Modeling And Simulation Needs For Seeding And Growth Of NTMs*

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Issues to be addressed:

1) 2/1 neoclassical tearing modes (NTMs) can cause disruptions.

2) NTMs are seeded by ELMs (or sawteeth), then grow in time.

3) New NTM issues: sequential ELM responses and flow effects.

4) What is needed for extended MHD simulations of NTMs?

***Based in part on DIII-D NTM seeding analysis in poster BP10.00028 by J.D. Callen, R.J. La Haye, R.S. Wilcox, E.J. Strait, C. Chrystal, M. Okabayashi, E.C. Howell, C.C. Hegna, "How Are NTMs Seeded," APS-DPP meeting, Fort Lauderdale, FL, October 21-25, 2019.**

NTM Is Induced By Sequential ELMs In DIII-D

- **Neoclassical tearing mode (NTM) issues: 1) 2/1 NTMs can grow into locked modes and disruptions; 2) how they are seeded is not understood; 3) DIII-D data shows ELMs can induce NTM growth.**

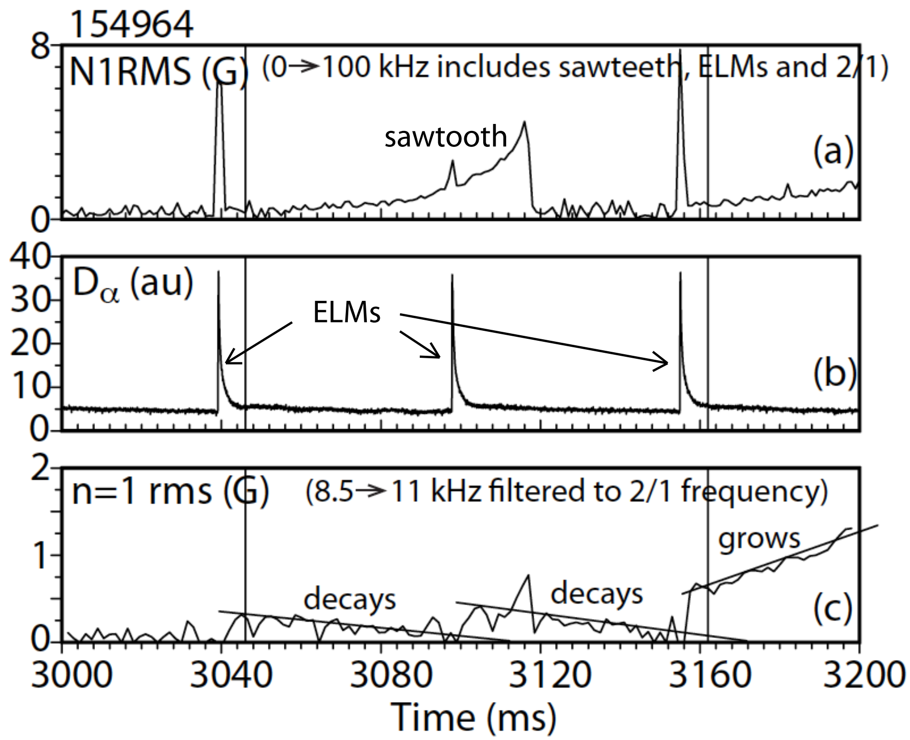

Figure 1: **Why do first two ELMs seed temporally decaying 2/1 modes? But third ELM seeds growing NTM that leads to a locked mode.**

MHD Transients Can Induce Effects At 2/1 Surface

- **Recent theory[1] predicts MHD transients abruptly induce radially local torque, radial electric field and flows that reduce the mode frequency and allow a metastable NTM to grow on the longer time scale determined by the modified Rutherford equation (MRE).**

[1] M. Beidler, J.D. Callen, C.C. Hegna, C.R. Sovinec, "Mode penetration induced by transient magnetic perturbations," Phys. Plasmas **25**, 082507 (2018).

- **Recent slab-model NIMROD modeling and theory[1] predict local responses to MHD transient δB_x, binormal torque, δE_x and flow:**

  $\delta B_x^{\rm MHD}(t)$ induces δB_x at 2/1 rational surface, which causes a local parallel current $\delta J_\|$ that produces a non-ambipolar $\delta J_\| \delta B_x$ torque "pulse;" this torque produces a radially outward electron particle flux that increases the radial electric field to maintain quasi-neutrality and local torque pulse (<1 ms) in + binormal direction (∼ poloidal).

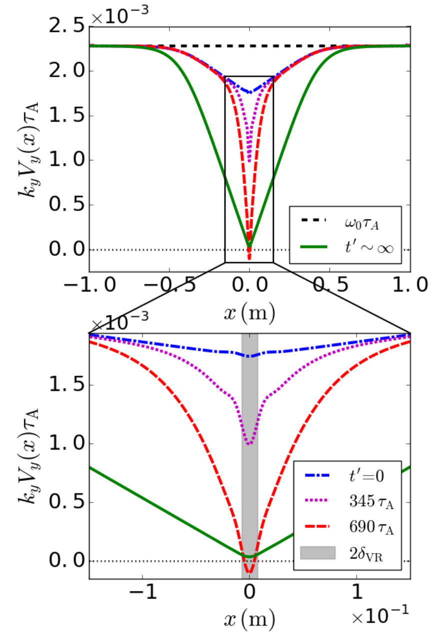

Figure 2: **$\delta x \sim 1$ cm flow response to MHD transient.**

Disruptions Caused By 2/1 NTMs Have Multiple Stages

General Comment: 2/1 NTMs cause the most problematic resistive-MHD-based disruptions.

- **First stage: slow evolution of current profile etc. which causes:**

  increase of NTM drives (classical Δ', bootstrap current) at $q = m/n$ surfaces, which for low q_{95} induces a sequence of NTMs, e.g., $4/3 \to 3/2 \to 2/1$.

- **Seeding stage: ELMs (or sawteeth) induce reconnection at $q = 2/1$:**

  MHD transients can induce reconnection, islands at $q = m/n$ surfaces,[1] successive ELMs can increase resonant perturbation δB_x and island width w, and if δB_x is large enough, NTM grows linearly in time, $\delta t \sim 30$ ms in DIII-D.

- **Third stage: Linearly growing NTM amplitude then precipitates**

  interaction with wall $\to$ reduced rotation $\to$ mode locking $\to$ disruption.

- **FOCUS HERE IS ON 2/1 NTM EVOLUTION IN DIII-D # 174446 WHERE n$^{\rm th}$ ELM EXCITES A ROBUSTLY GROWING 2/1 NTM.**

174446 Modespec Spectrogram Has Many Modes

- Top is Mirnov, middle is $\delta B_{\rm rms}$, and bottom is freq. spectrogram; green (4/3) & yellow (3/2) NTMs, red is $n = 1$ NTMs & sawteeth.

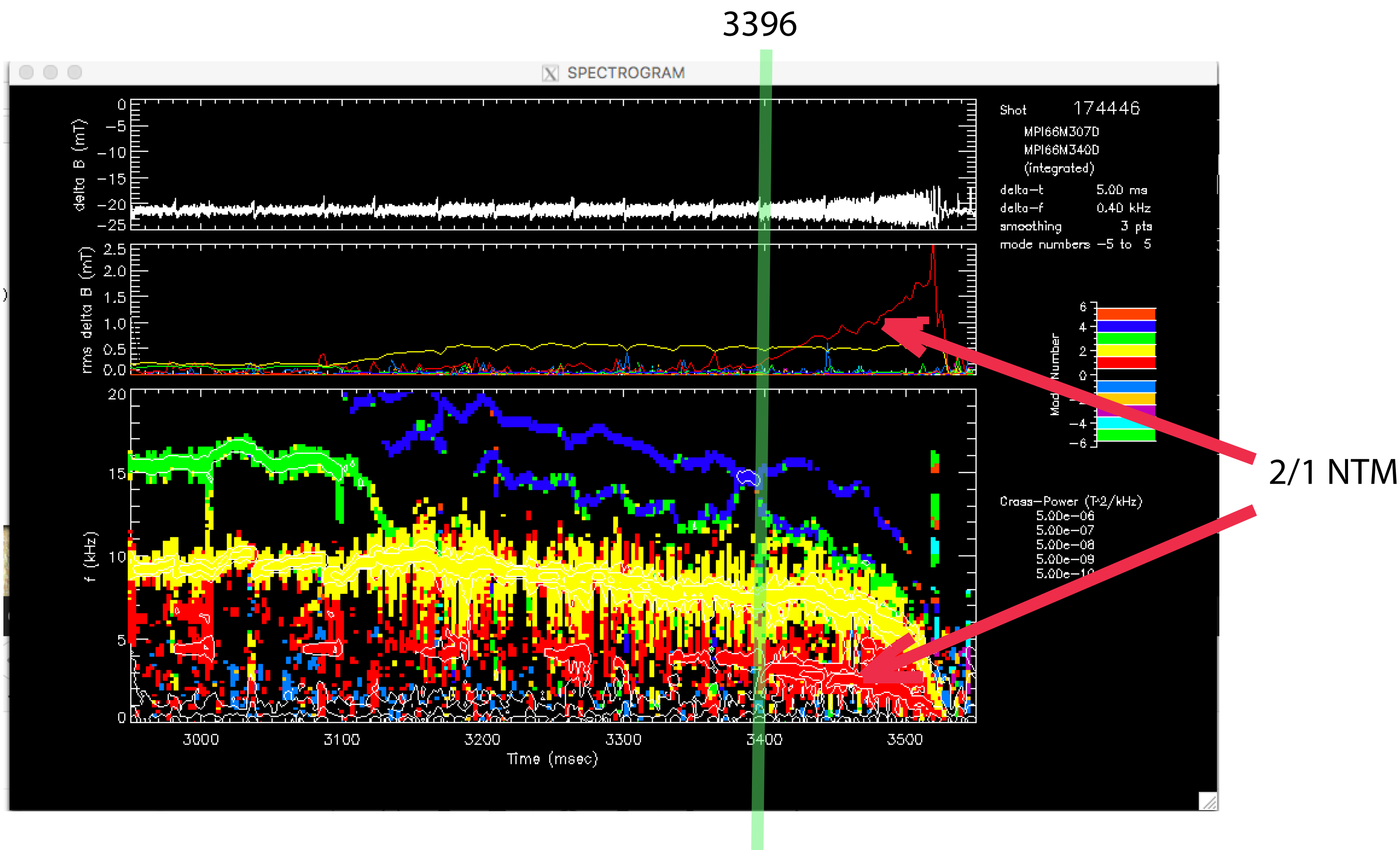

Figure 3: **2/1 NTM (red) begins growing robustly after ELM at $\sim$ 3396 ms.**

174446 Has Many ELMs Before NTM Grows

- Top is D_α (ELMs), second shows $n = 1$ mode frequency $f_{\rm m}$ (black) and 2/1 Er/RBp (blue) rotation frequency, third is $n = 1$ Mirnov $\delta B_{\rm rms}$ (black) and last is locked mode signal $\delta B_{\rm lock}$ (blue).

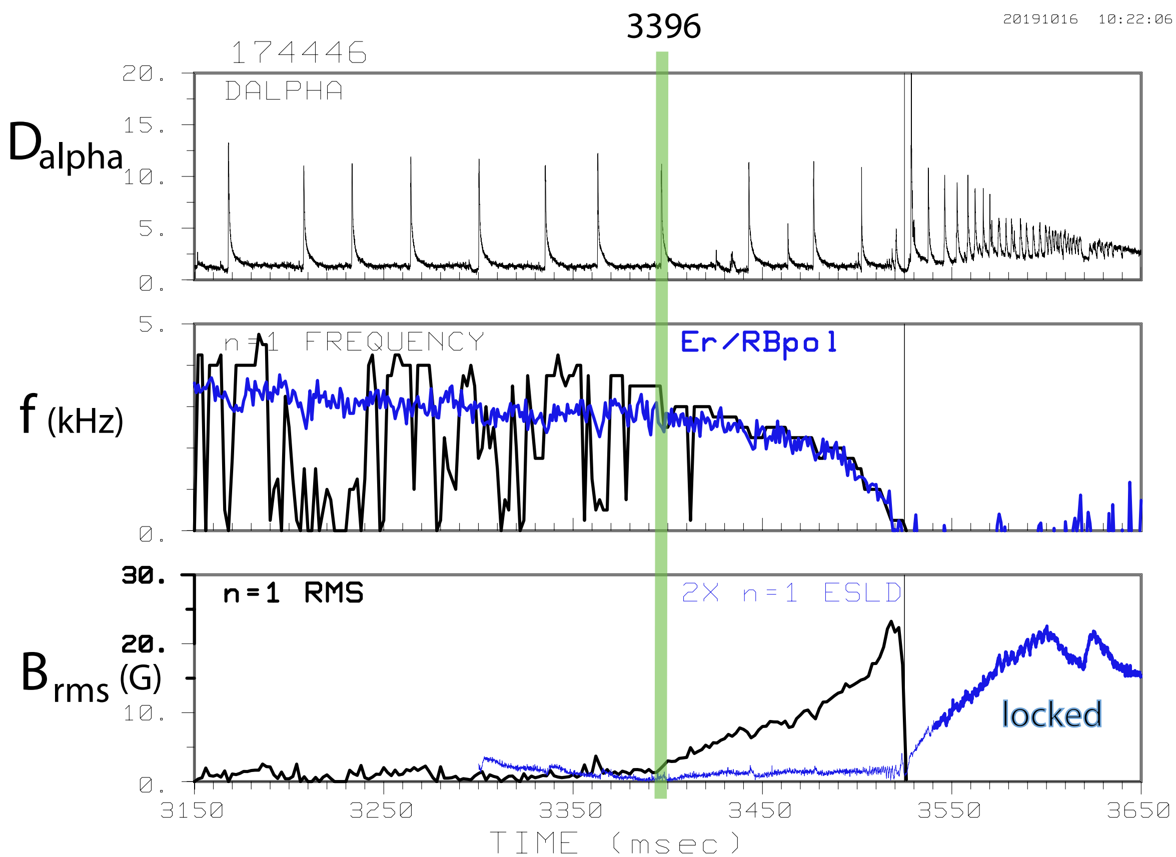

Figure 4: **174446 long time evolution; NTM grows robustly after ELM at 3396 ms.**

Final ELM At 3396 ms Excites Growing NTM

- Top is D_α (ELMs); second shows $n = 1$ frequency f_m (black) and 2/1 Er/RBp (blue) & plasma rotation (cyan) frequencies; third is $n = 1$ Mirnov δB_rms (black); and last is $d\,\delta B_\mathrm{rms}/dt$ (black) in G/s.

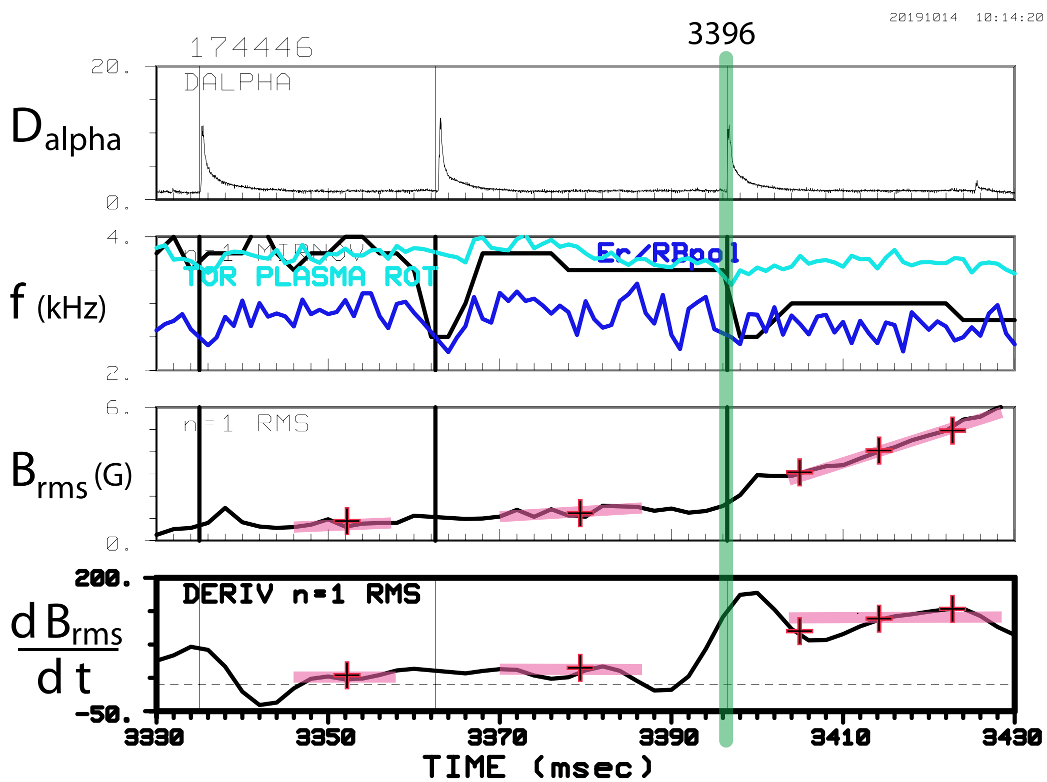

Figure 5: **As 2/1 NTM begins growing robustly, magnetics rotation drops close to *Er/RB*p. Pink lines highlight regimes of slow linear growth, then final robust growth. The + signs on pink curves are comparison times with theory on p 14.**

Magnetics Analysis Before 3396.5 & After 3396.8 ms Show 2/1 NTM, But Not During ∼ 0.4 ms ELM (green)

- **Top is D_α (ELM), bottom is externally measured $n = 1$ & $m = 2$ δB.**

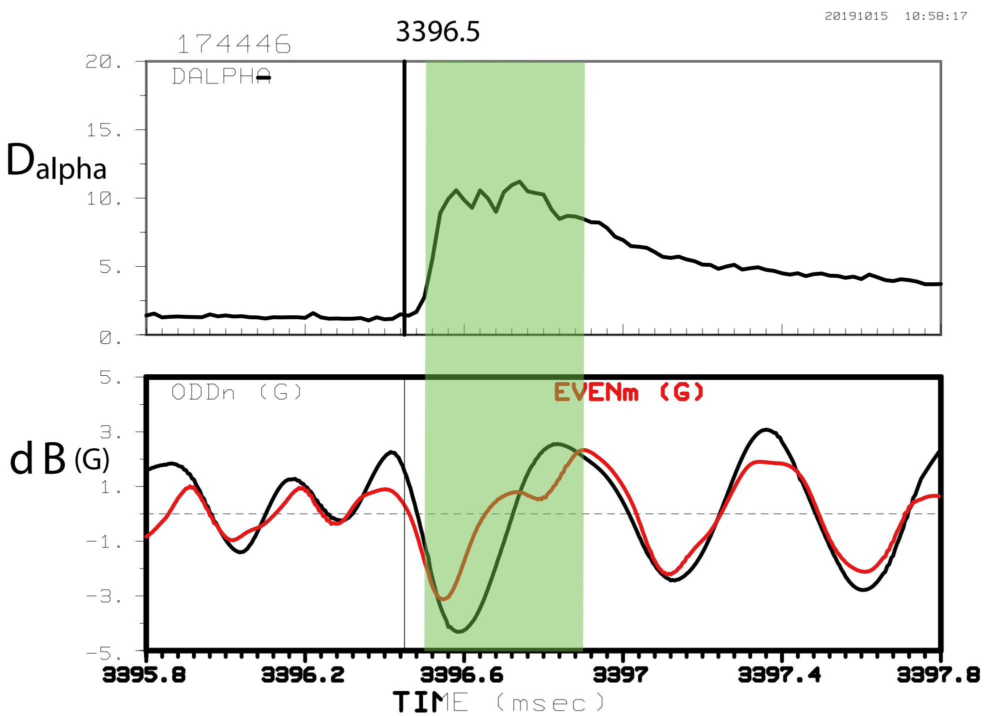

Figure 6: **Before and after the maximum ELM amplitude the EVENm (red, $m = 2$) and ODDn (black, $n = 1$) are in phase indicating a 2/1 NTM, which grows in amplitude and decreases in frequency from before to after ELM. During maximum ELM amplitude (green) they are not in phase — kink-tearing response?**

Toroidal Theory-Based NTM Model Has Many Aspects

- Parameters listed below are at 2/1 rational surface for discharge 174446 at 3390 ms — just before final ELM kicks off robust NTM; distances $\delta_{\rm VR}$, w etc. are 1.6 times smaller on outboard mid-plane.
- Resistivity causes resonant $\delta B_{\rm res}$ and magnetic reconnection in thin layers $\delta_{\rm VR} \simeq \rho_0 \, S_{\rm sh}^{-1/3} P_{\rm m}^{1/6} \lesssim 0.4$ cm at $q = m/n$ rational surfaces.
- External field errors cause very small flow-screened[1] equilibrium $\delta B_{\rm res}$ induced by $\delta B_{\rm FE}$ at 2/1 surface located at $\rho_0 \simeq 52$ cm:

$$\delta B_{\rm res} = \frac{\delta B_{\rm FE}}{-\rho_0 \Delta_0' + i\,\omega_0 \tau_{VR}} \simeq \frac{\delta B_{\rm FE}}{i\,\omega_0 \tau_{VR}} \simeq \frac{\delta B_{\rm FE}}{360\,i} \sim 3 \times 10^{-3}\, \delta B_{\rm FE}, \text{ in which}$$

$$\omega_0 \simeq k_\theta V_\theta + k_\zeta V_\zeta \simeq [(3.8)(-3.6) + (0.46)(50.9)] \times 10^3 \simeq 10^4\,{\rm s}^{-1},$$

$$\tau_{\rm VR} \equiv 2.104\, \tau_{\rm sh\,A} S_{\rm sh}^{2/3} P_{\rm m}^{1/6} \simeq 0.036\,{\rm s}.$$

- Fast ($\delta t \lesssim 1/|\omega_0| \sim 0.1$ ms) MHD events (ELMs, sawteeth) induce[1]

$$\delta B_{\rm res}(x,t) \simeq \delta B_{\rm x,0}\,(1 + i\omega_0 t + \cdots) + \delta B_{\rm MHD}\left[\,(t/\tau_{\rm VR}) + i\,(\omega_0 t^2/2\tau_{\rm VR}) + \cdots\right],$$

$\delta B_{\rm x,0}$ is initial resonant field and $\delta B_{\rm MHD} \equiv C_{\delta B}\, \delta B_{\rm rms}$ is MHD-induced transient, in which $C_{\delta B} \simeq 2.16$ is ratio of $\delta B_{\rm MHD}$ at 2/1 surface to $\delta B_{\rm rms}$ at Mirnov coil; purple terms represent out of phase terms that contribute to $\delta J_{\|} \delta B_{\rm res}$ torque.

NTM Model $\delta J_\| \delta B_{\rm res}$ Torque Causes $\delta V_{iy} \to \delta V_{i\theta}$

- Since $\delta J_\| \simeq (i/k_\theta \mu_0)\, \partial^2 \delta B_{\rm res}/\partial x^2$, the out-of-phase $\delta B_{\rm res}$ produces em torque pulse in visco-resistive (VR) reconnection layer $\delta_{\rm VR}$:[1]

$$\delta J_\| \delta B_{\rm res} \to -\frac{1}{2\,\mu_0 k_\theta}\left[\delta B_{x,0}\, \mathcal{I}m\{\delta B_{\rm res}\}\right] \simeq -\frac{(\delta B_{\rm MHD}/2 + \delta B_{x,0})\,(\delta B_{\rm MHD} + \delta B_{x,0})}{2\mu_0\, \delta_{VR}\, (k_\theta \rho_0)\, (\omega_0 \tau_{\rm VR})} < 0,$$

  within full $2\,\delta_{\rm VR} \simeq 0.8$ cm reconnection layer in time $\delta t \sim 1/|\omega_0| \lesssim 0.1$ ms, which induces an electron flux $\delta\Gamma_e \equiv -\,\delta J_\| \delta B_{\rm res}/e > 0$ that causes $\delta E_x > 0$, in order to preserve quasi-neutrality in the plasma.

- $\delta J_\| \delta B_{\rm res}$ transiently induces a non-ambipolar pulse in the ion flow:

$$\rho_m \frac{\delta V_{i\,y}}{\delta t} \simeq \delta J_\| \delta B_{\rm res} < 0, \quad \text{in slab binormal } (\vec{e}_y = \vec{e}_z \times \vec{e}_x) \text{ direction.}$$

- Binormal direction in DIII-D is in roughly poloidal $(-\,\theta)$ direction. Thus, poloidal flow evolution equation becomes $(D_\mu \simeq 1.3\ {\rm m^2/s})$

$$\rho_m \frac{\partial V_{i\theta}}{\partial t} \simeq -\,\rho_m \mu_i^{\rm nc}(V_{i\theta} - V_{i\theta}^{\rm nc}) + \rho_m D_\mu \frac{\partial^2 V_{i\theta}}{\partial x^2} - \delta J_\| \delta B_{\rm res}\,, \quad \mu_i^{\rm nc} \sim 1.6 \times 10^3/{\rm s}.$$

- There are two interesting limits of the poloidal flow equation:

  neoclassical poloidal flow damping dominates in equil.: $V_{i\theta} \simeq V_{i\theta}^{\rm nc} \simeq -\,1.3$ km/s,

  large $\delta J_\| \delta B_{\rm res} \Longrightarrow \delta V_{i\theta}(x,t) \simeq -\,V_y(x,t)$ in Fig. 2 (p 3): $\delta x \sim 2$ cm for $\delta t \lesssim 1$ ms.

NTM Model $\delta J_{\|}\delta B_{\rm res}$ Torque Affects Toroidal $\omega_{\rm t}$ Less

- Radial ion force balance is applicable for all times $\tau_A \gtrsim 10^{-7}$ s:

$$\omega_{\rm t} \equiv \vec{V}_i\cdot\vec{\nabla}\zeta = -\left[\frac{d\Phi_0}{d\psi_{\rm p}} + \frac{1}{n_{i0}q_i}\frac{dp_{i0}}{d\psi_{\rm p}}\right] + q\vec{V}_i\cdot\vec{\nabla}\theta \simeq \frac{E_\rho}{RB_{\rm p}} - \frac{1/(RB_{\rm p})}{n_i Z_i e}\frac{dp_i}{d\rho} + \frac{I}{R^2}U_{i\theta}.$$

- Toroidal rotation $\omega_{\rm t}$ is determined by toroidal momentum equation:[2]

$$\rho_m\frac{\partial\,\omega_{\rm t}}{\partial t} \simeq \rho_m D_\mu\frac{\partial^2\omega_{\rm t}}{\partial x^2} + \frac{S_{\zeta{\rm NBI}}}{R_0} + \frac{B_{\rm pol}}{R_0 B_{\rm t0}}\,\delta J_{\|}\delta B_{\rm res} + C_{\rm w}\,\frac{v_{Ti}}{R_0 q}\,\frac{{\rm w}(\delta B_{\rm res})}{\rho_0}\;.$$

- Transient $\omega_{\rm t}$ response to $\delta J_{\|}\delta B_{\rm res}$ torque induced by MHD events is similar to $\delta V_{i\theta}$, but smaller by factor $B_{\rm pol}/B_{\rm t0} \simeq \rho_0/R_0 q \simeq 0.15$.

- Equilibrium plasma rotation with no $\delta B_{\rm rms}$ effects is determined by NBI torque balanced by ITG - induced $\perp$ momentum diffusivity:

$$\rho_m\frac{\partial\,\omega_{\rm t}}{\partial t} \simeq \rho_m D_\mu\frac{\partial^2\omega_{\rm t}}{\partial x^2} + \frac{S_{\zeta{\rm NBI}}}{R_0} \simeq -\rho_m\frac{\omega_{\rm t} - \omega_{\rm t}^{\rm eq}}{\tau_\zeta}, \quad \text{in which } \tau_\zeta \simeq \frac{a^2}{4D_\mu} \sim 0.1 \text{ s}.$$

[2]Eq. (49) in A.I. Smolyakov, A. Hirose, E. Lazzaro, G.B. Re, J.D. Callen, "Rotating nonlinear magnetic islands in a tokamak," Phys. Pl. **2**, 1581 (1995).

NTM Model Modified Rutherford Eq. Predicts w(t)

- Island evolution governed by modified Rutherford equation (MRE):

$$\frac{d\mathrm{w}}{dt} = \frac{\overline{D}_\eta}{\rho_0}\left[\underbrace{\rho_0\,\Delta_0'}_{-\,0.1} + \overbrace{\underbrace{\frac{\rho_0}{\mathrm{w}}}_{11}\underbrace{\mathrm{d_{NTM}}}_{0.44}}^{**********} - \overbrace{\underbrace{\frac{\rho_0 \mathrm{w}_{\rm pol}^2}{\mathrm{w}^3}}_{5.5}\underbrace{F(f_{\rm m})}_{\lesssim\,1}}^{**********} + \underbrace{\cdots}_{\ll 1}\right],$$ when electron fluid is locked to Δ_0', $\mathrm{d_{NTM}}$,

$\overline{D}_\eta \equiv \langle|\vec{\nabla}\rho|^2\rangle\eta_\parallel^{\rm nc}/\mu_0$, effective magnetic field diffusivity, $\overline{D}_\eta \simeq 2200$ cm^2/s,

$\Delta_0' \simeq -\,0.002$ cm^{-1}, tearing response index, $\rho_0\,\Delta_0' \simeq 0.1$,

$\mathrm{d_{NTM}} \propto j_{\rm boot}/\langle j_\parallel\rangle$, bootstrap current drive (including GGJ), $\mathrm{d_{NTM}} \simeq 0.44$,

$\mathrm{w}(t) \equiv 4\,[L_{\rm sh}\,\delta B_{\rm res}(t)/k_\theta B_0]^{1/2}$, magnetic island width, $\mathrm{w}_0 \simeq 4.6$ cm,

$\mathrm{w_{pol}} = C_{\rm pol}\,\mathrm{w_{ib}}$, polarization current width, $C_{\rm pol} \simeq 2.3 \to$ $\mathrm{w_{pol}} \simeq 3.2$ cm,

$\mathrm{w_{ib}} \equiv \varrho_i\, q/\sqrt{\epsilon_B}$, ion banana width, $\mathrm{w_{ib}} = 1.4$ cm,

$F(f_{\rm m}) \equiv \frac{(f_{\rm m}-f_E)\,(f_{\rm m}-f_E-f_{*i})}{f_{*i}^2}$, $f_{\rm m} \equiv f_{\rm mag}$ effect on ion pol. current, $F(f_{\rm m}) \lesssim 1$,

$f_E \equiv \frac{E_\rho}{2\pi RB_{\rm p}}, \frac{Er}{RB_{\rm p}}$ freq., $f_{*i} = \frac{1}{n_i Z_i e}\frac{dp_i}{d\psi_{\rm p}}$ ion diamag., $f_E \simeq 2.5$ kHz, $f_{*i} \simeq -2.0$ kHz.

- Keeping dominant terms, lowest order MRE neglecting $\Delta_0', \cdots$ is

$$\boxed{\frac{d\mathrm{w}}{dt} \simeq \overline{D}_\eta\left[\frac{\mathrm{d_{NTM}}}{\mathrm{w}} - \frac{\mathrm{w}_{\rm pol}^2}{\mathrm{w}^3}F(f_{\rm m})\right],}$$ for both marginal and robust NTM growth.

The NTM Mode Frequency Is Important In The MRE

- NTM grows if magnetic island induced by $\delta B_{\rm res}$ is large enough:

$$\frac{d{\rm w}}{dt} \simeq \overline{D}_\eta \left[\frac{{\rm d_{NTM}}}{\rm w} - \frac{{\rm w}_{\rm pol}^2}{{\rm w}^3} F(f_{\rm m})\right] \qquad \boxed{> 0 \;\text{ if }\; {\rm w}^2 \equiv \left[\frac{16\, L_{\rm sh}}{k_\theta\, B_{\rm t0}}\right] \delta B_{\rm res} > \frac{{\rm w}_{\rm pol}^2\, F(f_{\rm m})}{{\rm d_{NTM}}}.}$$

- NTM mode frequency $f_{\rm m}$ is critical for determining $F(f_{\rm m})$ and hence magnitude of effect of stabilizing ion polarization current:

$$F(f_{\rm m}) \equiv \frac{(f_{\rm m} - f_E)\,(f_{\rm m} - f_E - f_{*i})}{f_{*i}^2}, \quad \text{mode freq. dependence of pol. current effect.}$$

- Toroidal torque balance determines NTM mode freq. $f_{\rm m}$ (prelim.):[2]

$$\frac{d\,f_{\rm m}}{dt} \simeq -\frac{(f_{\rm m} - f_{\rm t})}{\tau_\zeta} - \frac{(f_{\rm m} - f_{\rm offset})}{\tau_{\rm w}}, \quad \text{where } \frac{1}{\tau_\zeta} \sim 10, \ \frac{1}{\tau_{\rm w}} \sim \frac{\sqrt{\pi}\,\hat{s}}{2q^3}\left[\frac{r_0}{R_0}\right]^4 \frac{v_{Ti}}{R_0 q}\,\frac{\rm w}{r_0} \sim 67\frac{\rm w}{r_0}. \quad (1)$$

- $f_{\rm m}(t)$ response to $\delta j_\|\delta B_{\rm res}$ during ELM is determined differently.[1]

- There are two limiting regimes of (1) for NTM mode frequency $f_{\rm m}$:

  if island width w is small (i.e., $\tau_{\rm w} \gg \tau_\zeta$), equilibrium has $f_{\rm m} \simeq f_{\rm t} \to F(f_{\rm m}) \sim 1$;

  but if w is large enough (i.e., $\tau_{\rm w} \ll \tau_\zeta$), $f_{\rm m} \simeq f_{\rm offset} \sim f_E - f_{*i} \to F(f_{\rm m}) \lesssim 0.3 \ll 1$, then boxed equation is easily satisfied and MRE yields robust NTM growth.

Plots Of MRE $d\,\delta B_{\rm rms}/dt$ & F vs. $\delta B_{\rm rms}$ Are Useful

- **The 174446 data obtained from experimental data for $\delta B_{\rm rms}(t)$ in Fig. 5 on p 7 are shown as large + signs in Figs. 7 and 8 below.**

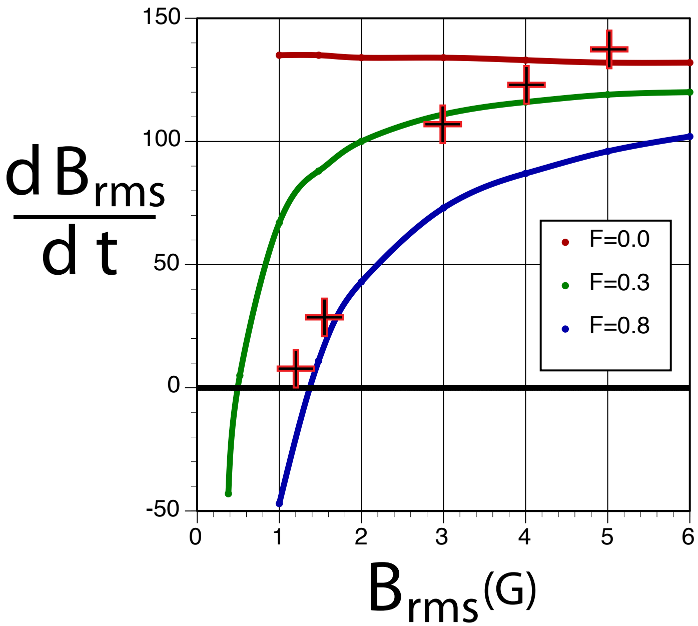

Figure 7: **Blue, $F = 0.8$, Green, $F = 0.3$, Red, $F = 0.0$ curves plot rate of change $d\,\delta B_{\rm rms}/dt$ as a function of the externally measured $\delta B_{\rm rms}$ obtained from modified Rutherford equation (MRE).**

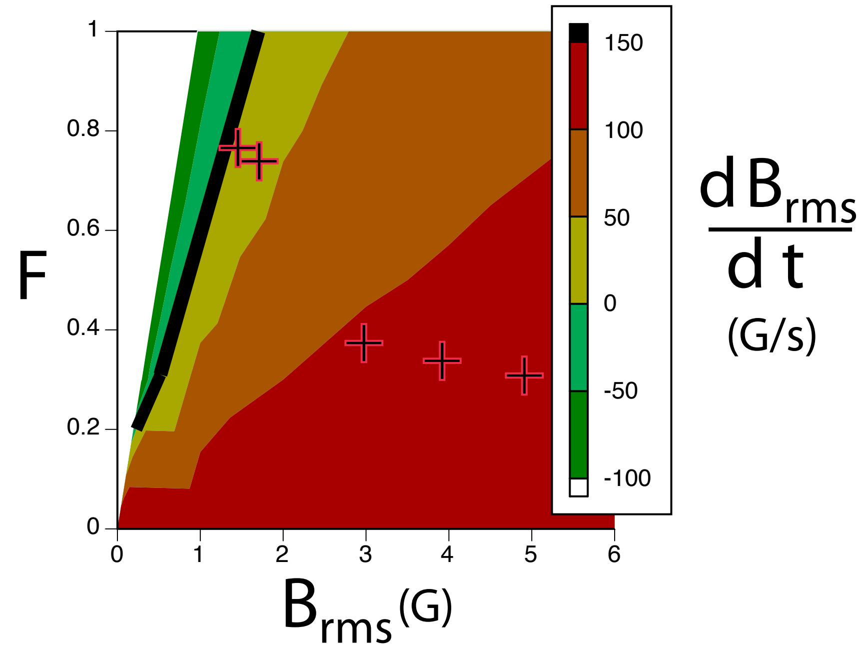

Figure 8: **Dependence of ion polarization current factor $F(f_{\rm m})$ on externally measured $\delta B_{\rm rms}$. Color scale represents rate of growth $d\,\delta B_{\rm rms}/dt$ in G/s. Rate of growth is largest when factor $F(f_{\rm m})$ is smallest. Black line is $d\delta B_{\rm rms}/dt = 0$.**

Criteria For NTM Robust Growth Are Identified

- **Two stages of NTM seeding by ELMs identified in 174446:**

  A) "marginal" slow NTM growth after a preceding ELM, then

  B) "final" robust NTM growth after last ELM reduces 2/1 mode rotation $f_{\rm m}$.

- **Approximate criteria for a particular ELM to produce NTM seeding in stages A and B have different scalings:**

  A) "marginal" NTM growth occurs for $d_{\rm NTM} \sim 2\,\dfrac{j_{\rm boot}}{\langle j_\parallel \rangle} \gtrsim \left[\dfrac{C_{\rm pol}\,w_{\rm ib}}{w_0}\right]^2 \propto \varrho_{*i}^2\,,$

  which can be written alternatively as $$\boxed{\frac{\delta B_{\rm rmp0}^{\rm crit}}{B_{\rm t0}} \gtrsim \frac{w_{\rm pol}^2\,/B_{\rm t0}}{C_{\rm m^2/T}\,d_{\rm NTM}} \propto \frac{\varrho_{*i}^2}{d_{\rm NTM}}} \quad (2).$$

  B) "robust" NTM growth more easily satisfies $d_{\rm NTM} \gtrsim 0.3\,C_{\rm pol}^2 w_{\rm ib}^2/w_0^2$, but island width $w_0(\delta B_{\rm rms0})$ from Eq. (1) can reduce 2/1 $f_{\rm m}$ below $f_{\rm t}$ reducing F or island growth continues; in both cases Eq. (2) becomes ultimate criterion.

- **Equation (2) predicts smaller $\delta B_{\rm rms}^{\rm crit}$ in larger, higher field tokamaks.**

SUMMARY: NTM Complete Modeling Has Many Facets

- **Transport evolution of discharge requires modeling of many things:**
  up to and including current diffusion time scale,
  including neoclassical poloidal ion flow damping,
  including not just transport of n, T but also $\omega_{\rm pol}$ and $\omega_{\rm tor}$,
  realistic representation of real tokamak geometry and control coils.

- **NTM seeding requires attention to responses to MHD transients:**
  means for introducing external MHD-type transient events,
  critical physics for NTMs — significant bootstrap current, ion bananas,
  $\chi_\parallel$ flattening of profiles in island, ion diamagnetic flow effects,
  experimental param. — resolve thin layers $\lesssim$ 1 cm, $S \sim 10^7$, $\chi_\parallel/\chi_\perp \sim 10^8$.

- **NTM evolution into large amplitude nonlinear regimes:**
  interaction with wall $\rightarrow$ reduced rotation $\rightarrow$ mode locking $\rightarrow$ disruption.

- **TWO QUESTIONS:**
  are NIMROD & M3D-C1 up to challenge of modeling 2/1 NTMs to disruption?
  who will develop useful theory-based reduced models of nonlinear NTMs?

Supplementary Viewgraphs

1 ms CER etc. Data In Hand For Some Discharges

- **DIII-D discharge 174446 has best data and a growing 2/1 NTM:**

  param.: LSN ISS ($\kappa = 1.9$, $\delta = 0.39$), $B_{t0} = 2$ T, $I_p = 1.51$ MA, $I_{bs} = 0.28$ MA, $\beta_p = 0.74$, $\beta = 0.025$, $\beta_N \simeq 1.7$, $\ell_i = 0.75$, $P_{NBI} = 5.6$ MW, $q_{95} = 3.4$, $q_0 \simeq 1$, $a_{eff} \simeq 75$ cm, $a_{EFIT} \equiv (R_{sep}^{out} - R_{sep}^{in})/2 \simeq 56$ cm, no β_N feedback,

  data: 1 ms vertical & tangential CER, great magnetics, MSE, Thomson, $\cdots$

- **There are other discharges with NTMs we are beginning work on:**

  154964 (ELM excited, motivating slide on p 4), 154986 (sawtooth driven),

  174452 (NBI pulse excites), 174454 (sawtooth driven).

- **FOCUS HERE IS ON 2/1 NTM EVOLUTION IN 174446 WHERE n^{th} ELM EXCITES A ROBUSTLY GROWING 2/1 NTM.**

Model Prediction For $\delta B_{\rm res}$ Grows Linearly In Time

- The square of the island width w can be written in terms of $\delta B_{\rm res}$:

$$w^2 = \frac{\partial\, w^2}{\partial\, \delta B_{\rm res}}\, \delta B_{\rm res}, \quad \text{in which } \frac{\partial\, w^2}{\partial\, \delta B_{\rm res}} \equiv \frac{16\, L_{\rm sh}}{k_\theta\, B_{\rm t0}} \simeq 6.35 \text{ m}^2\ \text{T}^{-1}.$$

- Approximating w & f_m in polarization current term by ELM-induced values w_0 & f_{m0} at t_0, lowest order MRE on p 16 can be written as an equation for externally measured $\delta B_{\rm rms} = \delta B_{\rm res}/C_{\delta B}$ ($C_{\delta B} \simeq 2.16$):

$$\frac{d\,\delta B_{\rm rms}}{dt} \simeq \left.\frac{d\,\delta B_{\rm rms}}{dt}\right|_{t_0}, \quad \text{in which the constant is } \left.\frac{d\,\delta B_{\rm rms}}{dt}\right|_{t_0} \equiv \frac{2\, \overline{D}_\eta / C_{\delta B}}{\partial w^2/\partial\, \delta B_{\rm res}} \left[d_{\rm NTM} - \frac{w_{\rm pol}^2}{w_0^2}\, F(f_{m0}) \right].$$

- Predictions for $d\,\delta B_{\rm res}/dt$ in robustly growing as well as marginally growing 2/1 NTM regimes roughly agree with those observed in discharge 174446 in Fig. 1 (p3) (prelim. + numbers, see also Fig. 8):

marginal 3346–3358 ms: $F(f_{m0}) \sim 1 \to \left.\frac{d\,\delta B_{\rm rms}}{dt}\right|_{\rm th} \sim 0$ G/s vs. $\left.\frac{d\,\delta B_{\rm rms}}{dt}\right|_{\rm exp} \simeq 10$ G/s,

slow 3370–3386 ms: $F(f_{m0}) \sim 1 \to \left.\frac{d\,\delta B_{\rm rms}}{dt}\right|_{\rm th} \sim 25$ G/s vs. $\left.\frac{d\,\delta B_{\rm rms}}{dt}\right|_{\rm exp} \simeq 30$ G/s,

robust 3405–3430 ms: $F(f_{m0}) \sim 0.3 \to \left.\frac{d\,\delta B_{\rm rms}}{dt}\right|_{\rm th} \sim 127$ G/s vs. $\left.\frac{d\,\delta B_{\rm rms}}{dt}\right|_{\rm exp} \simeq 135$ G/s.

NTM Physics Is Different From Slab Model Physics

- **Magnetic geometry is different — toroidal 3D not 2D (slab):**
  however, most magnetic reconnection physics is captured by slab model, and the 3D geometry mainly just changes geometry metric element coefficients.

- **ELM/sawtooth-induced resonant field $\delta B_{x,2/1}$ dynamics is similar.**[a]

- **But additional physical processes are needed for describing NTMs:**
  poloidal and toroidal flows are different and respond differently to em forces, poloidal ion flow equilibrium is governed by neococlassical flow damping, and perturbed bootstrap current drives temporal growth of NTM islands.

- **Key seeding physics for NTMs is different from RMP locking:**[3]
  MHD seeding of NTM needs $\delta J_\| \delta B_x$ force to cause $\omega - \omega_E \sim \omega_{*i}$ (NTM freq.), instead of having the $\delta J_\| \delta B_x$ force lock toroidal flow to external RMP field.

- **Nonlinear evolution governed by more complicated MRE that adds bootstrap current drive and polarization-current-induced threshold.**

[3] C. Paz-Soldan et al., "Observation of a Multimode Plasma Response and its Relationship to Density Pumpout and Edge-Localized Mode Suppression," Phys. Rev. Lett. **114**, 105001 (2015).

Status and Current Issues For Modeling Of NTMs

- **Analysis since mid 1990s[4] is mostly based on cylindrical models.**

- **Analysis of NTMs in various tokamaks has been carried out[5] over the past two decades mostly using cylindrically-based models.**

- **Toroidal models of tokamak plasma flows[6] and transport equations[7] with 3D field effects are available → more precision & details feasible with kinetic EFIT05 equilibria, ONETWO, OMFIT analyses?**

- **In our NTM seeding studies both cylindrical and developing toroidal models are being used. Their main points agree within ∼ 20 %.**

- **Notable current issues for understanding, quantifying NTMs are:**
  can transient flows induced by MHD events (ELMs, sawteeth) be predicted?
  can δB_θ(t) be predicted? — marginal growth and robust growth regimes?
  how are they seeded? — by MHD transients, grow "out of noise," or ?
  is magnetic rotation drop from plasma rotation key for seeding robust growth?

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[4] O. Sauter, R.J. La Haye, Z. Chang, D.A. Gates, Y. Kamada, H. Zohm et al., "Beta limits in long-pulse tokamak discharges," Phys. Pl. **4**, 1654 (1997).
[5] R.J. La Haye, "Neoclassical tearing modes and their control," Phys. Plasmas **13**, 055501 (2006).
[6] J.D. Callen, A.J. Cole, C.C. Hegna, "Toroidal flow and radial particle flux in tokamak plasmas," Phys. Pl. **16**, 082504 (2009); Err. **20**, 069901 (2013).
[7] J.D. Callen, C.C. Hegna, and A.J. Cole, "Transport equations in tokamak plasmas," Phys. Plasmas **17**, 056113 (2010).

NTM Modeling: Nonlinear Magnetic Islands Occur

- **Key parameters of the local sheared magnetic field geometry are**

  magnetic shear length $L_{\rm sh} \equiv \frac{R_0 q_0}{\hat{s}_0} \simeq 3.1$ m, for $q_0 = 2$, $\hat{s}_0 \equiv \frac{\rho_0}{q_0} \frac{dq}{d\rho}\big|_{\rho_0} \simeq 1.1$,
  poloidal wavenumber $k_\theta \equiv m/\rho_0 = 2/0.52 \simeq 3.8\ {\rm m}^{-1}$.

- **Full magnetic island width induced by $\delta B_{\rm res}$ at 2/1 surface is**

  $$\mathrm{w}(t) \equiv 4\sqrt{\frac{L_{\rm sh}}{k_\theta}\frac{\delta B_{\rm res}(t)}{B_{\rm t0}}} \simeq 4.7\,{\rm cm} \text{ in 174446 at 3390 ms where } \delta B_{\rm res} \simeq 3.4\times 10^{-4}\,{\rm T}.$$

- **When island width w is larger than the full magnetic reconnection layer width $2\,\delta_{\rm VR}$, one is in the nonlinear, Rutherford regime:**

  criterion $\mathrm{w} > 2\,\delta_{\rm res} \longrightarrow \delta B_{\rm res} > \frac{k_\theta}{L_{\rm sh}} \frac{B_{\rm t0}}{4} \delta_{\rm VR}^2 \simeq 10^{-5}\,{\rm T} = 0.1\,{\rm G}$ (very small)
  that is typically below noise-determined δB_θ detection level ~ 0.5 G (@ 3 kHz).

- **Since $\mathrm{w}/2\,\delta_{\rm VR} \sim 6$, we must use the modified Rutherford equation to determine temporal evolution of the magnetic island width $\mathrm{w}(t)$ and predict external Mirnov perturbation $\delta B_{\rm rms}(t) \equiv \delta B_{\rm res}(t)/C_{\delta B}$.**