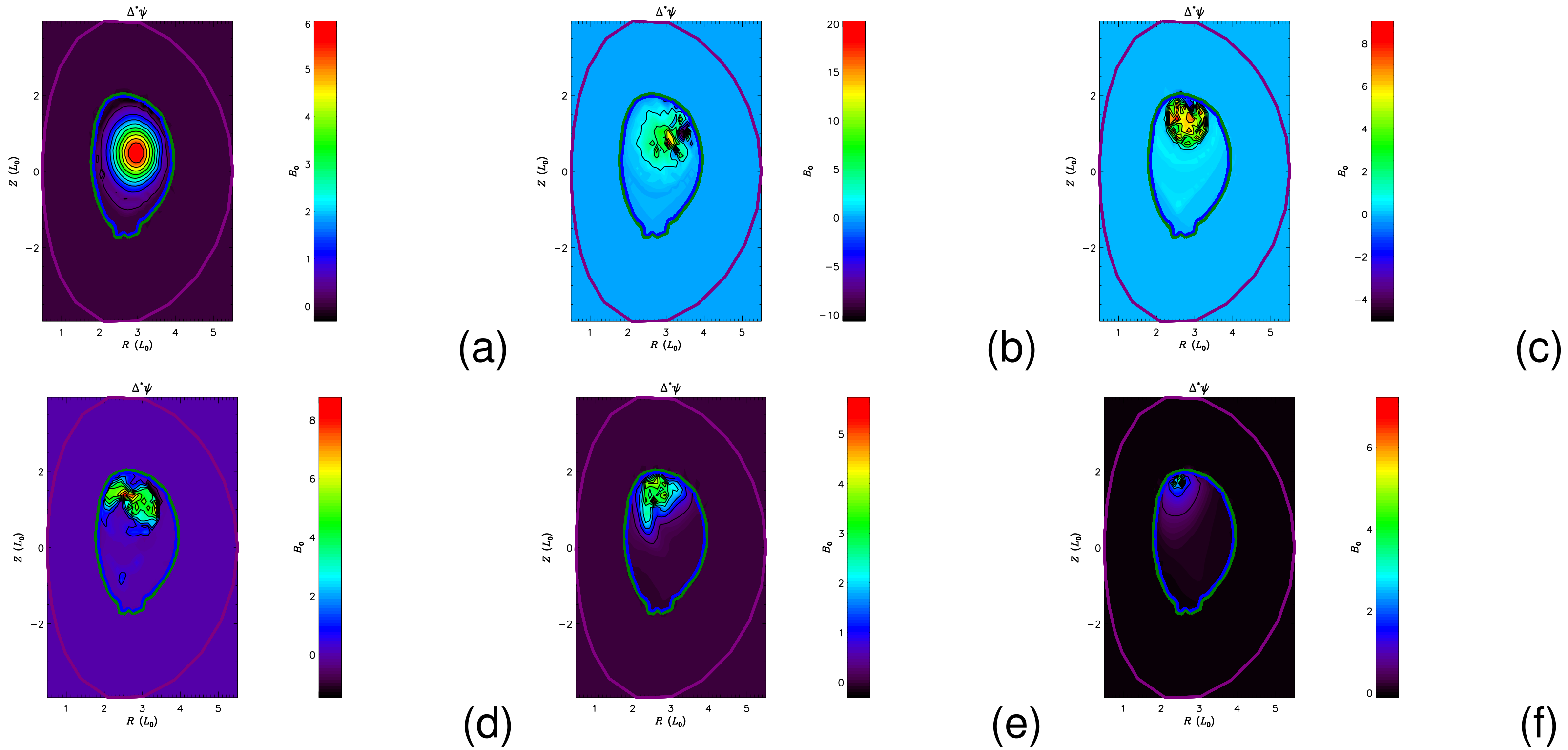
M3D-C1 JET simulation Progress Report

H. Strauss

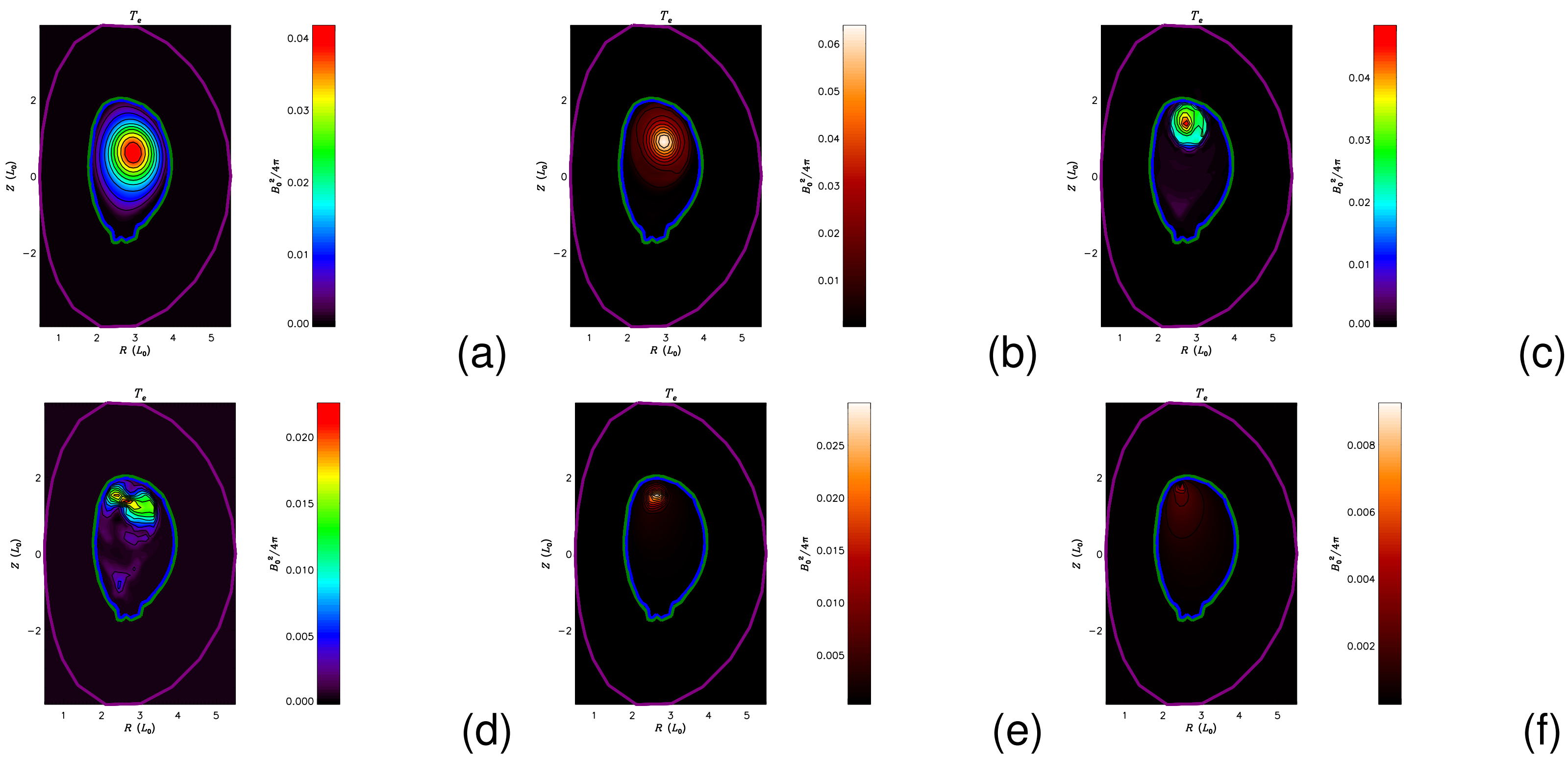
- simulation of JET shot 71985
- implemented modifications of M3DC1
- toroidal dealiasing
- thin wall

Simulation of JET shot 71985 - j_ϕ



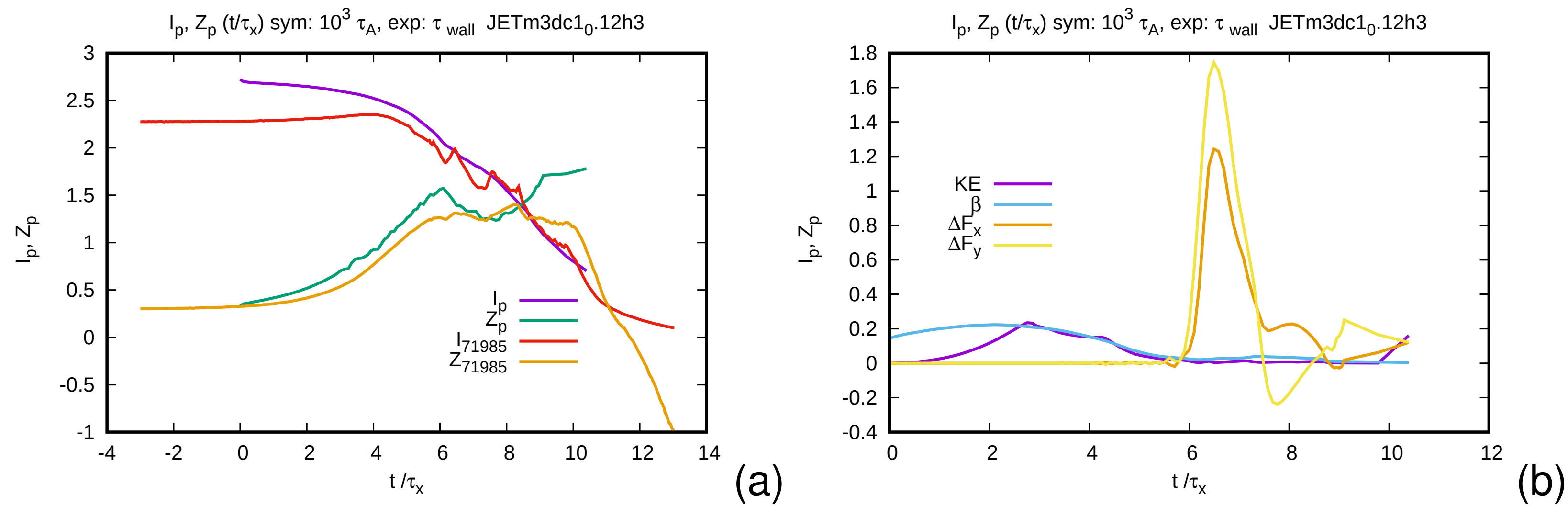
Current density j_ϕ at (a) $t = 2\tau_x$ where $\tau_x = 10^3\tau_A$. (b) $t = 4\tau_x$ (c) $t = 6\tau_x$ (d) $t = 8\tau_x$ (e) $t = 10\tau_x$ (f) $t = 12\tau_x$ and $\tau_{wall} \approx \tau_x/2$. parameters: $\eta_0 = 1.e-5, eta_{max} = 1.e-2, eta_{wall} = 1.e-3$, $\mu_0 = 1.e-5$, $\mu_{max} = 1.e-2$, $\kappa_\perp = 1e-2, \kappa_\| = 1.$

Simulation of JET shot 71985 - T_e



Electron temperature T_e at (a) $t = 2\tau_x$ where $\tau_x = 10^3\tau_A$. (b) $t = 4\tau_x$ (c) $t = 6\tau_x$ (d) $t = 8\tau_x$ (e) $t = 10\tau_x$ (f) $t = 12\tau_x$ and $\tau_{wall} \approx \tau_x/2$.

Time history



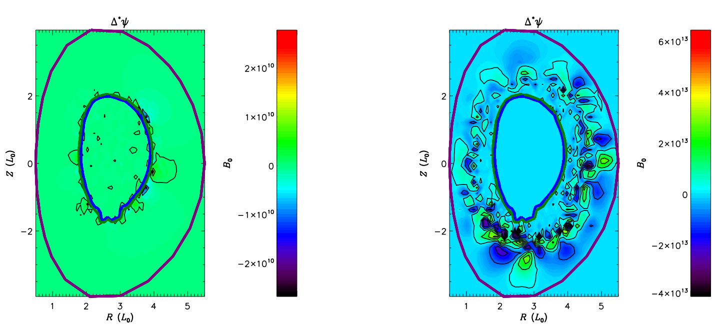
(a) total current in simulation I_p, total current in experiment I_{71985}, vertical displacement in simulation Z_p, vertical displacement in experiment Z_{71985},

(b) KE, 100β, components of sideways F_x, F_y in MN.

Code modifications

- pre programmed voltage for CQ modelling
- input η_{min} and η_{max} rather than T_{emax}, T_{emin}
- restart at specified previous timestep
- to do: modify resistivity profile in the wall to reduce resistivity gradient at the plasma wall boundary
- Taylor Galerkin upwind diffusion, helps stabilize advection, used in M3D

$$\frac{\partial T}{\partial t} = \ldots + \mu \nabla \cdot \frac{\mathbf{vv}}{|v|} \cdot \nabla T \tag{1}$$



Dealiasing

During TQ there can be loss of equilibrium, MHD turbulence, which generates very fine scales.

Viscosity and hyper viscosity can damp the highest wavenumber modes, but will have too much damping at lower wavenumbers.

Dealiasing truncates the toroidal Fourier spectrum, removing the highest $1/3$ of the modes. [S. Orszag, J. Atmos. Sci **28** 1074 (1971)] The lower modes are unchanged.

Very effective in M3D.

wavenumber	viscous	hypervisc	dealias
k_{max}	1	1	1
$k_{max}/2$	0.25	0.0625	0

Toroidal Dealiasing

An element is bounded toroidally by planes ϕ_n, ϕ_{n-1}, where $\phi_n - \phi_{n-1} = \Delta\phi = 2\pi/N$. A function V has values at the ends of the element V_n, V_{n-1} and V_n', V_{n-1}', where the prime is the ϕ derivative. There are N such elements. This makes the elements C_1 continuous. Take the discrete Fourier transform

$$\tilde{V}_k = \sum_{n=1}^{N} \exp[ik(n-1)\Delta\phi]V_n \tag{2}$$

truncate, and back transform

$$V_m = \sum_{k=0}^{N/3} C_k \exp[-ik(m-1)\Delta\phi]\tilde{V}_k \tag{3}$$

Here the maximum mode number is $N/2$, and $2/3$ of the maximum mode number is $N/3$, where $C_k = 2/N$, except $C_0 = 1/N$.

Toroidal Dealiasing

There is no need for FFTs. The dealiasing operation can be written

$$V_m^* = \sum_{n=1}^{N} O_{mn} V_n \tag{4}$$

where matrix O_{mn} is precomputed,

$$O_{mn} = Re\{\sum_{k=0}^{N/3} C_k \exp[ik(n-m)\Delta\phi]\} = \sum_{k=0}^{N/3} C_k \cos[k(n-m)\Delta\phi] \tag{5}$$

The V_n' values can be treated the same way,

$$V^{*\prime}{}_m = \sum_{n=1}^{N} O_{mn} V_n' \tag{6}$$

This determines smoothed values of V_n, V_n' at the ends of the elements.

Higher order toroidal dealiasing

Each element end has 2 degrees of freedom, V_n and V_n', so resolution could be improved by interpolating to the midpoint of each element, using the unique Hermite cubic polynomials of the elements. If $V(x)$ is a cubic polynomial, $x = (\phi - \phi_n)/\Delta\phi$,

$$V(x) = V_n(1-3x^2+2x^3)+V_n'(x-2x^2+x^3)+V_{n+1}'(-x^2+x^3)+V_{n+1}(3x^2-2x^3) \tag{7}$$

Then for $x = 1/2$, V_n can be defined at the midpoints of the elements,

$$V_{n+1/2} = (V_{n+1} + V_n)/2 + (V_n' - V_{n+1}')/8 \tag{8}$$

and the dealiasing is done over an array of $2N$ points. The resolution is better: now $2N/3$ harmonics are discarded, leaving $4N/3$ harmonics, twice as many as before. To restore V' back solve (8)

$$V_{n+1}' = V_n' + 4(V_n + V_{n+1}) - 8V_{n+1/2} \tag{9}$$

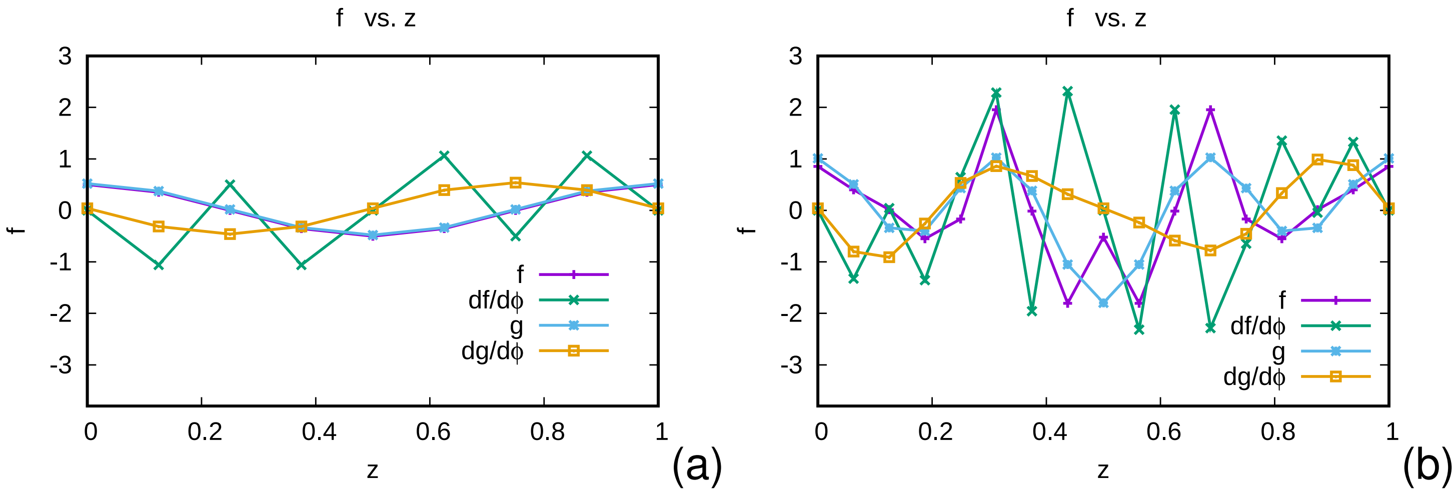
To ensure periodicity,

$$\sum_n V_n + V_{n+1} - 2V_{n+1/2} = 0 \tag{10}$$

To eliminate an arbitrary constant from the choice of V_1' in (9) require that

$$\sum_n V_n' = 0 \tag{11}$$

Example



f(z), f'(z), g(z), g'(z) (a) Method 1, low resolution (b) Method 2, high resolution Fig.(a) uses Method 1 on a mesh of 8 elements. The function $f(\phi)$ is

$$f(\phi) = \cos(\phi) + \cos(3\phi)/3 \tag{12}$$

$$\frac{df}{d\phi} = -\sin(\phi) - \sin(3\phi) \tag{13}$$

In the plots g and $dg/d\phi$ are the dealiased version of $f, df/d\phi$. Method 2 is illustrated in Fig.(b). The initial functions are

$$f(\phi) = \cos(\phi) + \cos(3\phi)/3 + \cos(6\phi)/6 \tag{14}$$

$$\frac{df}{d\phi} = -\sin(\phi) - \sin(3\phi) - \sin(6\phi) \tag{15}$$

Thin resistive wall

A similar approach can be used for a thin resistive wall. For toroidally symmetric wall, the Green's function [GRIN, Pletzer] can be Fourier analyzed

$$\frac{\partial \tilde{\psi}_{j,k}}{\partial n} = \sum_{l} G_{j,l,k} \tilde{\psi}_{l,k} \tag{16}$$

where

$$\tilde{\psi}_{l,k} = \sum_{n=1}^{N} \exp[ik(n-1)\Delta\phi]\psi_{l,n} \tag{17}$$

Taking the reverse transform

$$\frac{\partial \psi_{l,m}}{\partial n} = \sum_{m,j} G_{l,m,j,n} \psi_{j,n} \tag{18}$$

where indices j, l are poloidal points on the wall and m, n are toroidal locations,

$$G_{l,m,j,l} = Re\{\sum_{k} G_{l,m,k} \exp[ik(j-l)\Delta\phi]\} \tag{19}$$

The matrix $G_{l,m,j,l}$ only needs to be computed once.

There is no need for FFTs, no vacuum field solve at each time step.

Summary

- simulation of JET shot 71985
  - agreement with experiment is improving
- Code modifications
- modify resistivity profile in the wall to reduce resistivity gradient at the plasma
- dealiasing without FFT
- Thin resistive wall without FFT
- goal: to make AVDE simulations more robust