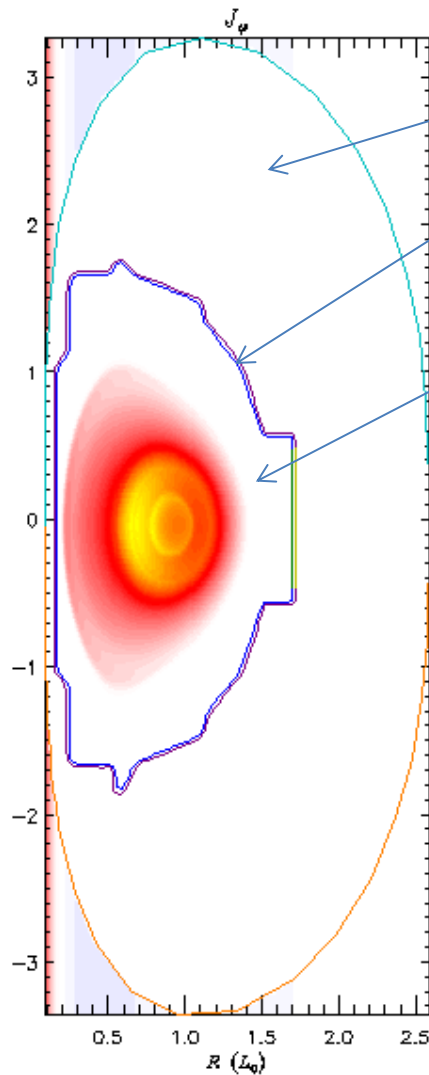
M3D-C¹ uses unique 3D high-order finite elements

- M3D-C¹ uses high-order curved triangular prism elements
- Within each triangular prism, there is a polynomial in (R, φ, Z) with 72 coefficients
- The solution *and 1st derivatives* are constrained to be continuous from one element to the next.
- Thus, there is much more resolution than for the same number of linear elements
- Error $\sim h^5$



Also, implicit time-stepping allows for very long time simulations

*M3D-C¹ has been extended to 3 regions for RW**



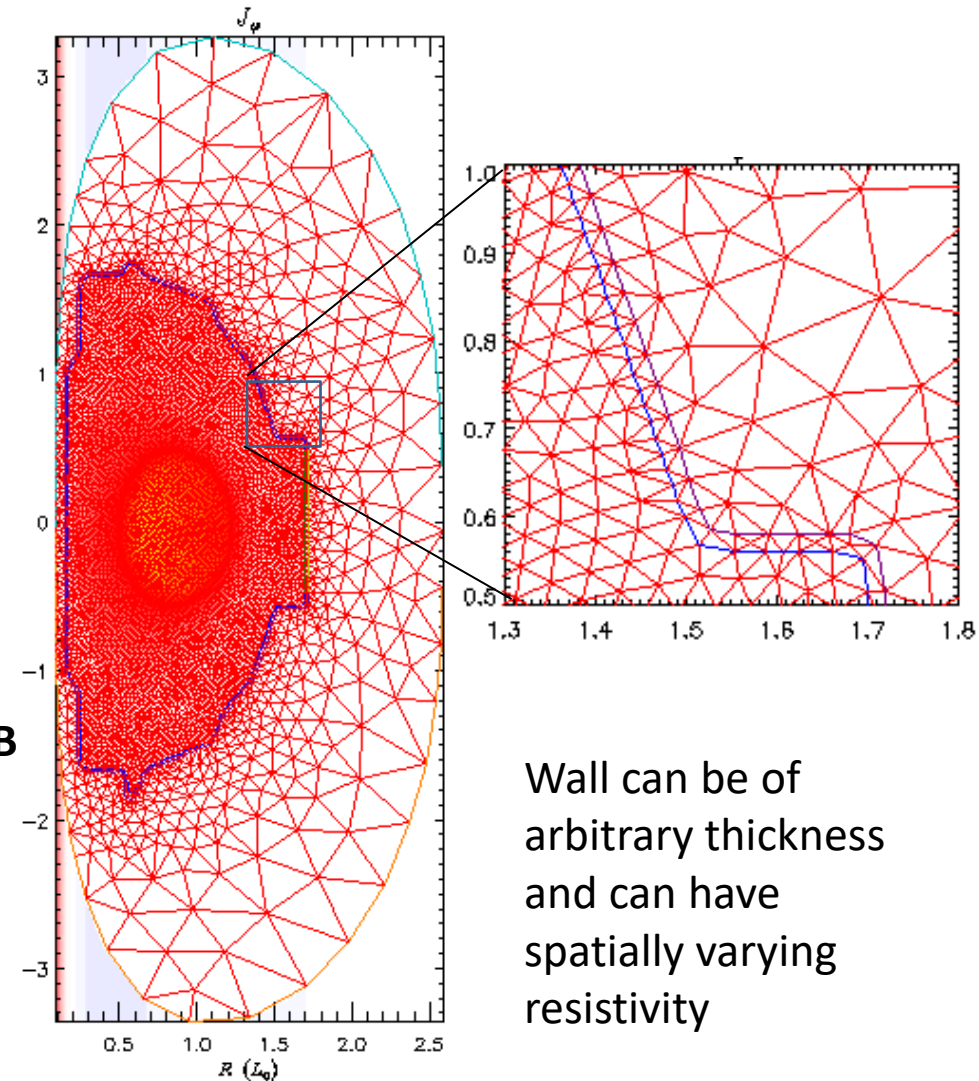
Vacuum ($\mathbf{J}=0$)

RW ($\mathbf{E} = \eta_w \mathbf{J}$)

Plasma (X-MHD)

BC:

- \mathbf{v} , p , n set at inner wall
- \mathbf{B} set at outer (ideal) wall
- No boundary conditions on \mathbf{B} or \mathbf{J} at the resistive wall
- (halo) Current can flow into and out of the wall



Wall can be of arbitrary thickness and can have spatially varying resistivity

*Ferraro, et al. ,Phys Plasma**23** 056114 (2015)

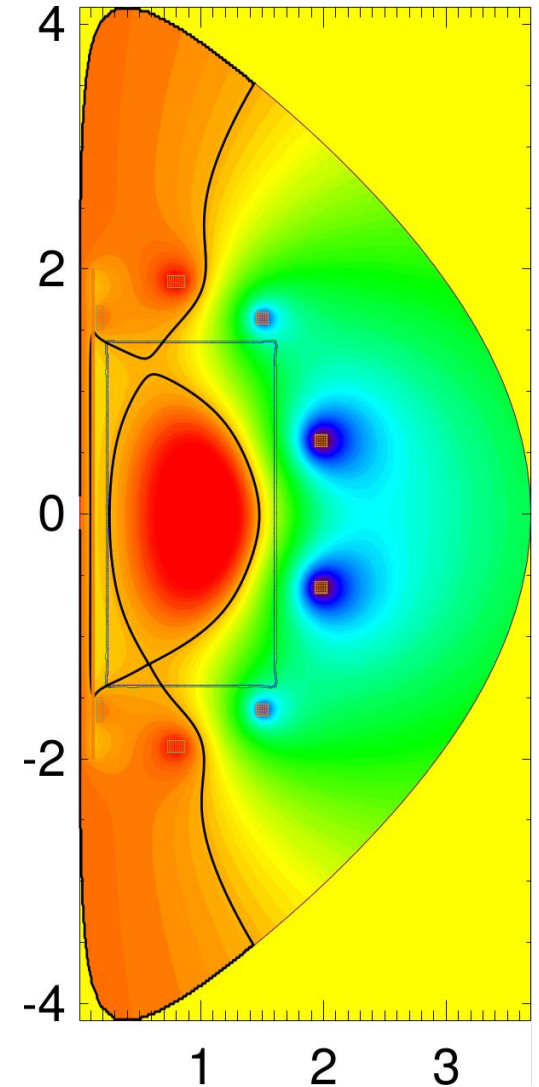
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- Features of the M3D-C¹ code
- **Benchmark Studies with NIMROD, JOREK, CarMa0NL**
- ITER VDE Studies

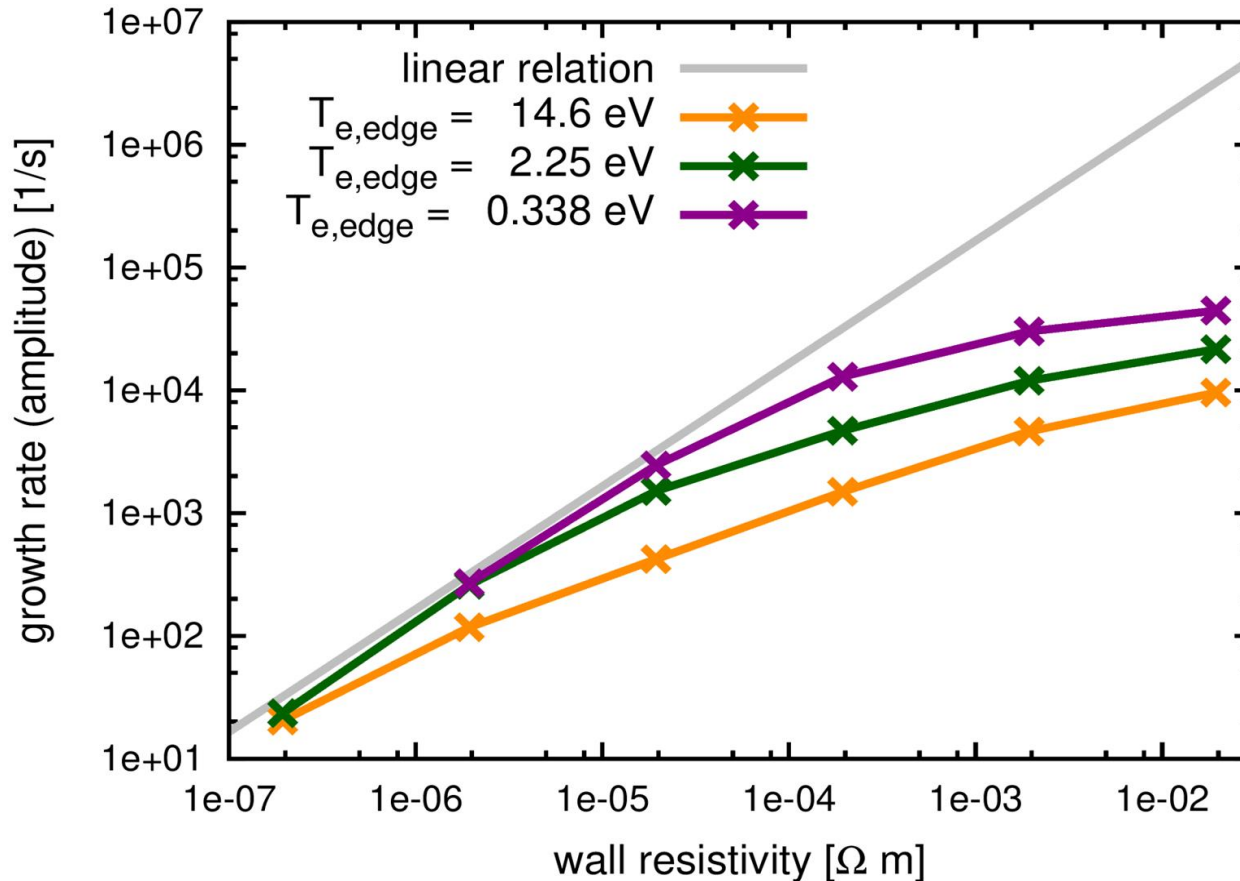
Benchmark M3D-C¹, NIMROD & JOREK

- Compare results of all three codes for the same VDE case
 - Based on NSTX VDE discharge #139536*
 - Axisymmetric rectangular resistive wall that all codes can handle
- Linear, 2D axisymmetric nonlinear & 3D nonlinear simulations
 - Compare evolution, wall currents & forces

*D. Dfefferle, et al.: Phys. Plasmas **25** (2018)

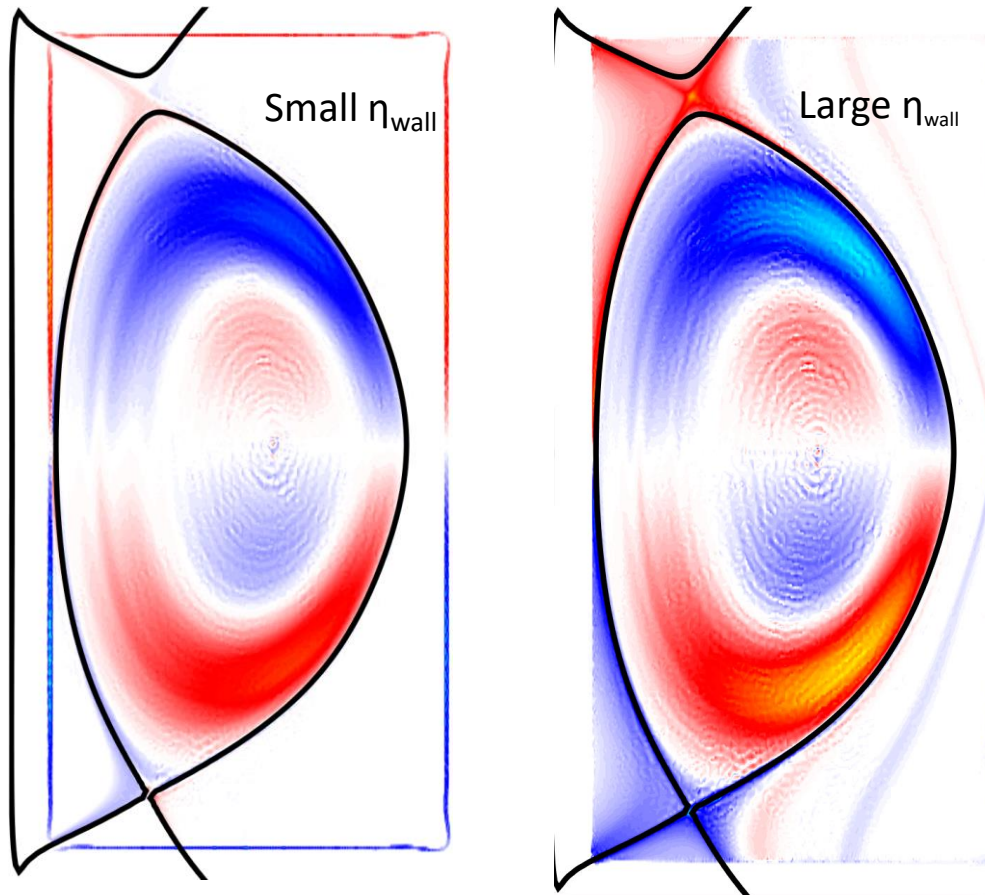


Linear VDE growth vs. η_{wall} depends on T_{edge}



- Small η_{wall} , small T_{edge} :
VDE growth rate $\sim \eta_{wall}$
- Large η_{wall} , large T_{edge} :
VDE slowed down by
response currents in
open field line region

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VDE slowed down by
response currents in
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Toroidal current density eigenfunctions

Linear benchmark with NIMROD

M3D-C¹

NIMROD

Poloidal Direction

Tri. C^1 Reduced Quintic FE

High. Order quad C^0 FE

Toroidal Direction

Hermite Cubic C^1 FE

Spectral

Magnetic Field

$$\mathbf{B} = \nabla \psi \times \nabla \varphi - \nabla_{\perp} f' + F \nabla \varphi$$

$$\mathbf{B} = B_r \hat{R} + B_z \hat{Z} + B_{\varphi} \hat{\varphi}$$

Velocity Field

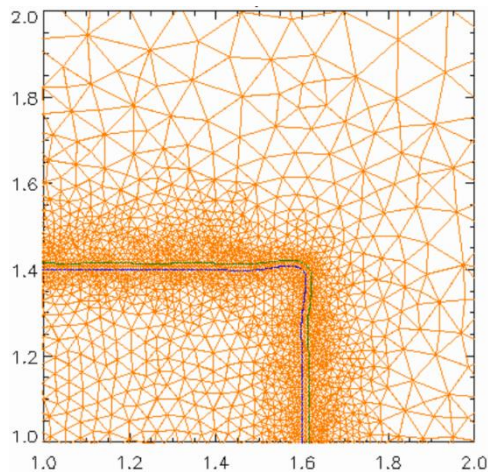
$$\mathbf{V} = R^2 \nabla U \times \nabla \varphi + \omega R^2 \nabla \varphi + R^{-2} \nabla_{\perp} \chi$$

$$\mathbf{V} = V_r \hat{R} + V_z \hat{Z} + V_{\varphi} \hat{\varphi}$$

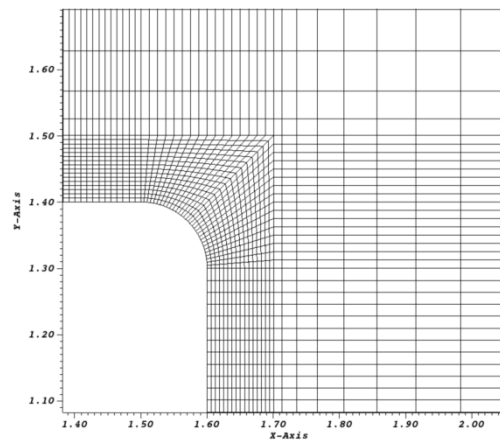
Coupling to Conductors

same matrix

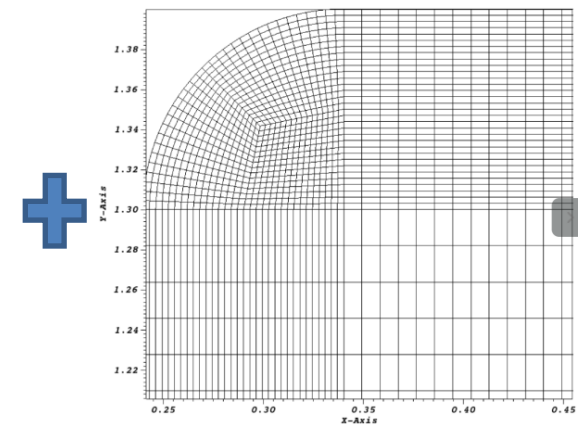
Separate matrices w interface



M3D-C¹ 3 regions, thick wall

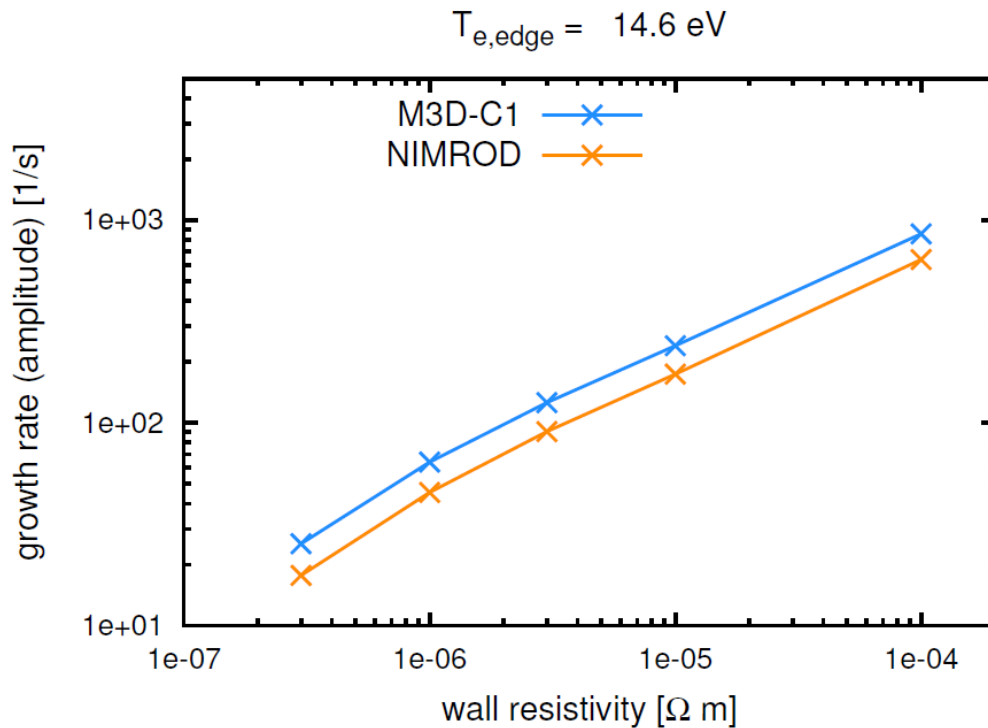


NIMROD vacuum region

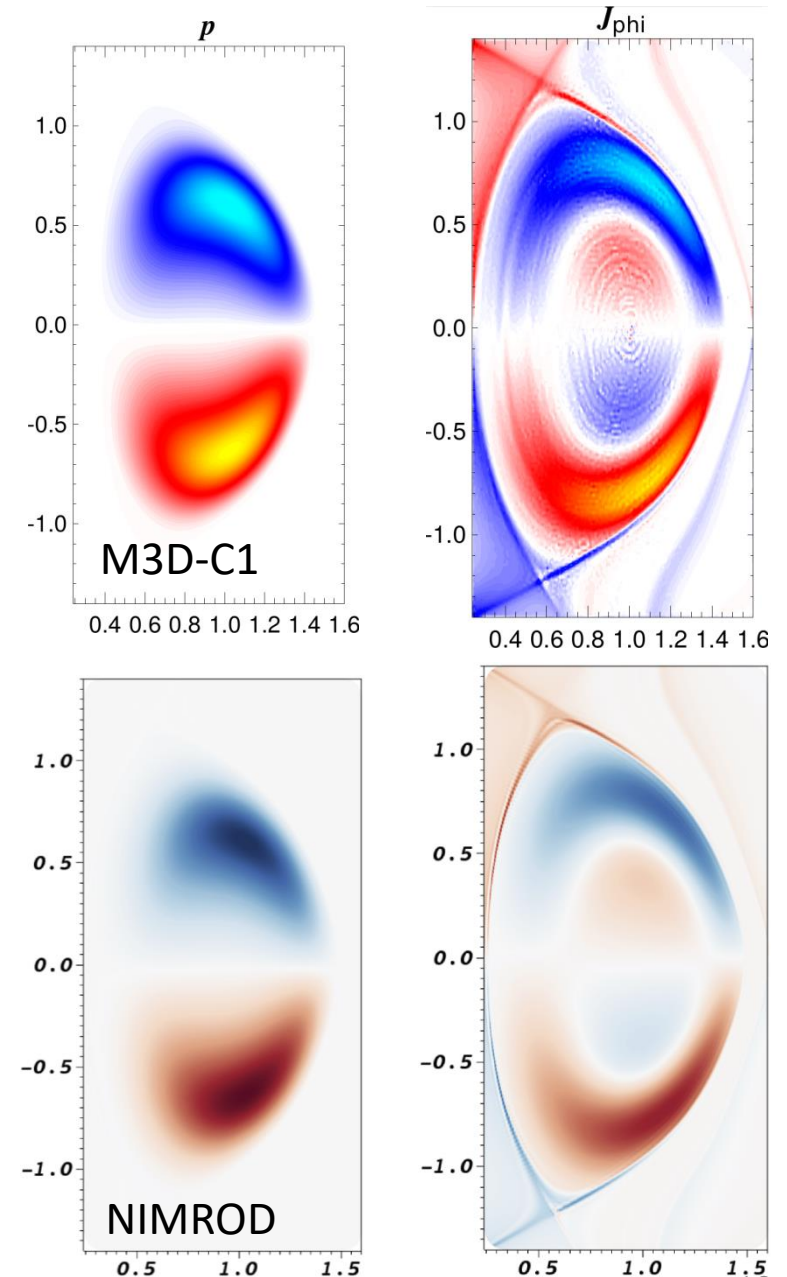


NIMROD plasma region

Linear benchmark with NIMROD (preliminary)

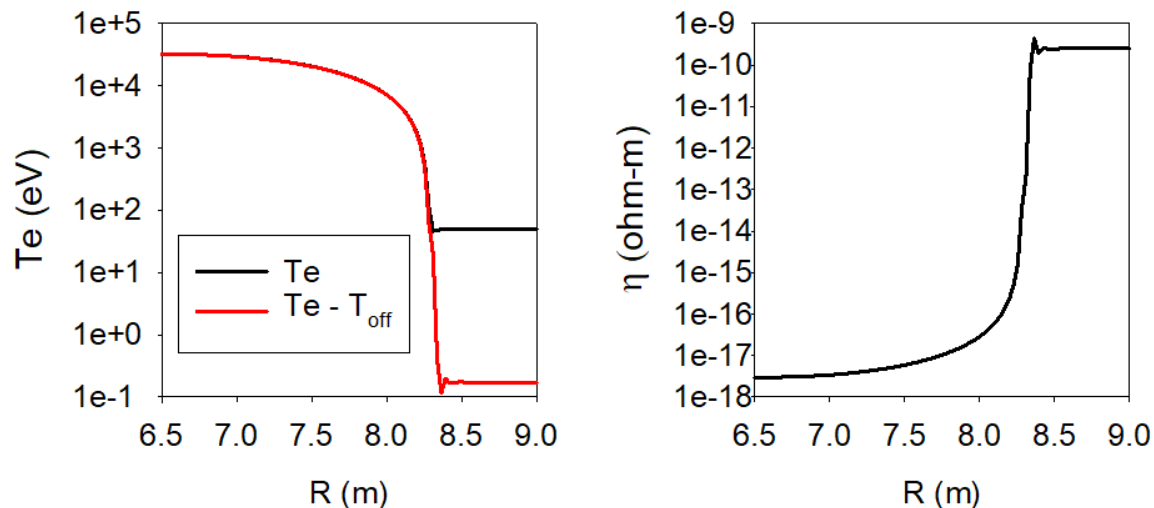


- Growth rates differ by $\sim 30\%$
- Slight differences in diffusion parameters



Linear benchmark with JOREK-STARWALL

- Comparison of linear phase of 2D nonlinear simulations
 - To avoid negative temperatures from developing, we use an offset in resistivity calculation so open-field-line resistivity is not constrained by T_{edge} : $\eta = \eta_{\text{spitzer}}(T_e - T_{\text{off}})$



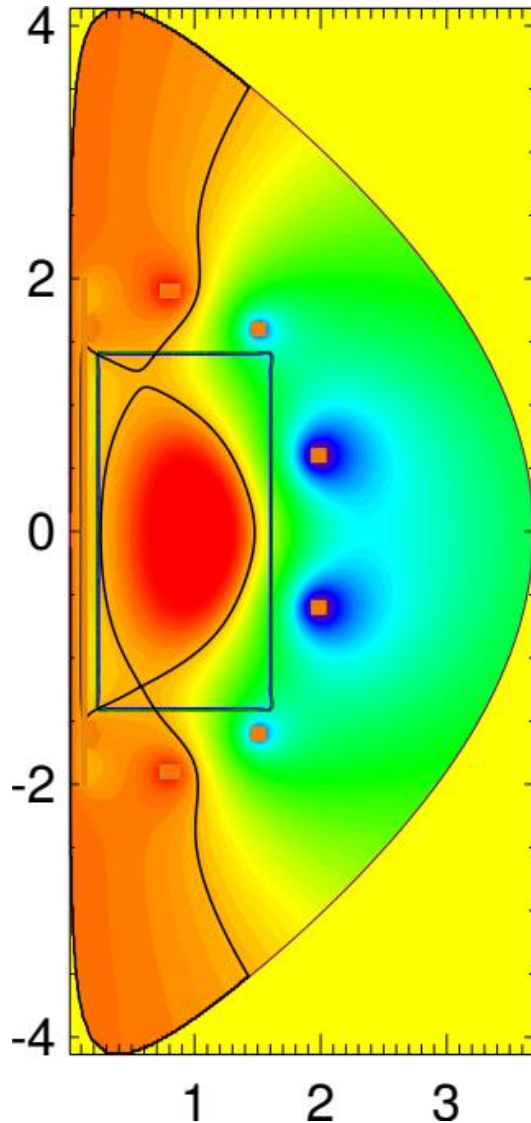
8 orders variation
in resistivity from
center to wall !!!

- Differences between JOREK & M3D- C^1 /NIMROD models:
 - JOREK has full MHD model, but uses *reduced MHD* for VDEs
 - No ideal wall BCs at domain boundary
 - Only normal velocity component vanishes at resistive wall

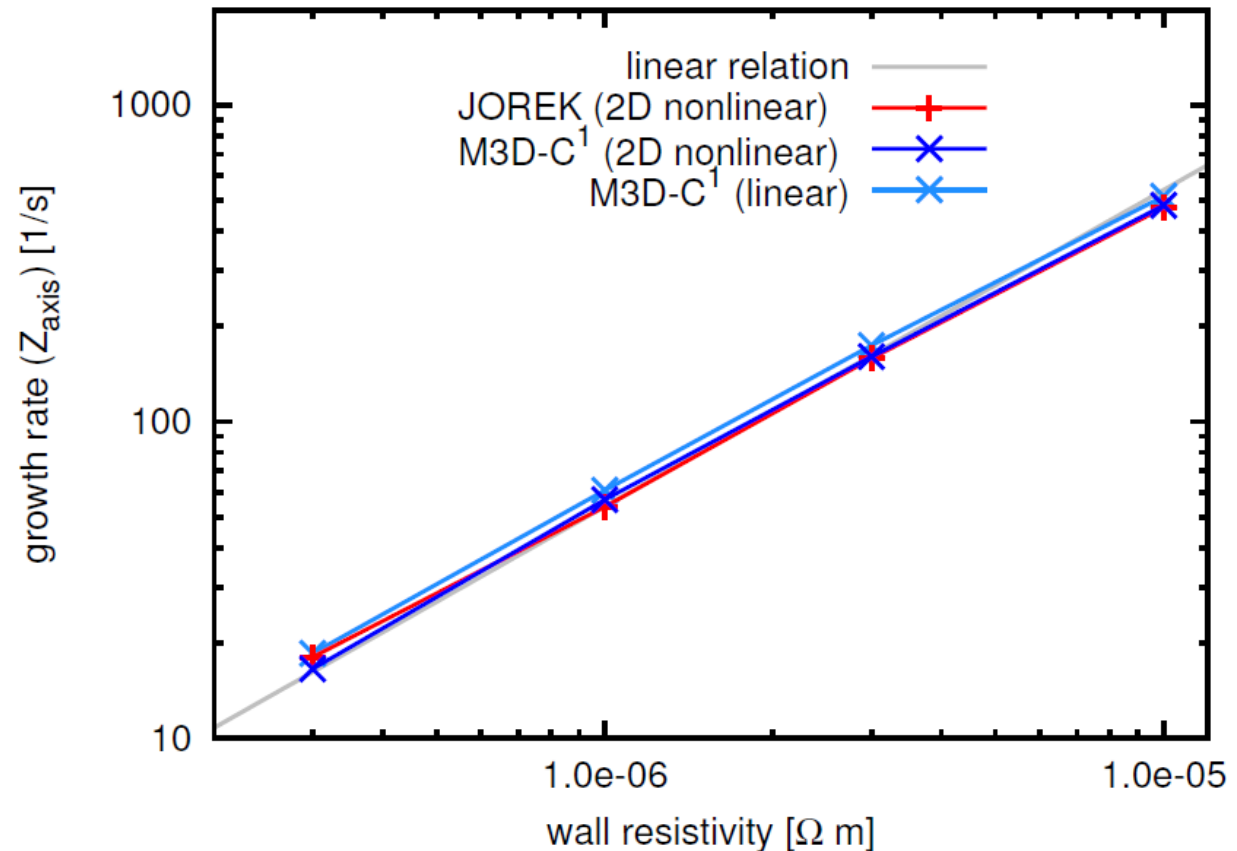
Progress on VDE benchmark between M3D-C¹ & JOREK

– linear phase of 2D nonlinear simulations

Equilibrium Ψ



$Z_{\text{axis}} = -1.64\text{cm to } -3.04\text{cm}$, effective $T_{e,\text{edge}} = 1\text{eV}$



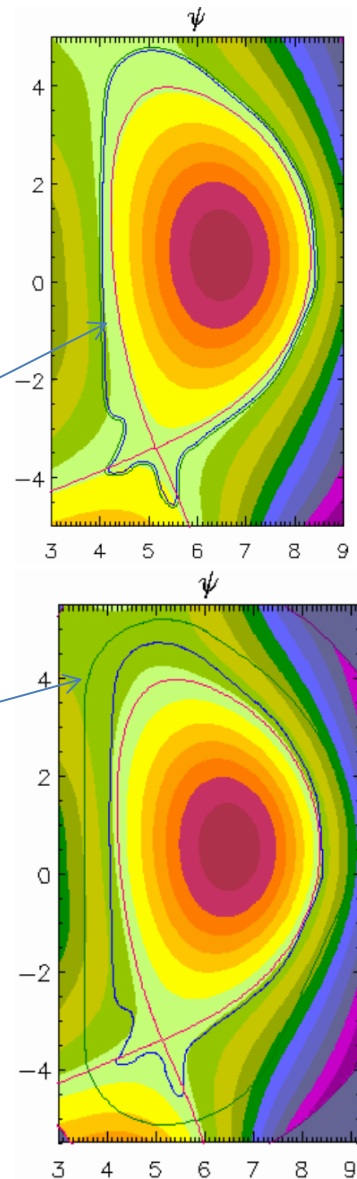
- Based on simplified NSTX VDE case
- Difference between linear growth rates $\leq 9\%$

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2D nonlinear ITER VDE simulation

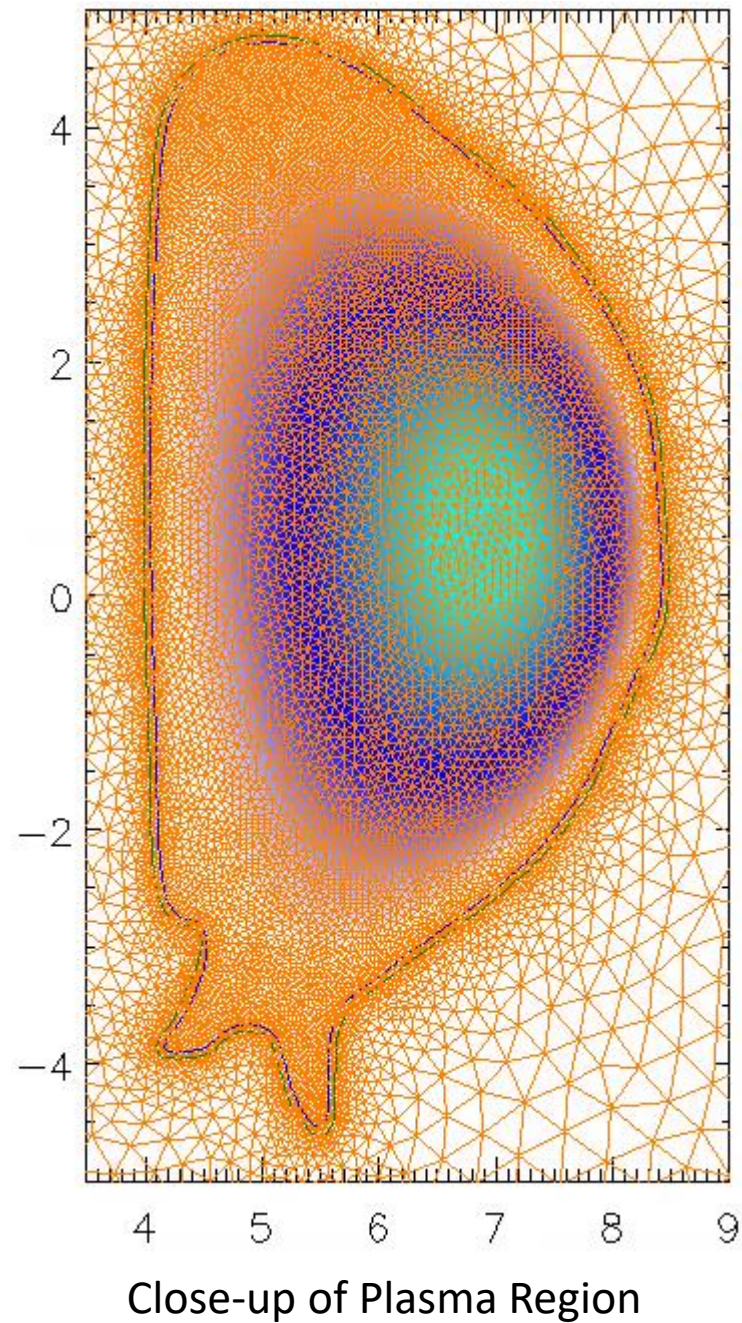
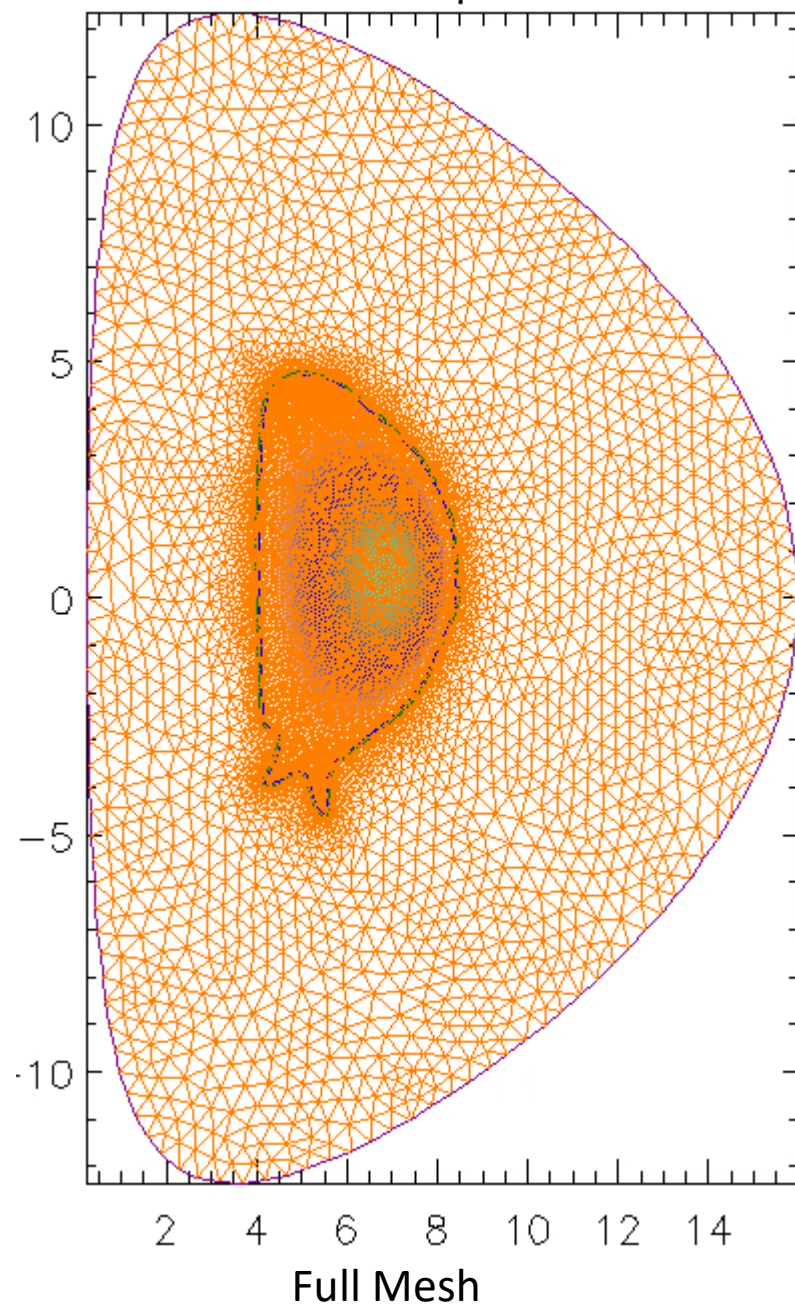
- Based on standard 5.3 T / 15 MA ITER scenario
- Used realistic parameters for *wall resistivity*, *plasma resistivity*, *plasma mass* (no scaling: $1,150,000 \tau_A$!!)
- 2D benchmark with CarMaONL in progress
 - Comparison of 2D evolution & wall currents/forces
 - with ITER first wall as resistive wall
 - with first wall as boundary & vessel wall as resistive wall
- Coupling M3D-C¹ & CARIDDI (3D conducting structures)
 - 2D M3D-C¹ simulations
 - 3D M3D-C¹ simulations



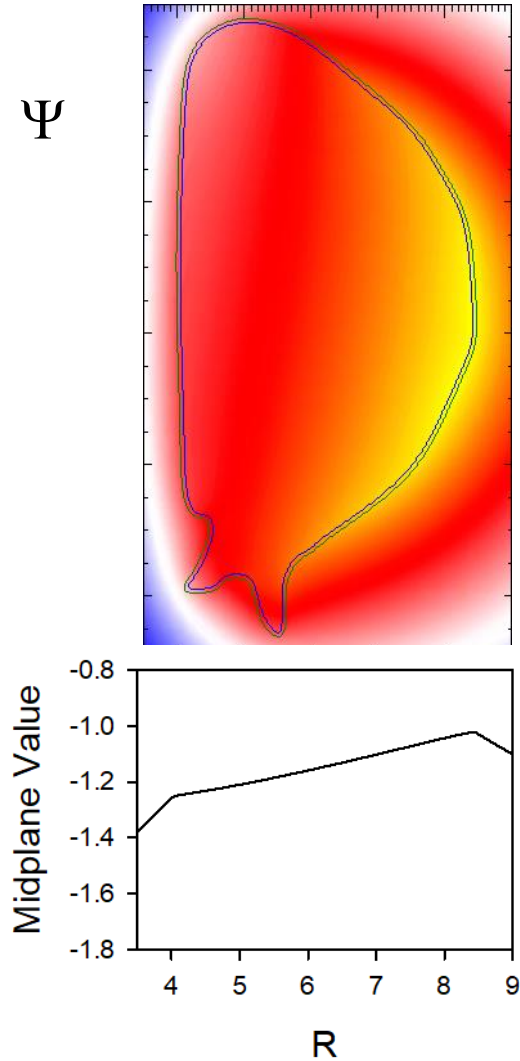
I. Krebs
F. Villone

/p/m3dc1/nferraro/data/test/mesh/iter_mesh

Poloidal unstructured mesh used in ITER calculation

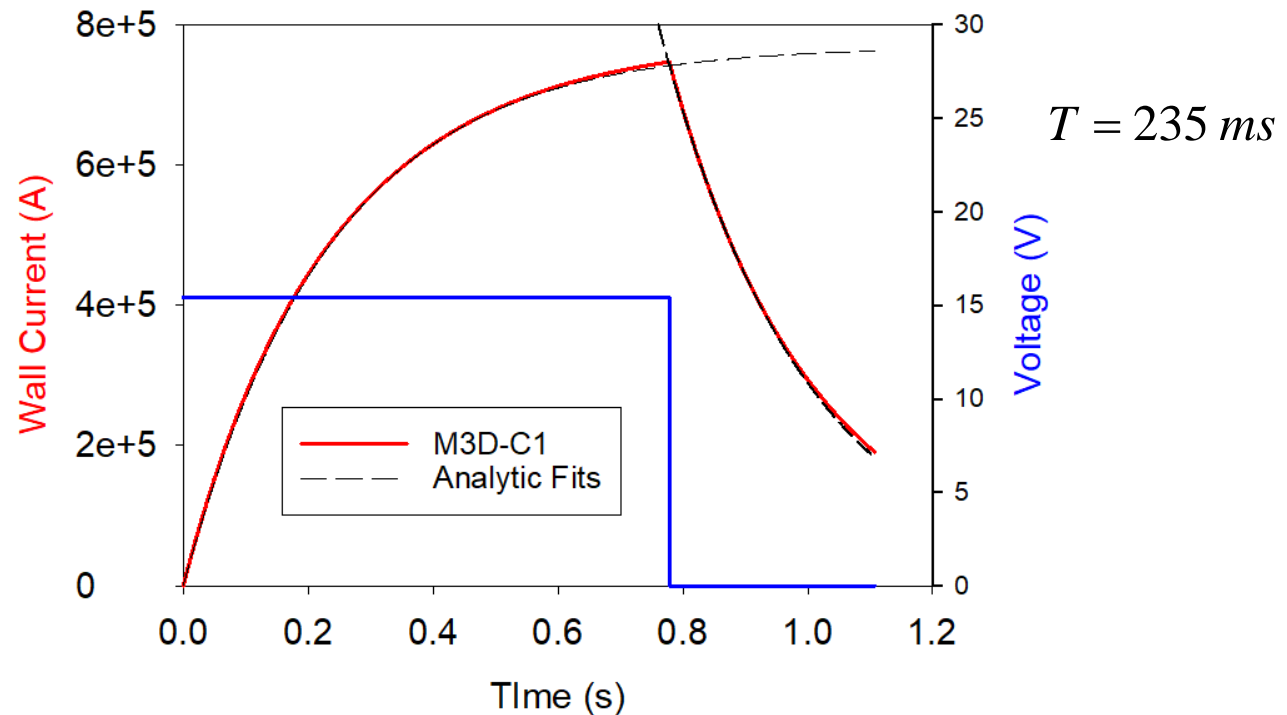


L/R time from simulation without plasma



- Simulation with constant loop voltage applied at $t=0$ & no plasma
- Wall resistivity adjusted to give correct L/R time

$$I(t) = \begin{cases} I_0(1 - e^{-t/T}) & : t < t_M \\ I_M e^{-(t-t_M)/T} & : t \geq t_M \end{cases}$$



Simulation time: $1,800,000 \tau_A$

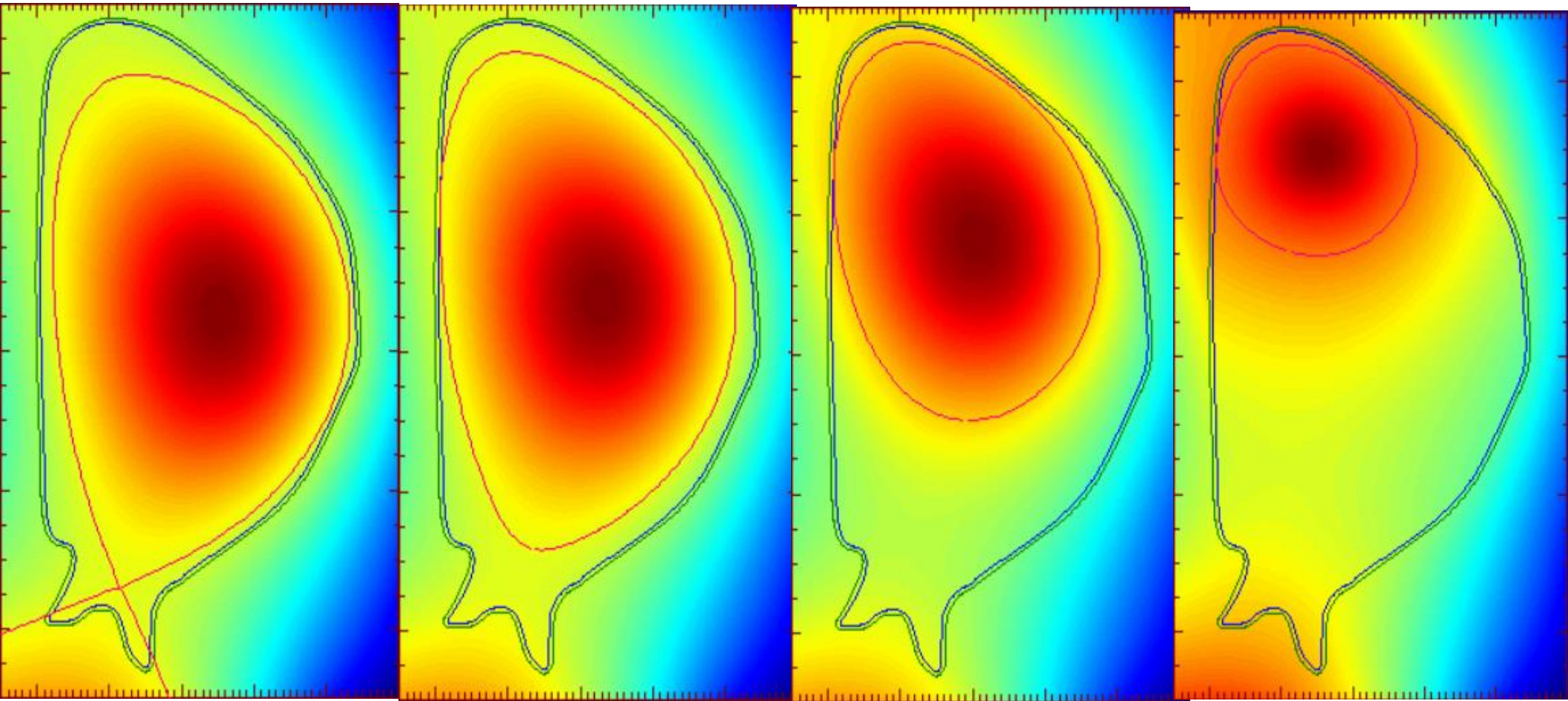
2D nonlinear Hot ITER VDE simulation with single wall

Equilibrium (t=0ms)

Wall contact (t=380 ms)

TQ (t=643 ms)

Max Force (t=673 ms)

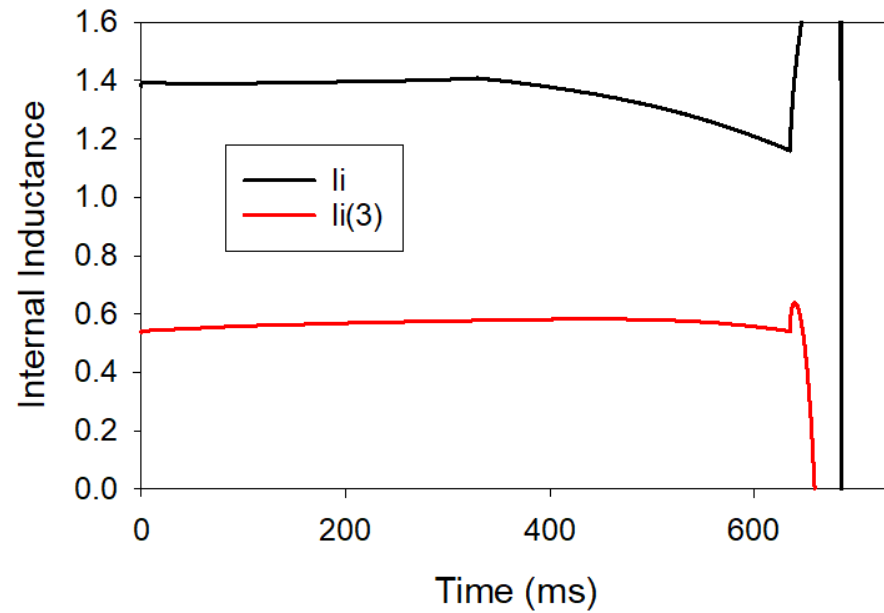
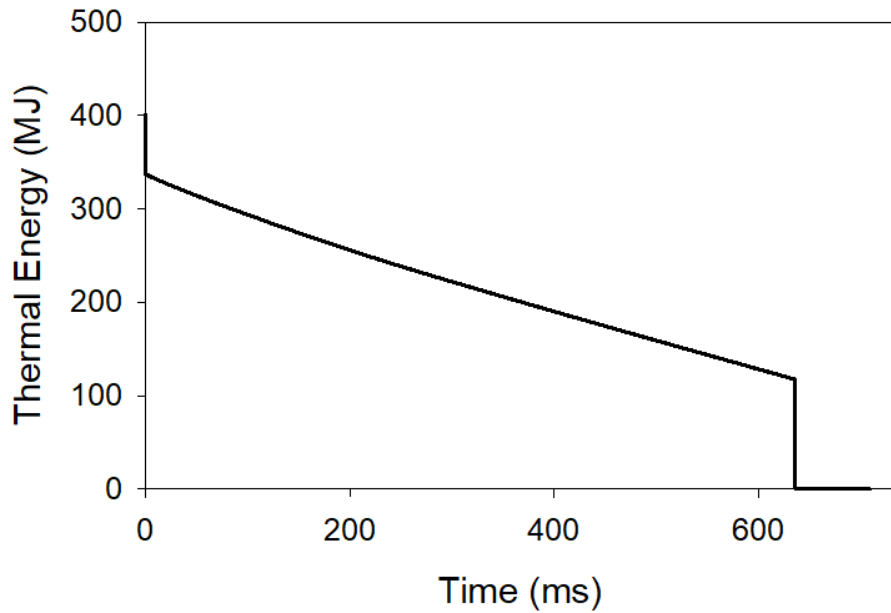
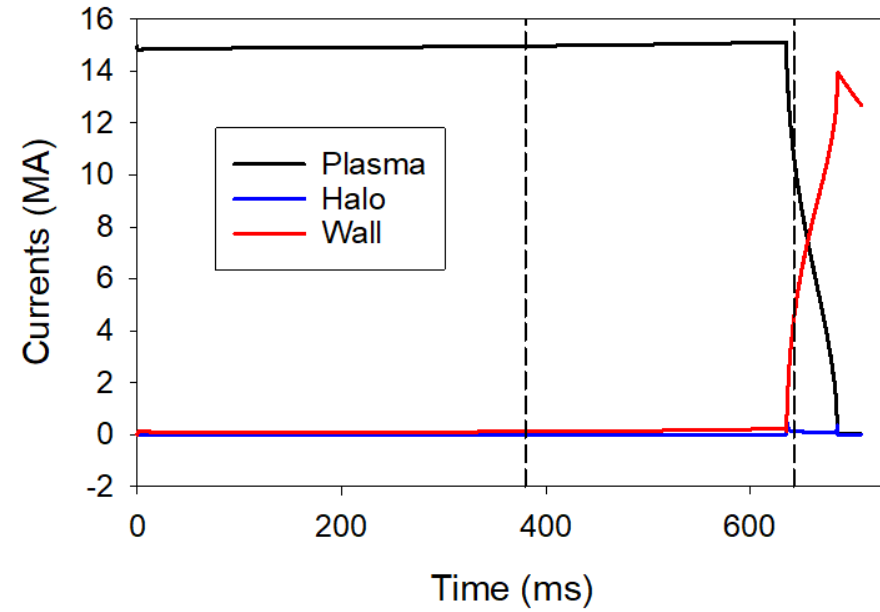
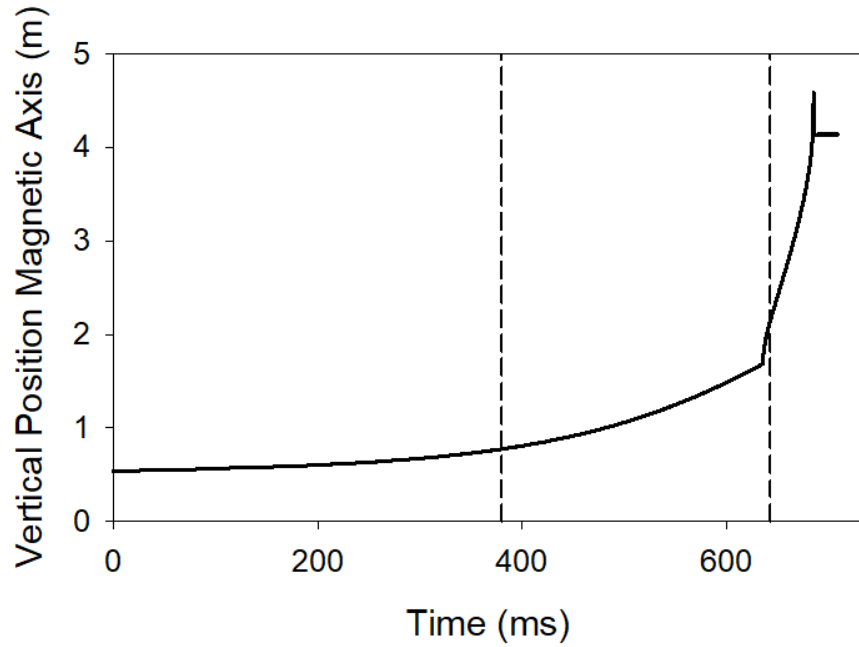


Poloidal Magnetic Flux

Table=33

19

2D nonlinear ITER Hot VDE simulation



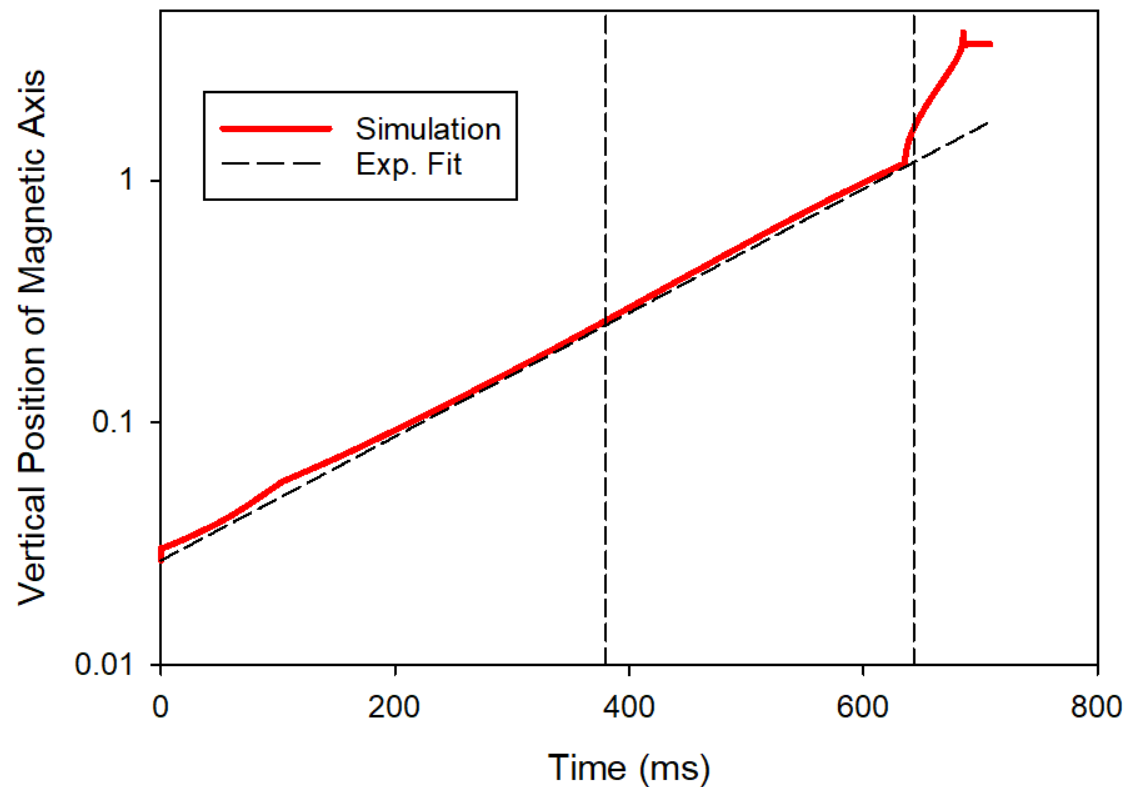
2D nonlinear ITER Hot VDE simulation

Z-position of magnetic axis well fit by linear growth rate until start of the thermal quench

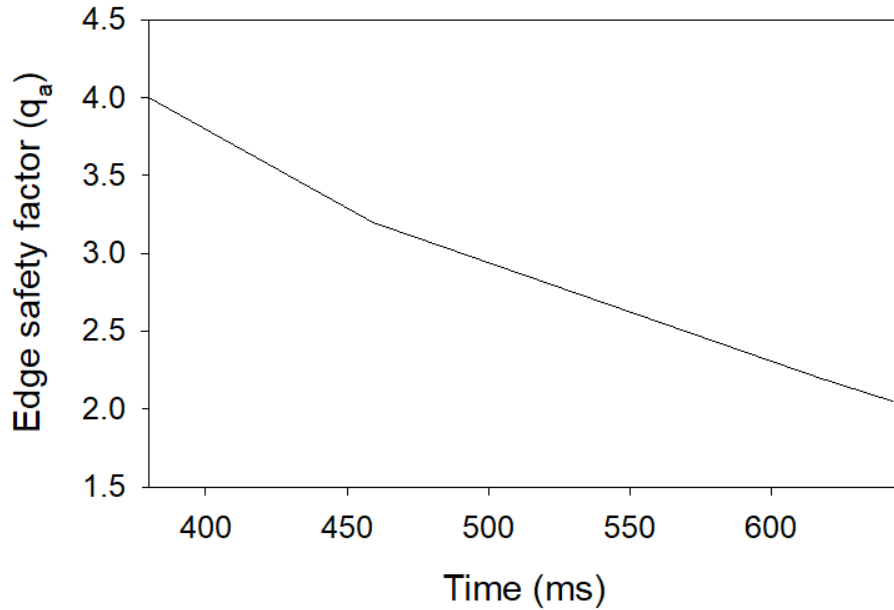
$$Z_{mag} - Z_0 = .027 \exp\left(\frac{t(ms)}{170}\right)$$

Touches wall

TQ

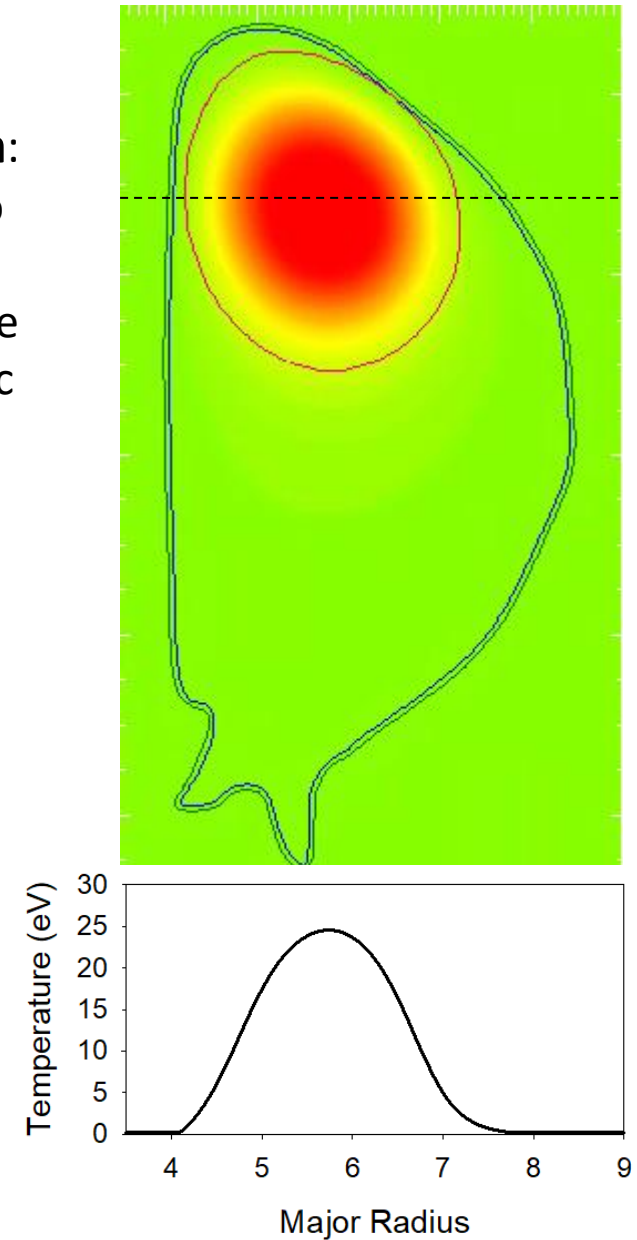
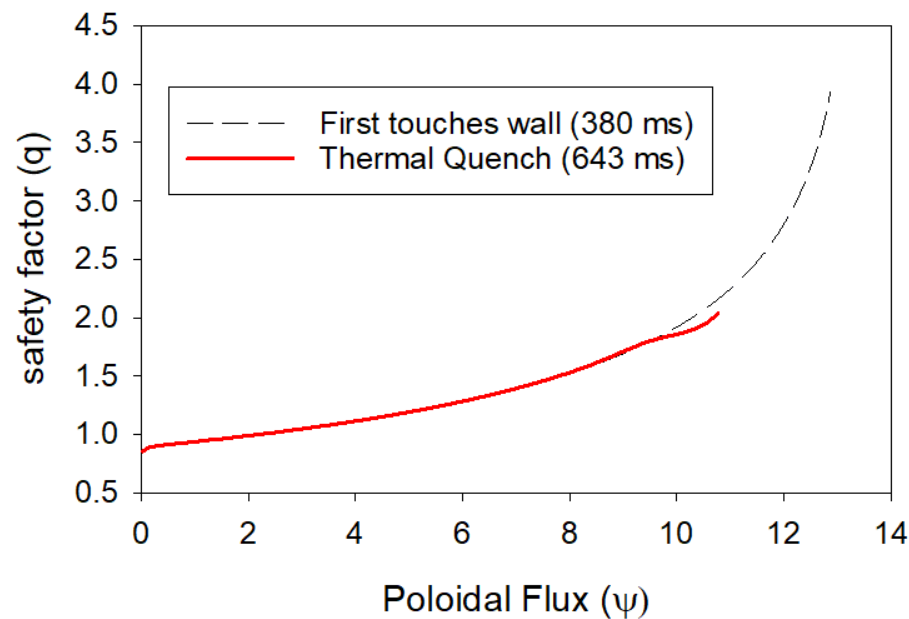


Thermal Quench at $t = 643 \text{ ms}$ ($q_a = 2$)

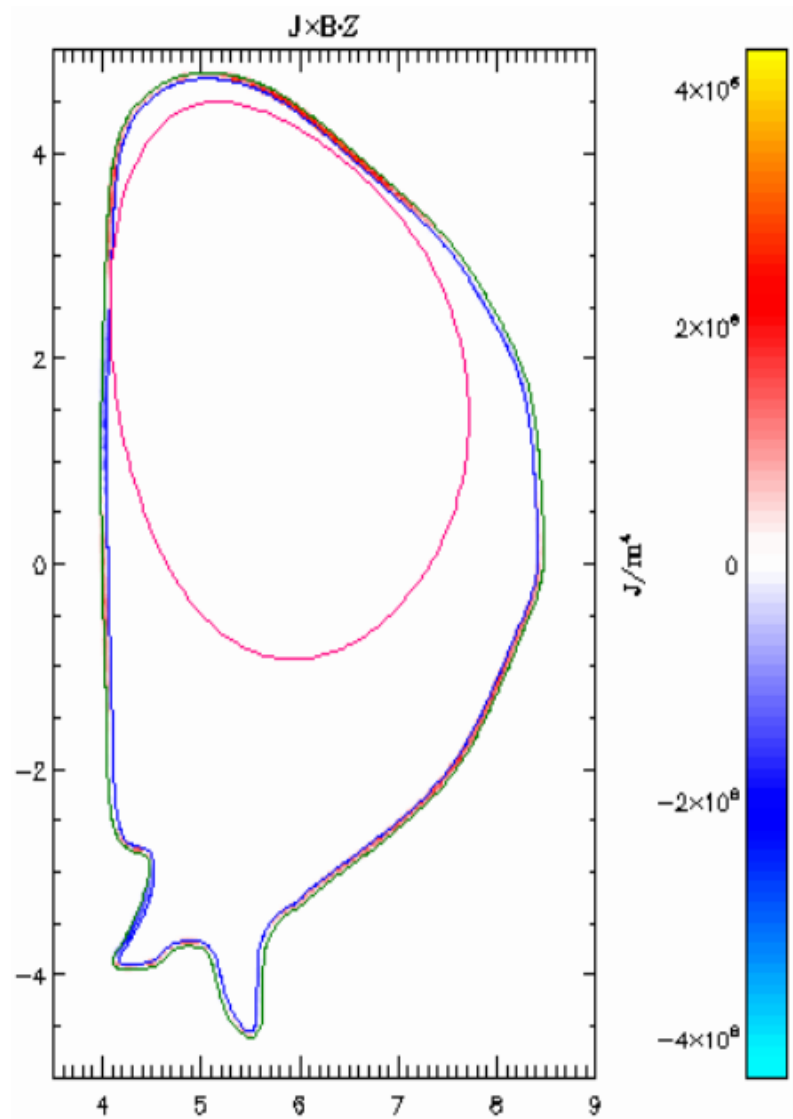
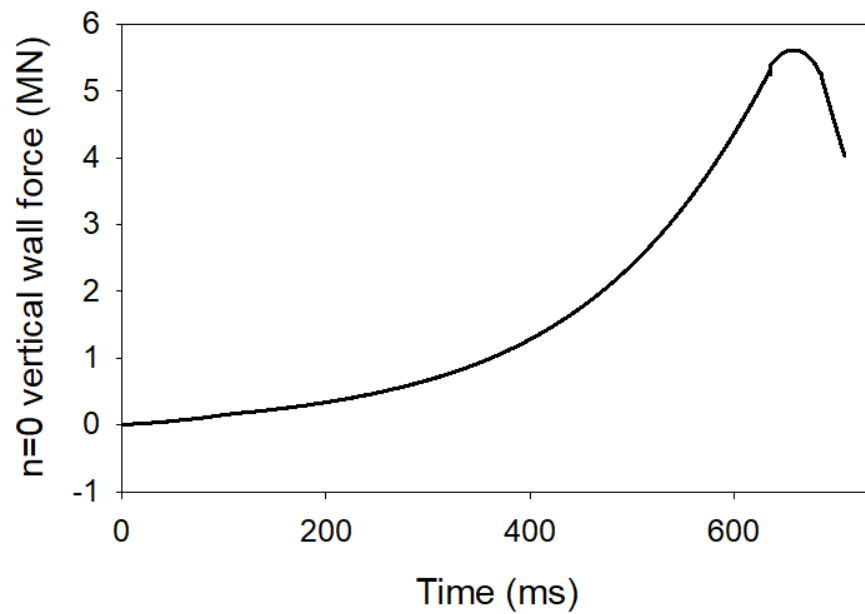


Thermal quench:

→ κ_{\perp} increased to $10^6 \text{ m}^2/\text{s}$ so that $T_e(0) \sim 25 \text{ eV}$ in the presence of Ohmic heating

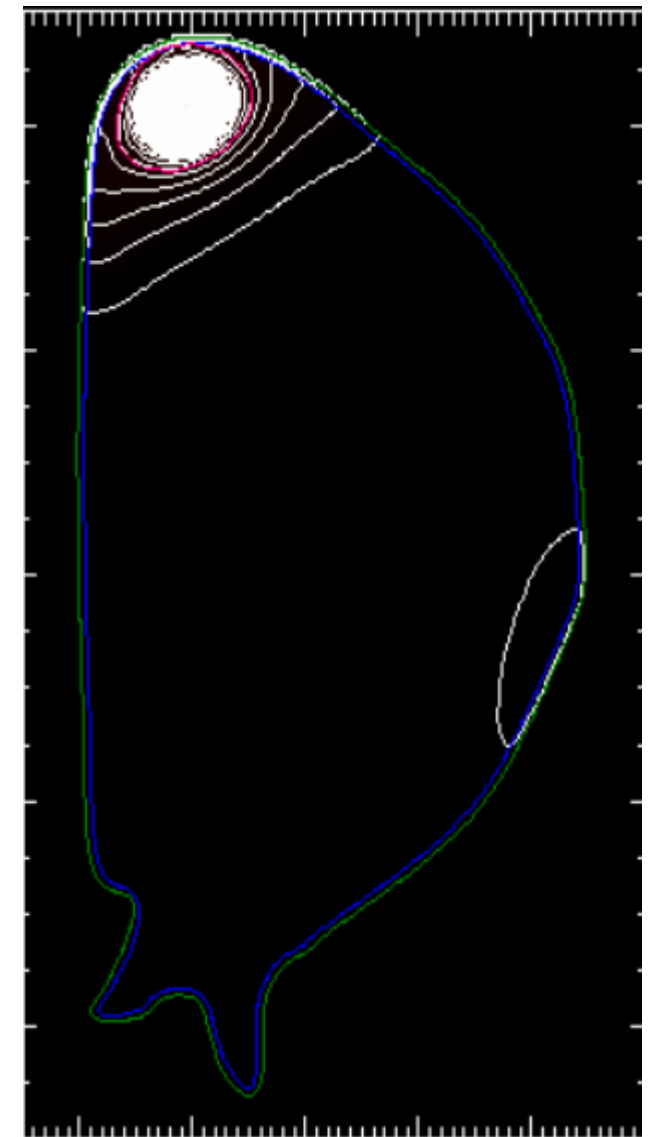
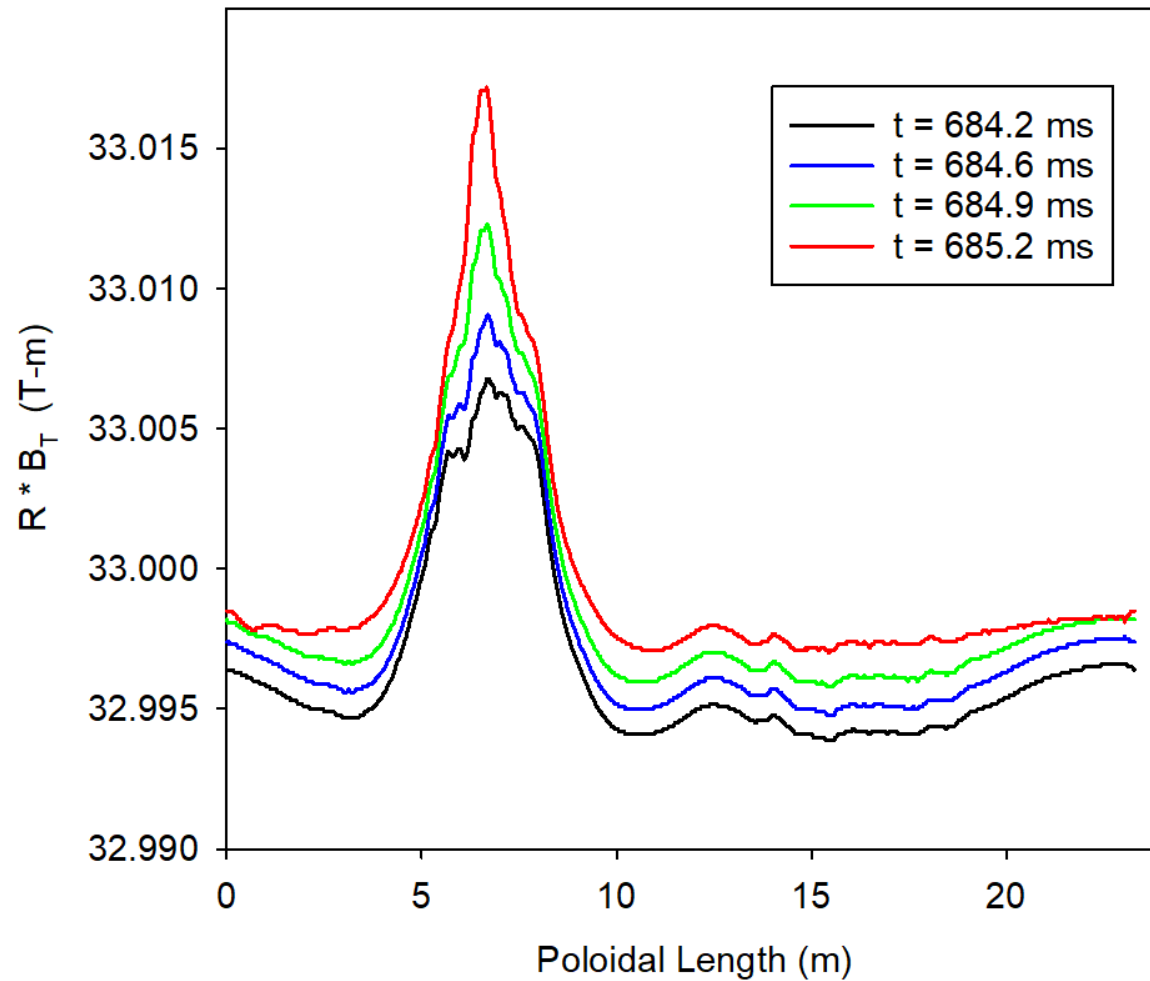


Vertical Force on Wall



Vertical wall force @ $t = 646$ ms

Halo current at time of maximum force



$R * B_T$ at $t = 684.6$ ms

2D nonlinear Cold ITER VDE simulation with single wall

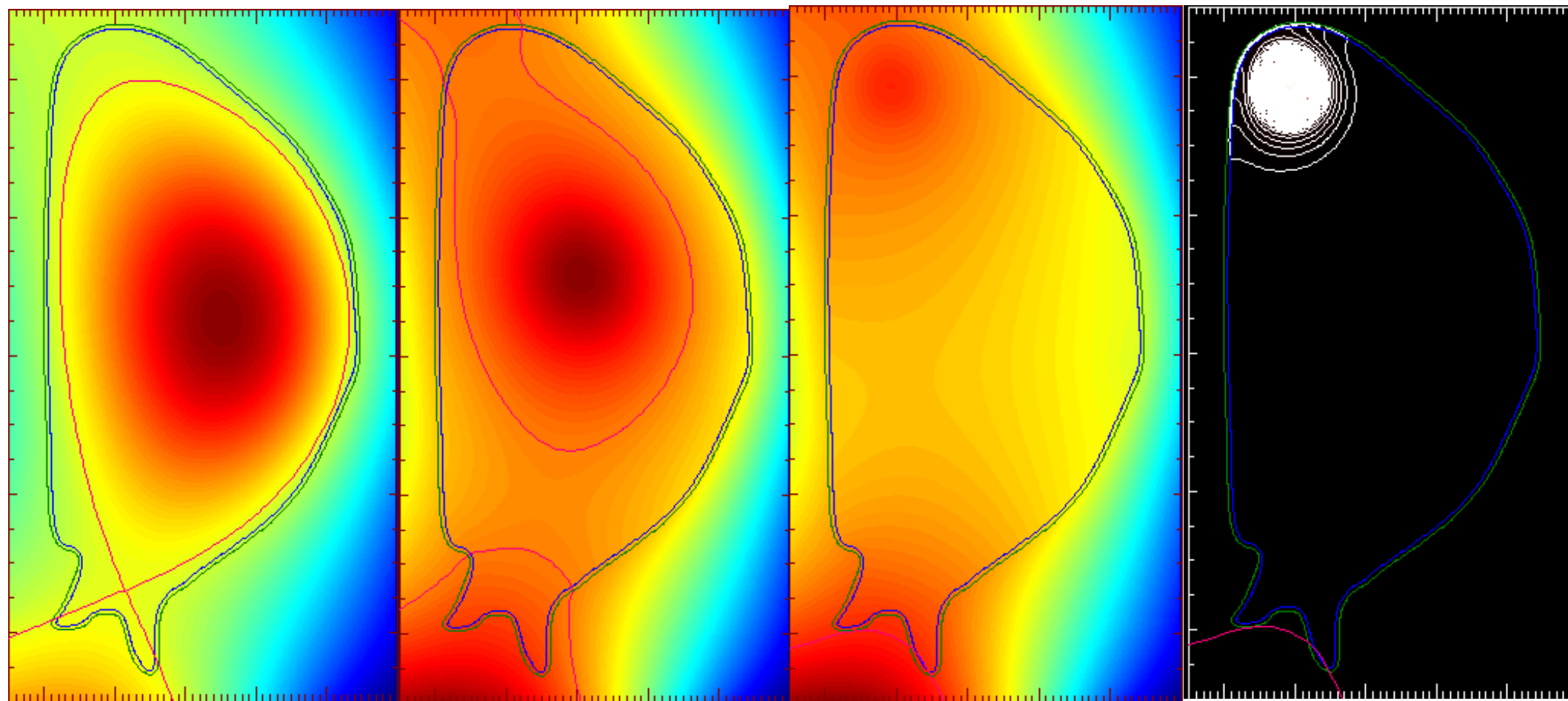
Poloidal Magnetic Flux

Equilibrium & TQ (t=0ms)

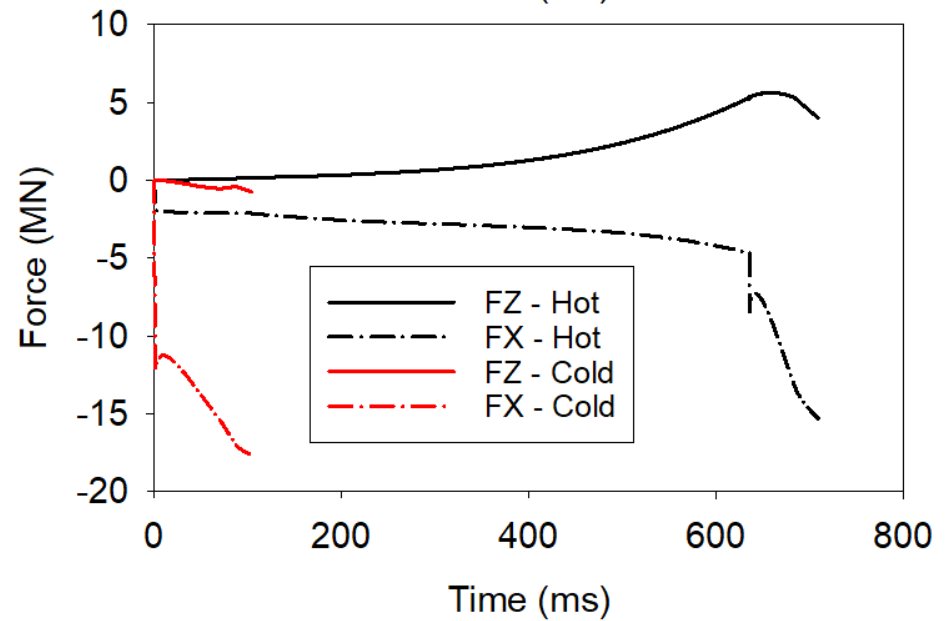
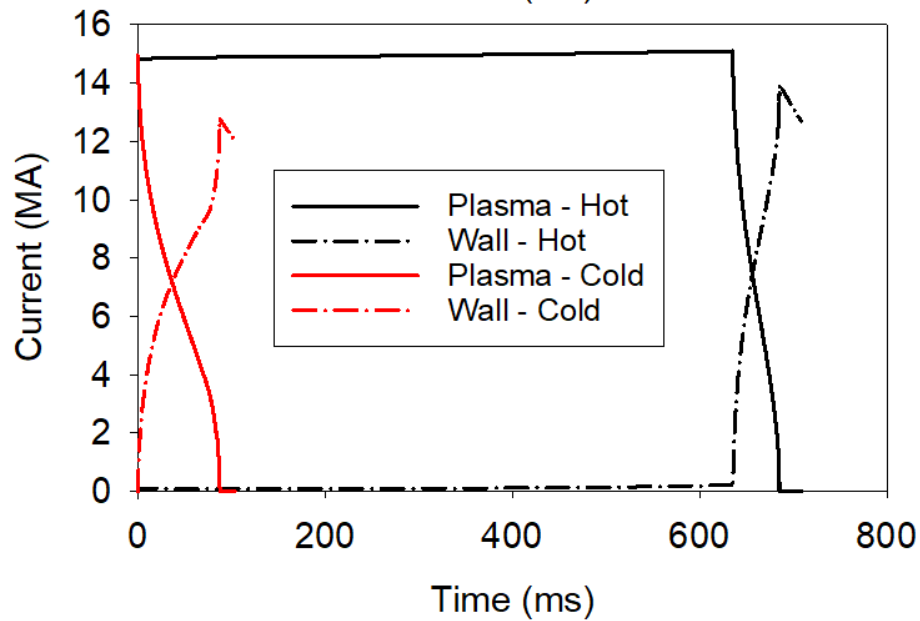
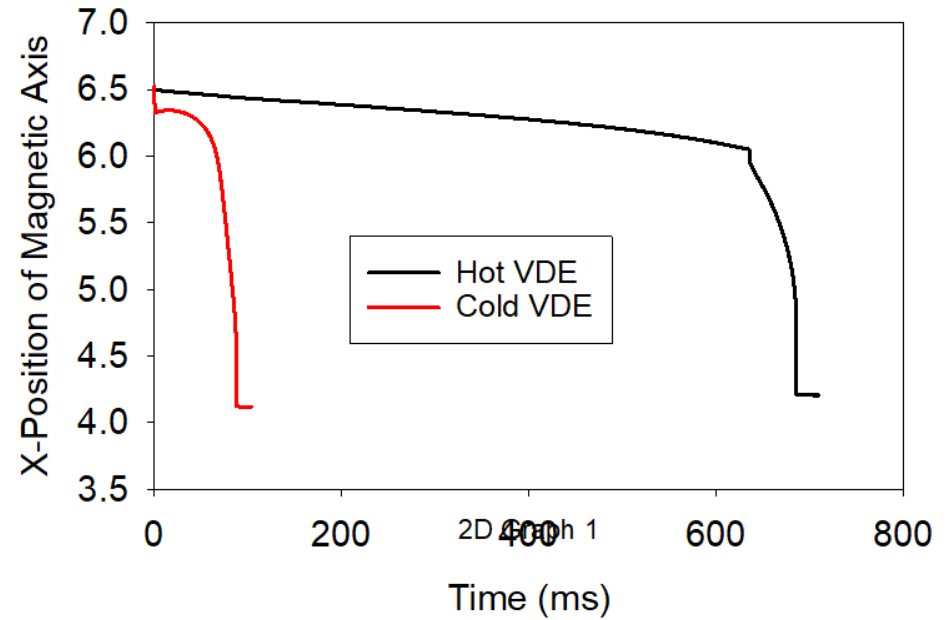
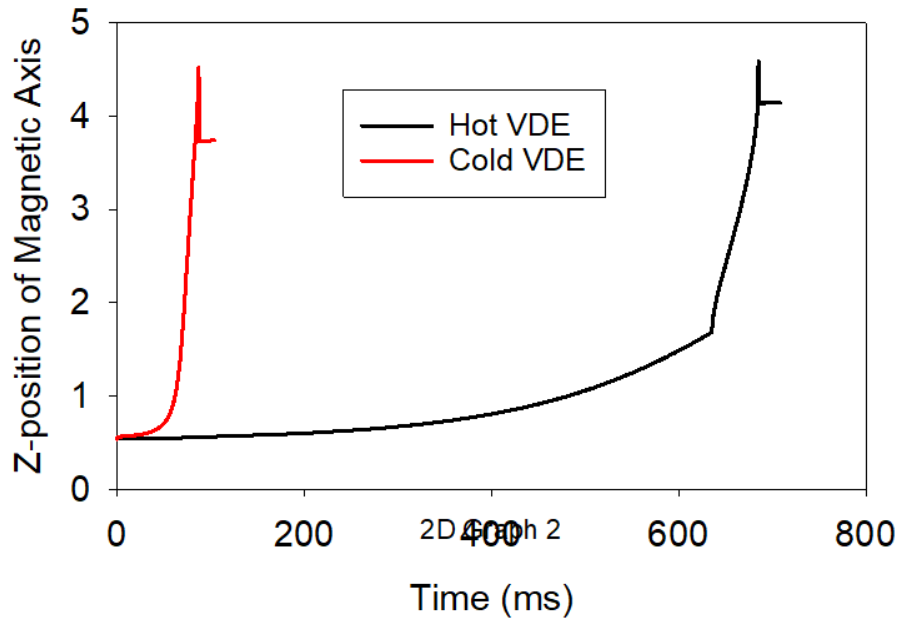
Psi (t=66 ms)

Psi (t=85 ms)

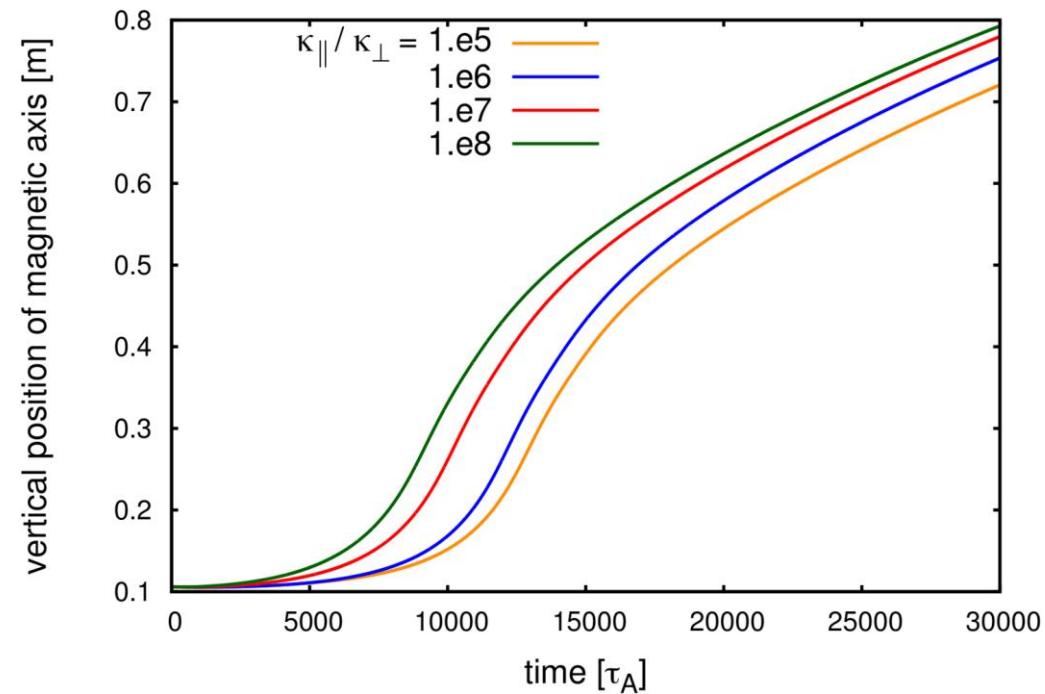
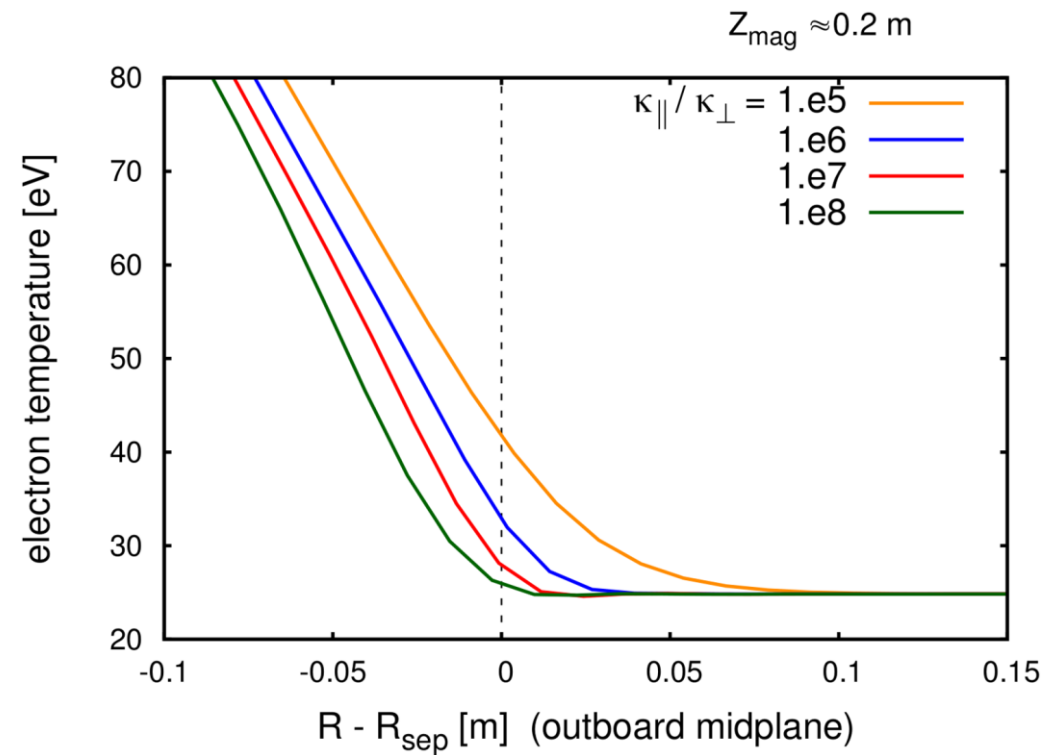
$R \cdot B_T$ (t=85 ms)



Comparison of 2D nonlinear Hot and Cold ITER VDE

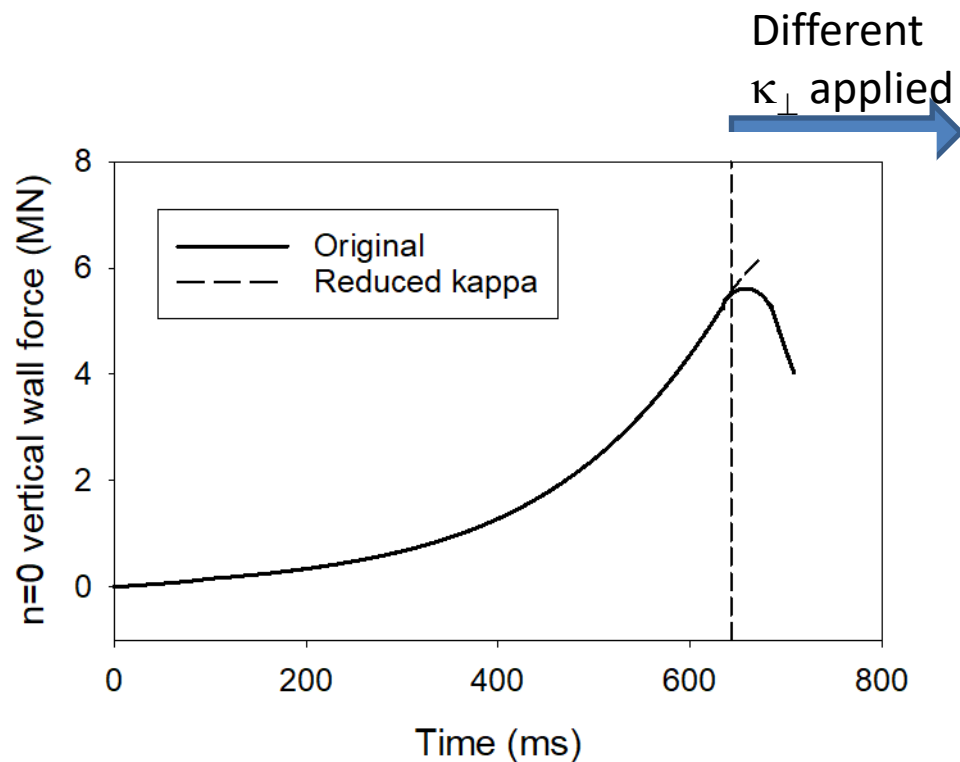


Halo width self-consistently determined by $\kappa_{\parallel}/\kappa_{\perp}$



→ Halo width & temperature at LCFS determined by T_{edge} & $\kappa_{\parallel}/\kappa_{\perp}$

Dependence of Maximum Vessel Force on Post-TQ T_e



Higher post-TQ electron temperature led to slower current decay and larger vertical force on vessel.

Ongoing and Future Work

- **Benchmark Studies with NIMROD, JOEK, CarMa0NL**
 - Continue these to 2D NL and 3D NL
- **ITER VDE Studies**
 - 2D parameter studies on $\kappa_{\parallel}/\kappa_{\perp}$, TQ time
 - Thick Vessel with varying resistivity
 - Couple to Cariddi (3D conducting structures)
 - Fully 3D calculations (with SPI)