A continuum approach to the relativistic electron drift kinetic equation in NIMROD*

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Goal: consistent and comprehensive coupling between plasma fluid codes and relativistic electron kinetics.

- NIMROD can be used to solve non-relativistic drift kinetic equations (DKEs).
- Can this capability be adapted to study relativistic electrons generated during a disruption?
- What's been done so far?
- What's next?

Propose NIMROD as a tool for doing continuum relativistic drift kinetics.

Outline

- Existing capability: non-relativistic.
- Existing capability: relativistic.
- Plans.

A Tale of Two Cities

- Göteborg: Get the kinetics right. Study dynamics in a 2D relativistic phase space but use simplified models of what's happening spatially.
- Ft. Lauderdale: Get the fluid (MHD) right. Capture 3D magnetic evolution but use simplified, relativistic electron fluid model (0-D phase space).

Chapman-Enskog-like (CEL) approach provides consistent closure for fluid equations.

- In the Ramos theory(Ramos, *Phys Plasmas* **17**, 082502 (2010)), low-order fluid moments written as

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{u} = 0$$

$$mn(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}) - qn(\mathbf{E} + \mathbf{u} \times \mathbf{B}) + \nabla(nT) + \nabla \cdot [\pi_{\|}(\mathbf{bb} - \mathbf{I}/3)] - \mathbf{F}^{\text{coll}} = 0$$

$$\frac{3n}{2}\frac{dT}{dt} + nT\nabla \cdot \mathbf{u} + \nabla \cdot (q_{\|}\mathbf{b} + \frac{5nT}{2qB}\mathbf{b} \times \nabla T) - G^{\text{coll}} - \pi_{\|}[\frac{1}{3}\nabla \cdot \mathbf{u} - \mathbf{bb} \cdot \nabla \mathbf{u}] = 0$$

Closure moments computed from solution to CEL-DKE.

- Assume $f = f_{\mathrm{M}} + f_{\mathrm{NM}}$ with $\bar{f}_{\mathrm{NMe}} = O(\delta^2 f_{\mathrm{Me}})$ and $\bar{f}_{\mathrm{NMi}} = O(\delta f_{\mathrm{Mi}})$.
- Write CEL-DKE in the fluid frame (Ramos, *Phys Plasmas* **17**, 082502 (2010)):

$$
\begin{aligned}
&\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial t} + v'_{\parallel}\mathbf{b}\cdot\nabla\bar{f}_{\mathrm{NM}} - \frac{1-\xi^2}{2\xi}v'_{\parallel}\mathbf{b}\cdot\nabla\ln B\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial \xi} \\
&+\frac{v_0}{2}(\mathbf{b}\cdot\nabla\ln n)[\xi\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial s} + \frac{1-\xi^2}{s}\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial \xi}] - s[\xi\mathbf{b}\cdot\nabla + \frac{\partial}{\partial t}]\ln v_0\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial s} = \langle C(f)\rangle \\
&+\Big[(\frac{5}{2} - s^2)v'_{\parallel}\mathbf{b}\cdot\nabla\ln T + \frac{v'_{\parallel}}{nT}\mathbf{b}\cdot[\frac{2}{3}\nabla\pi_{\parallel} - \pi_{\parallel}\nabla\ln B - \mathbf{F}^{\mathrm{coll}}] \\
&+2s^2(\frac{3}{2}\xi^2 - \frac{1}{2})[\frac{1}{3}\nabla\cdot\mathbf{u} - \mathbf{bb}\cdot\nabla\mathbf{u}] + \frac{2}{3nT}(s^2 - \frac{5}{2})[\mathbf{b}\cdot\nabla q_{\parallel} - q_{\parallel}\mathbf{b}\cdot\nabla\ln B - G^{\mathrm{coll}}] \\
&+\frac{2}{3eB}s^2(\frac{3}{2}\xi^2 - \frac{1}{2})[(\frac{5}{2} - s^2)(\nabla\ln B - 2\kappa) + \nabla\ln n]\cdot\nabla T\times\mathbf{b} \\
&+\frac{4}{3eB}(\frac{s^4}{2} - \frac{5}{2}s^2 + \frac{15}{8})(\nabla\ln B + \kappa)\cdot\nabla T\times\mathbf{b}\Big]f_M
\end{aligned}
$$

Random questions

- Is a non-relativistic, electron CEL-DKE (and associated closures) compatible with evolving a separate population of kinetically-treated, relativistic electrons?
- Does a relativistic CEL-DKE theory for the entire distribution of electrons exist?
- Or, should entire electron population be treated relativistically using kinetics?

Adapt existing velocity space representation to relativistic phase space.

- With $s = v/v_0$ and $\xi = v_{\|}/v$, expansion for non-relativistic f is

$$f(R, Z, \phi, \xi, s, t) = \sum_i f_{i,n=0}(\xi, s, t)\alpha_{i,n=0} + \sum_{i,n>0} f_{i,n}(\xi, s, t)\alpha_{i,n} + f^*_{i,n}(\xi, s, t)\alpha^*_{i,n},$$

where $\alpha_{i,n} \equiv \psi_i(x, y)\exp(in\phi)$ and (R, Z, ϕ) are cylindrical spatial coordinates.

- With $s = \gamma mv/mc = p/mc = \gamma v/c$ and $\xi = p_{\|}/p$, expansion for relativistic f is

$$f(R, Z, \phi, \xi = p_{\|}/p, s = \gamma v/c, t) = \sum_i f_{i,n=0}(\xi = p_{\|}/p, s = \gamma v/c)\alpha_{i,n=0}$$
$$+ \sum_{i,n>0} f_{i,n}(\xi = p_{\|}/p, s = \gamma v/c, t)\alpha_{i,n} + f^*_{i,n}(\xi = p_{\|}/p, s = \gamma v/c, t)\alpha^*_{i,n},$$

Nonclassical quadrature scheme for “unboosted" relativistic Maxwellian

$$\tilde{f}_{MR} \sim e^{-z\sqrt{1+s^2}}$$

- Relativisitc parameter: $z \equiv m_e c^2/kT$.

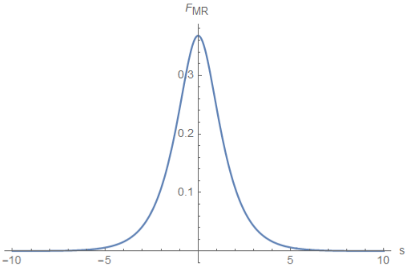


Figure: $\tilde{f}_{MR}$ vs normalized momentum

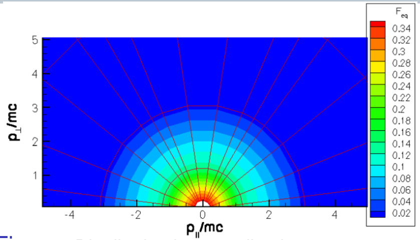


Figure: Distribution in normalized momentum space for z=1.

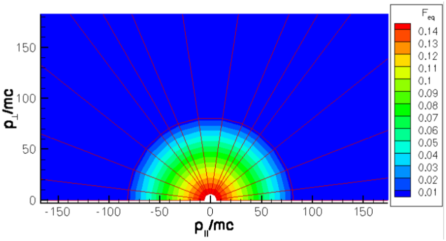


Figure: Distribution in normalized momentum

Preliminary Relativistic DKE for NIMROD

Start by implementing some of the terms in the NORSE Code*:

$$\frac{\partial f}{\partial t} - \frac{e\mathbf{E}}{m_e c} \cdot \boldsymbol{\nabla}_p f + \boldsymbol{\nabla}_p \cdot (\mathbf{F}_s f) = C_{fp}(f) + S$$

- Implement and test acceleration and synchrotron radiation reaction force, $\mathbf{F}_s$, in NIMROD. Future code development for

  - $C_{fp}(f)$: fully nonlinear, relativistic collision operator, and
  - S: sources and sinks (heat or particles).

*A. Stahl, *et al.*, "NORSE: A solver for the relativistic non-linear Fokker-Planck equation for electrons in a homogeneous plasma",Computer Physics Communications **212** (2017) 269-279.

Electric Field Acceleration and Synchrotron Radiation Reaction Force

- 2D case: $f = f(s, \xi, t)$ where $\xi = p_{\parallel}/p = s_{\parallel}/s$
- Collision-less and source-less plasma kinetic equation with radiation reaction force:

$$\frac{\partial f}{\partial t} - \frac{e\mathbf{E}}{m_e c} \cdot \boldsymbol{\nabla}_p f + \boldsymbol{\nabla}_p \cdot (\mathbf{F}_s f) = 0$$

- Expansion of 2nd and 3rd terms in (s, ξ):

$$\frac{\mathbf{E}}{E} \cdot \boldsymbol{\nabla}_p f = \xi \frac{\partial f}{\partial s} + \left(\frac{1-\xi^2}{s} \right) \frac{\partial f}{\partial \xi}$$

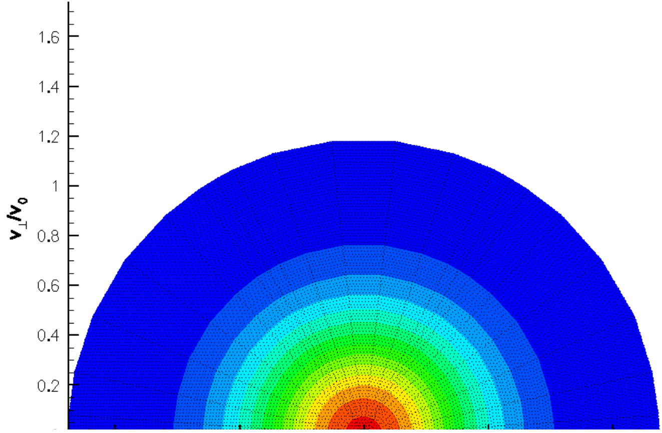
$$\boldsymbol{\nabla}_p \cdot (\mathbf{F}_s f) = -\frac{1}{\tau_r} \left(\frac{1-\xi^2}{\gamma} \right) \left[\gamma^2 s \frac{\partial f}{\partial s} - \xi \frac{\partial f}{\partial \xi} + \left(4s^2 + \frac{2}{1-\xi^2} \right) f \right]$$

$$\tau_r \equiv \frac{6\pi\epsilon_0 (m_e c)^3}{e^4 B^2}$$

Time Evolution of a Relativistic Maxwellian

Synchrotron Radiation

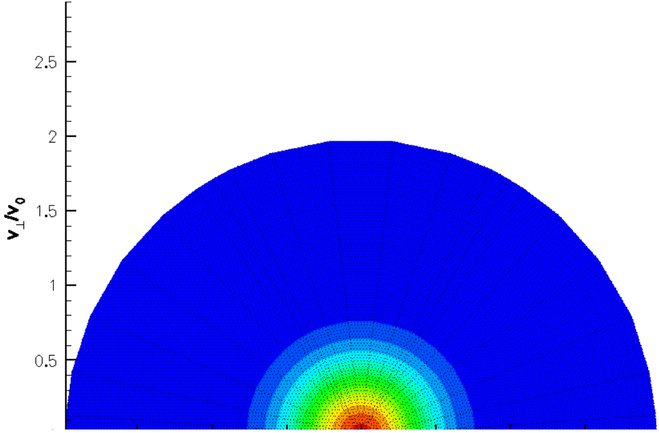
$E = 0$ V/m, $B = 20$ T, $\Delta t = 5 \times 10^{-6}$s



Time Evolution of a Relativistic Maxwellian

$\mathbf{E}$ acceleration and synchrotron radiation

$E = 0.22$ V/m, $B = 20$ T, $\Delta t = 5 \times 10^{-6}$s



Implement additional terms in NIMROD's relativistic electron DKE.

$$\frac{\partial f}{\partial t} + \mathbf{v}_{gc} \cdot \boldsymbol{\nabla} f + \dot{p_{\|}} \frac{\partial f}{\partial p_{\|}} = C(f) + C_s(f) + S_A$$

where

$$\mathbf{v}_{gc} \equiv \frac{p_{\|}}{\gamma m} \frac{\mathbf{B}^*}{B_{\|}^*} + \frac{\mathbf{E}^* \times \mathbf{B}_{\|}^*}{\left(B_{\|}^*\right)^2}, \quad \mathbf{B}^* \equiv \mathbf{B} + \frac{p_{\|}}{q} \boldsymbol{\nabla} \times \mathbf{b}, \quad \mathbf{E}^* \equiv \mathbf{E} - \frac{1}{q}\left(\boldsymbol{\nabla}\mathcal{E} - p_{\|}\frac{\partial \mathbf{b}}{\partial t}\right)$$

J. R. Cary and A. J. Brizard, "Hamiltonian theory of guiding-center motion", Reviews of Modern Physics **81** (2009) 694-723.

$$C(f) = \frac{1}{s^2}\frac{\partial}{\partial s}\left[s^2\left(C_F f + C_A \frac{\partial f}{\partial s}\right)\right] + \frac{C_B}{s^2}\frac{\partial}{\partial \xi}\left[\left(1-\xi^2\right)\frac{\partial f}{\partial \xi}\right]$$

$$C_F = 2(c/v_{Te})^2 \Psi(x)$$

$$C_A = (\gamma/s)\Psi(x)$$

$$C_B = \frac{1}{2}(\gamma/s)\left[Z + \phi(x) - \Psi(x) + \frac{1}{2}(v_{Te}/c)^4 x^2\right]$$

$$x = v/v_{Te}$$

Z. Guo, C. J. McDevitt, and X. Tang, "Phase-space dynamics of runaway electrons in magnetic fields" Plasma Phys. Control. Fusion 59 (2017) 044003

Future work.

- Implement full guiding center velocity and test particle collision operator.
- Implement coupling scheme.
- Participate in benchmark exercise by comparing with relativistic electron fluid approach.