Reduced models of runaway electrons in NIMROD

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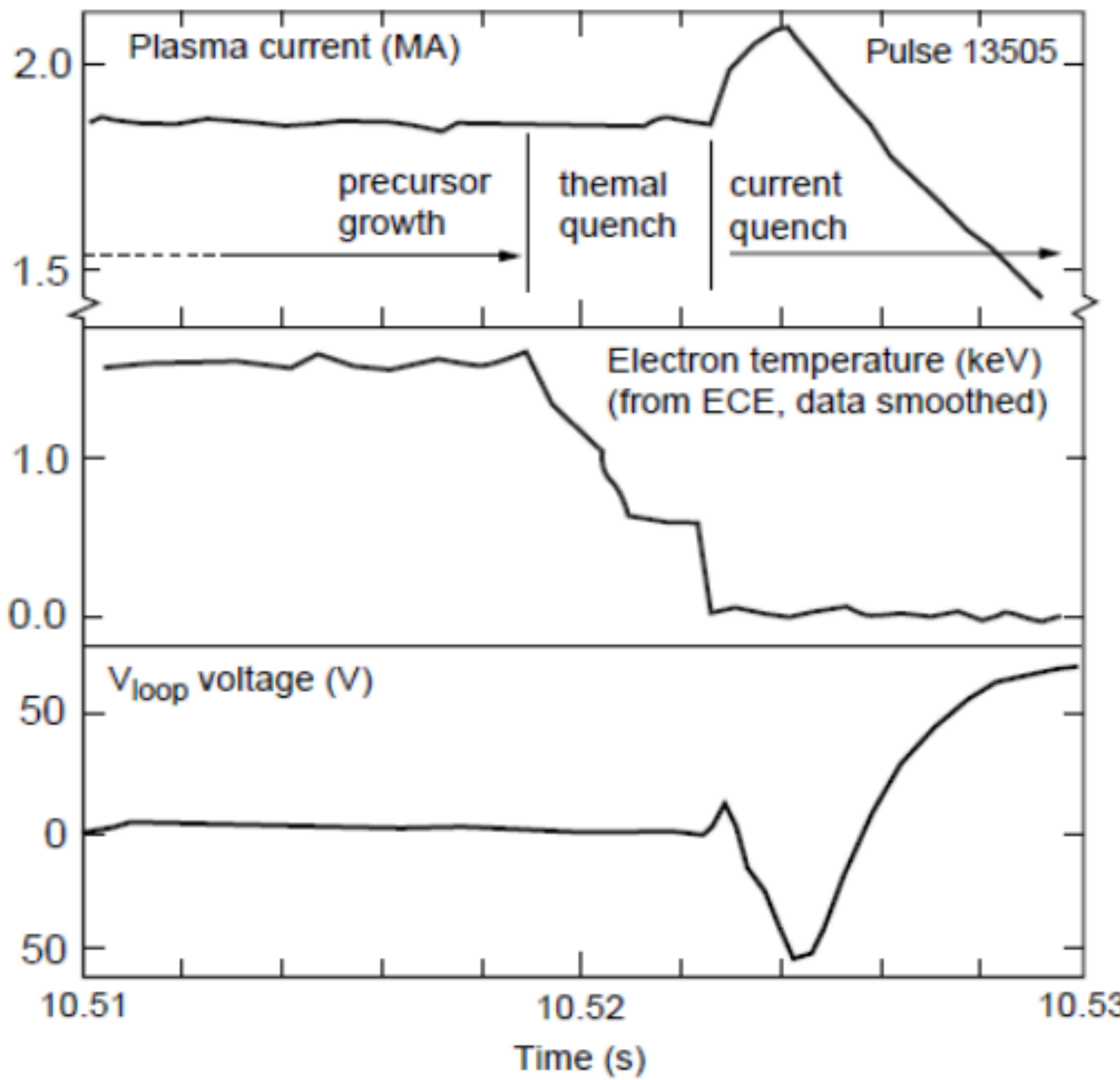
Content of the research

- Reduced models of runaway electron
  1. w/o source term: dynamic scale is much faster than the runaway generation;
  2. w/ source term: the self-consistent initial value simulation.

- The least-squares finite element method and the implicit scheme is used to solve the reduced model of runaways , which is shown to release the CFL time step constraint;

- Additional iterations at each time step are applied on the reduced model with the nonlinear source terms.

Typical disruption (JET)

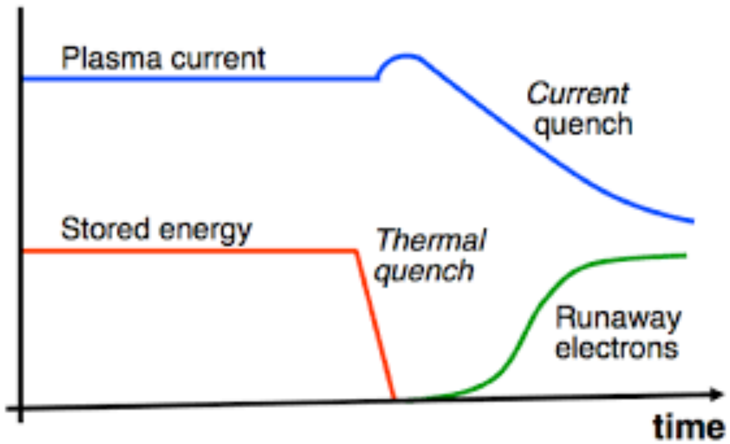


The runaway current becomes dominant at post-disruption plasma.

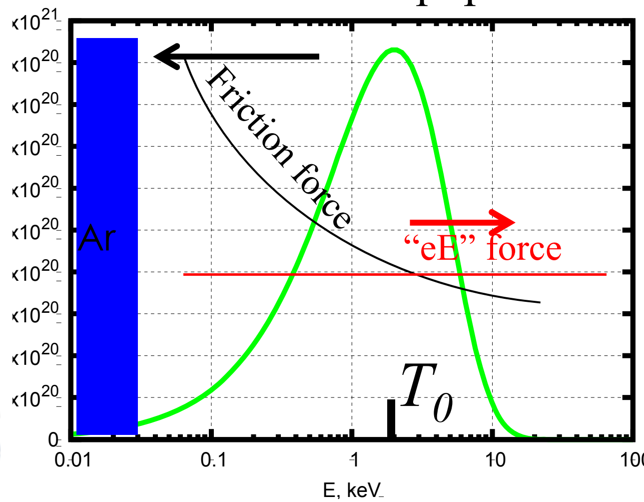
The increasing electric field during the current quench generates runaways

ITER Physics Basis, 1999

Runaway electron generation is an important phenomenon of disruptions



Initial hot and cold populations



MHD instability (tearing mode, kink mode, fishbone….) and whistler waves driven by runaway current

Thermal quench:

$$T_e, T_i \downarrow$$

$$\sigma \uparrow$$

$$E_\varphi = \sigma J_\varphi \uparrow$$

Runaway electron avalanche:

$$E_\varphi > E_D$$

Electrons are accelerated!

Current quench:

$$I_p \downarrow$$

$$I_{re} \uparrow$$

P.Helander et.al. Phys. Plasma, 122102(2007)
H.S.Cai and G.Y.Fu, Nucl.Fusion, 022001(2015)

Previous work has investigated linear resistive MHD instabilities with runaway electrons

Runaways can drive or suppress MHD instabilities during the current quench phase, which in turn may improve or deteriorate the confinement.

- The post-disruption runaway current profile could be more strongly peaked in the center of the discharge than the pre-disruption current. Plasmas with steep current profiles can be prone to tearing-mode instability. *(P.Helander, et.al. Phys.Plasma 14, 122102(2007))*

- Such a peaked runaway current can also drive other resistive magnetohydrodynamic instabilities in the plasma with high resistivity. H.S.Cai et.al. investigated the resistive internal kink mode in a tokamak plasma with runaway current using M3D. *(H.S.Cai and G.Y.Fu, Nucl. Fusion, 55 022001(2015))*

Numerical difficulty: a small time step is often used to push the relativistic runaway electrons, which causes the expensive nonlinear simulations of the MHD modes.

Energetic particle component and runaway electron are essential different

- In the hybrid model, energetic particles treat as a perturbative component; but the runaway electrons become dominant during the current quench;

- Relativistic speed of runaway electrons disables the kinetic resonance with the low frequency MHD modes. So the kinetic effect in the momentum space is not essentially important for the runaways;

- During the current quench, parallel electric field can not be ignored even in the study of the interaction with the low frequency modes.

Several approximations are appropriate for the RE reduced model

- Runaway electrons are collisionless and the inertia of runaway electrons is neglected;
- Runaway electrons are highly relativistic and their velocities are mainly parallel to the magnetic field as the passing particles;

$$v_{\|} \sim c \gg v_{\perp}$$

- The time scale of MHD instabilities of the post-disruption plasma is much shorter than that of runaway electron, so the generation source of runaway electrons is not considered to study the instability. However, the equilibrium distribution of runaways is difficult to calculate, so we take the account of the source and sink terms for modeling discharge evolution;

- The energy variation of the runaway electrons is ignored in the studying of a low frequency MHD modes.

The RE drift-kinetic equation is the basis for a fluid model

Hazeltine's drift kinetic equation with a low frequency MHD mode:

$$\frac{\partial f}{\partial t} + (\vec{v}_{\|} + \vec{v}_D) \cdot \nabla f + \left[e\frac{\partial \Phi}{\partial t} + \mu\frac{\partial B}{\partial t} - \frac{e\vec{v}_{\|}}{c} \cdot \frac{\partial \vec{A}}{\partial t} \right] \frac{\partial f}{\partial E} = C$$

$$\vec{v}_D = \underbrace{\boxed{\frac{\vec{E} \times \vec{B}}{B^2}}}_{\text{ExB drift}} + \frac{\gamma}{\omega_{ce}} \hat{b} \times \left(\underbrace{\boxed{\frac{\mu}{m_e} \nabla B}}_{\text{grad drift}} + \underbrace{\boxed{v_{\|}^2 \vec{\kappa}}}_{\text{curvature drift}} + v_{\|} \frac{\partial \hat{b}}{\partial t} \right)$$

$$\frac{\gamma}{\omega_{ce}} v_{\|} \hat{b} \times \frac{\partial \hat{b}}{\partial t}$$

the electron drift due to the changing of the direction of the magnetic field, which is missed in previous work.

The reduced RE drift kinetic equation is being implemented in NIMROD

- Apply our model approximations on Hazeltine's drift kinetic equation with the low frequency MHD mode:

$\frac{\partial f}{\partial E} \sim 0$	$C \sim 0$	$v_{\parallel} \sim c$	$\frac{\mu}{m_e} \nabla B \sim \frac{v_{\perp}^2}{2} \vec{\kappa} \ll v_{\parallel}^2 \vec{\kappa}$
constant energy	collisionless	constant $v_{\parallel}$	ignore the gradient drift of REs

where, E is the total guiding center energy:

$$E = \frac{m_e v_{\parallel}^2}{2} + e\Phi + \mu B$$

A reduced model of runaway electron density:

$$\frac{\partial n_{re}}{\partial t} + c\hat{b} \cdot \nabla n_{re} + \vec{v}_D \cdot \nabla n_{re} = 0$$

A case is tested in cylindrical geometry

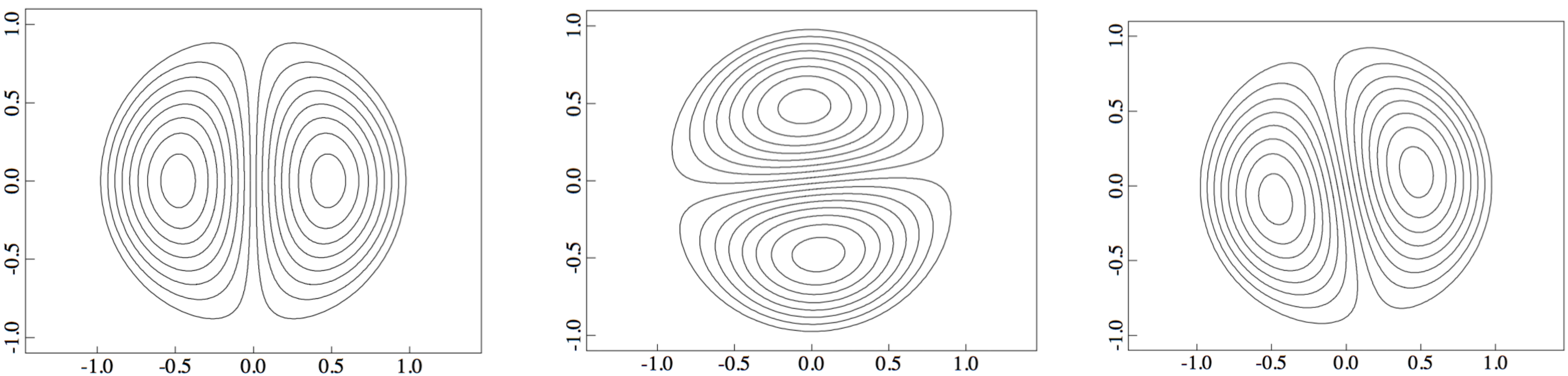
$$\vec{v}_D = 0$$

$$\vec{J}_{RE\perp} = 0$$

$$\Delta n_{re} = -c\hat{b}_0 \cdot \nabla n_{re}$$

The runaway electrons are highly relativistic and their velocities are parallel to the equilibrium magnetic field. With the uniform q=1.2, rigid rotation is expected from parallel motion.

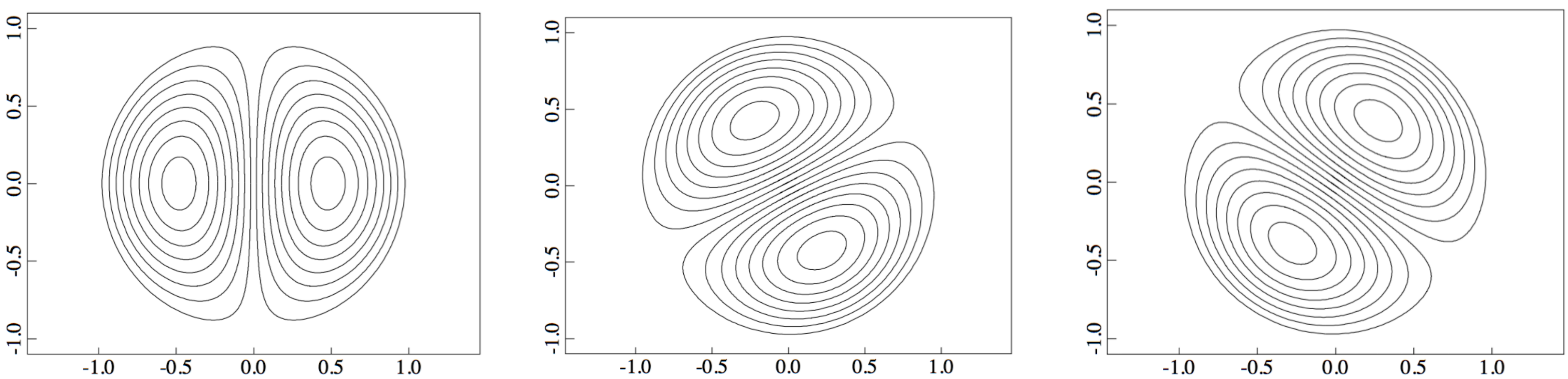
$$\Delta t = t_A \quad c/v_A \sim 10^3$$



Contours of runaway electron density Δn_{re} rotates along the equilibrium magnetic field

A larger time step is tested

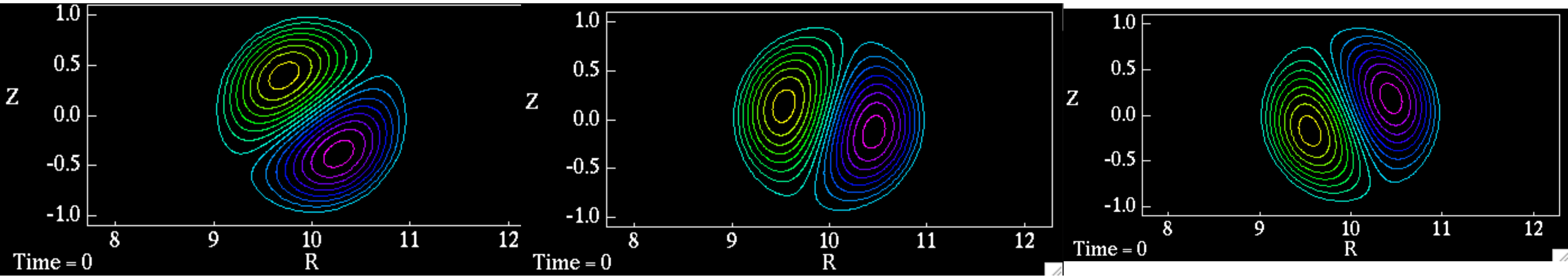
$$\Delta t = 5t_A$$



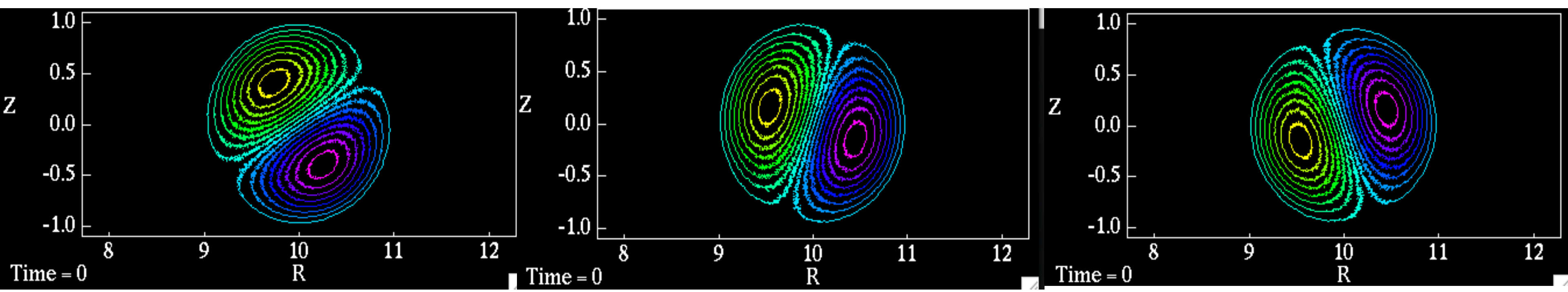
Time step used in the simulation is found to be much larger than the CFL constrained time step:

$$c\Delta t \gg L$$

A case is tested in toroidal geometry with constant q



$\Delta t = 10t_A$



$\Delta t = 100t_A$

When the time step increases up to 100 Alfven time, the contours of runaway density becomes diffusive after ten thousands of rotations.

The RE drift-kinetic equation couples with the MHD field

$$\frac{\partial n_{re}}{\partial t} + c\hat{b}\cdot\nabla n_{re} + \vec{v}_D\cdot\nabla n_{re} = 0$$

RE drift velocity:

$$\vec{v}_D = \frac{\vec{E}\times\vec{B}}{B^2} + \frac{\gamma}{\omega_{ce}}\hat{b}\times\left(\frac{\mu}{m_e}\nabla B + v_\parallel^2\vec{\kappa} + v_\parallel\frac{\partial\hat{b}}{\partial t}\right)$$

- To calculate RE drift velocity, we need in update the field variables in the framework of NIMROD. Here the runaway electrons contribute a resistance-free current, which is removed from the total current in Ohm’s law.

$$\vec{E} = -\vec{v}\times\vec{B} + \eta(\vec{J} - \vec{J}_{RE})$$

$$\vec{J}_{re} = -en_{re}\left(c\hat{b} + \frac{\gamma}{\omega_{ce}}\hat{b}\times(c^2\vec{\kappa} + c\frac{\partial\hat{b}}{\partial t})\right)$$

$$\frac{\partial\vec{B}}{\partial t} = -\nabla\times\vec{E} + \kappa_b\nabla\nabla\cdot\vec{B} \qquad \vec{J} = \frac{1}{\mu_0}\nabla\times\vec{B}$$

$$mn\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\vec{v} = (\vec{J} - \vec{J}_{RE})\times\vec{B} - \nabla\sum_\alpha nT_\alpha - \nabla\cdot\Pi$$

$$B = |\vec{B}|$$

$$\hat{b} = \frac{\vec{B}}{B}$$

$$\vec{\kappa} = \hat{b}\cdot\nabla\hat{b}$$

The predict-correct scheme is used in NIMROD

$$v^{n-1/2} \qquad\qquad v^{n+1/2}$$

$$\longrightarrow t$$

$$\{B, n, T, n_{re}\}^{n-1} \qquad\qquad \{B, n, T, n_{re}\}^{n}$$

Receive $\vec{v}^{n+1/2}$ from NIMROD code

$$\Delta\vec{B} = \Delta t \nabla \times \left(\vec{v}^{n+1/2} \times \vec{B}^* - \frac{\eta}{\mu_0} \nabla \times \vec{B}^* + \eta \vec{J}^*_{RE} \right)$$

where $\vec{B}^* = \vec{B}^n$(prediction), $\vec{B}^* = f\vec{B}^n + (1-f)\vec{B}^{n+1}$ (correction)

$$\Delta\hat{b} = \frac{1}{B^{n+1}}(\Delta\vec{B} - \hat{b}^*|\Delta\vec{B}|)$$

$$\Delta n_{re} = -\Delta t(c\hat{b}^{n+1} \cdot \nabla n^*_{re} + \vec{v}^{n+1}_D \cdot \nabla n^*_{re})$$

$$\vec{J}^{n+1}_{re} = -ecn^{n+1}_{re}\hat{b}^{n+1} - \frac{ecn^{n+1}_{re}}{\omega^{n+1}_{ce}}\hat{b}^{n+1} \times (c\vec{\kappa}^{n+1} + \frac{\Delta\hat{b}}{\Delta t})$$

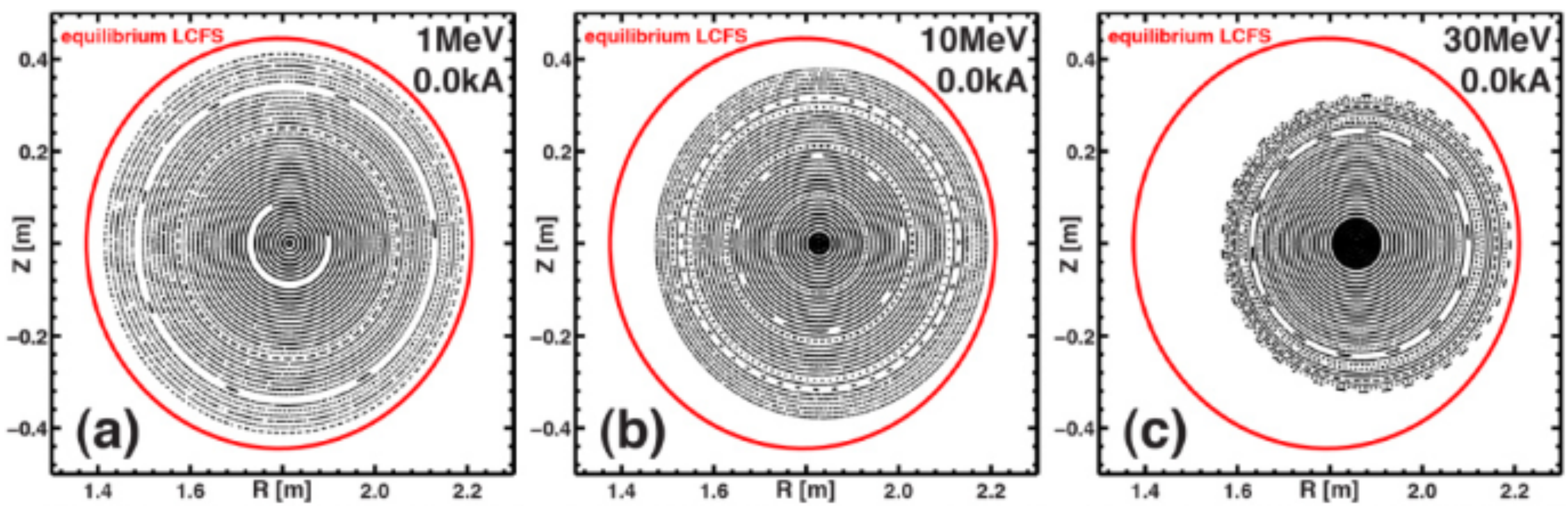
Send to NIMROD flow equation

$$mn\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\vec{v} = (\vec{J} - \vec{J}_{RE}) \times \vec{B} - \nabla \sum_\alpha nT_\alpha - \nabla \cdot \Pi$$

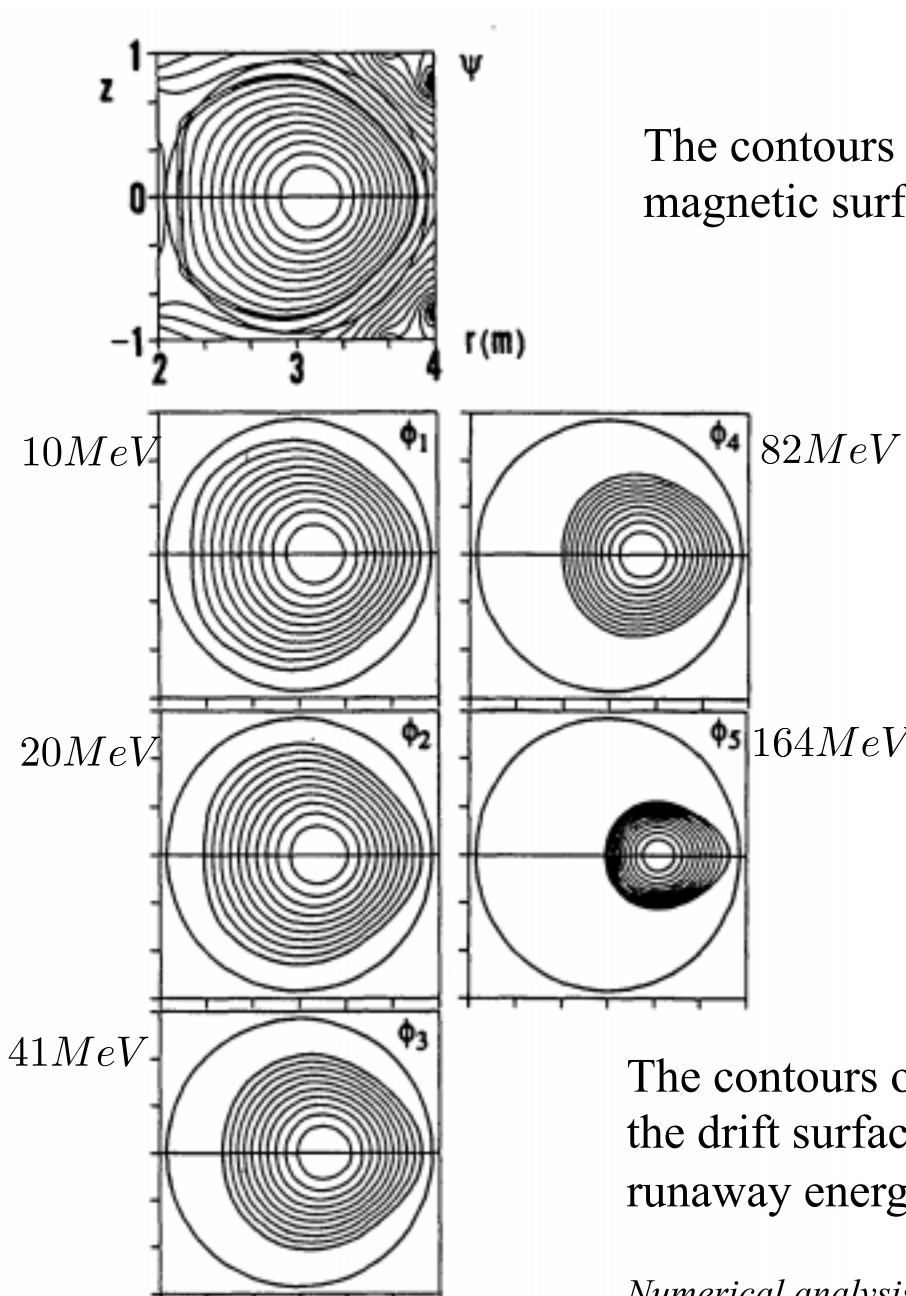
However, equilibrium orbits of runaways is difficult to calculate

- The electric fields generated during the current quench phase can give rise to runaway electrons with energies as high as 10's-100's of MeV.

- The confinement volume shrinks with increasing particle energy. The orbits (in the unperturbed field) of the particles are circles that are displaced horizontally with respect to the flux surfaces, with a displacement that is proportional to the energy.



G. Papp, M. Drevlak, T. Fulop, P. Helander and G. I.Pokol, Plasma phys. and controlled fusion, 2011



The contours of the flux function show the magnetic surfaces at equilibrium.

The contours of the stream functions show the drift surfaces of beams with the different runaway energies.

Numerical analysis of runaway tokamak equilibrium Z. Yoshida Nucl.Fusion, p317(1990)

Initial value nonlinear calculation is needed

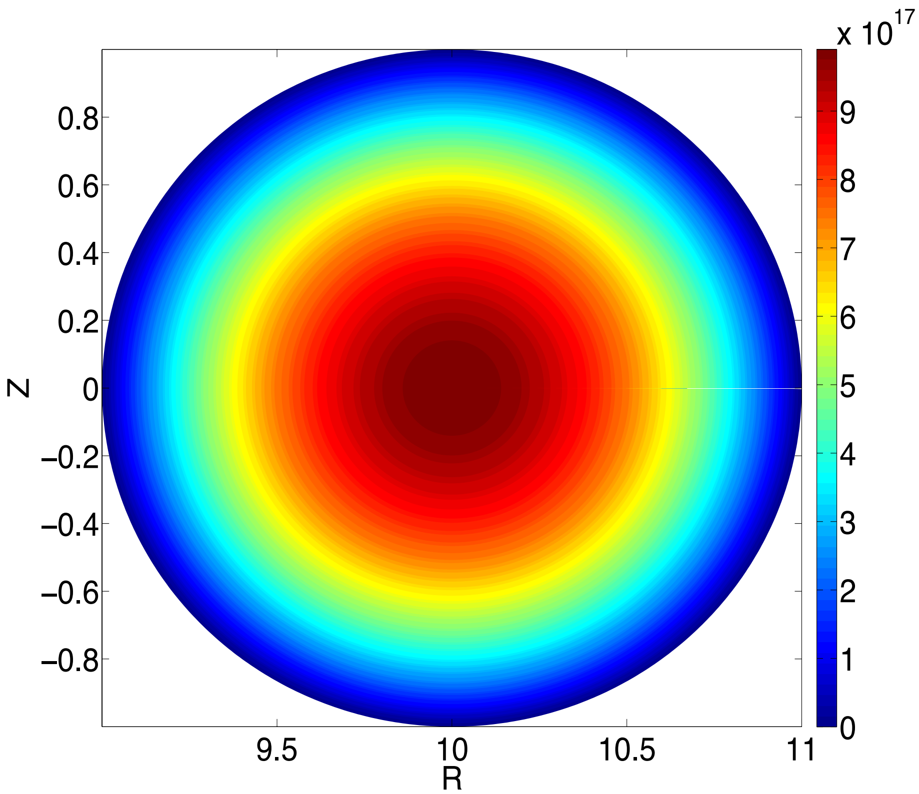
The uncertainties on determining the equilibrium distribution of runaway electrons and MHD equilibrium at the post-disruption:

- The shrinkage of the confinement zone of the relativistic runaways;
- The MHD equilibrium is modified by the runaway current, which becomes dominant at post-disruption plasma.

The linear calculation relies on the accurate equilibriums of both MHD and runaway electrons. However, the initial value nonlinear calculation can start with an inaccurate runaway electron distribution and low beta MHD equilibrium.

Runaway electron source term

$$S_{re} = n_e n_{re} f(E_\parallel) = (Z_{eff} n - n_{re}) n_{re} f(E_\parallel)$$



$$n_0 = 5 \times 10^{19} m^{-3}$$

$$E_{\parallel 0} = 80 V/m$$

$$T_e = 2.35 eV$$

$$n_{re0} = 10^6 m^{-3}$$

Runaway reduced model takes account of source

$$mn\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u}\right) = \underline{en_{re}E_{||}\hat{b}} + \vec{J}\times\vec{B} - \nabla p$$

The force due to parallel electric field

$$\partial_t n_{re} = -c\hat{b}\cdot\nabla n_{re} + \underline{S(n_{re}, E_{||})}$$

Runaway electron source

$$E_{||} = \eta(J_{||} + \underline{ecn_{re}})$$

Parallel runaway electron current

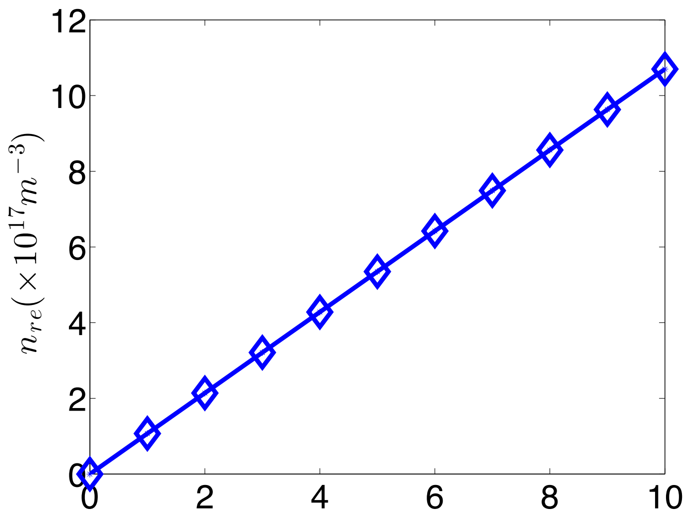
Numerical scheme is unconditionally stable

$$(1 + \Delta t f(E_{\|}) n_{re}) \Delta n_{re}^{k+1} + \frac{\Delta t}{2} c\hat{b} \cdot \nabla \Delta n_{re}^{k+1} =$$
$$-\Delta t c\hat{b} \cdot \nabla n_{re}^{k} + \Delta t f(E_{\|}) \left[n_{re}(n - n_{re}) + \frac{\Delta n_{re}^{k}}{2} \left(n - \frac{\Delta n_{re}^{k}}{2} \right) \right]$$

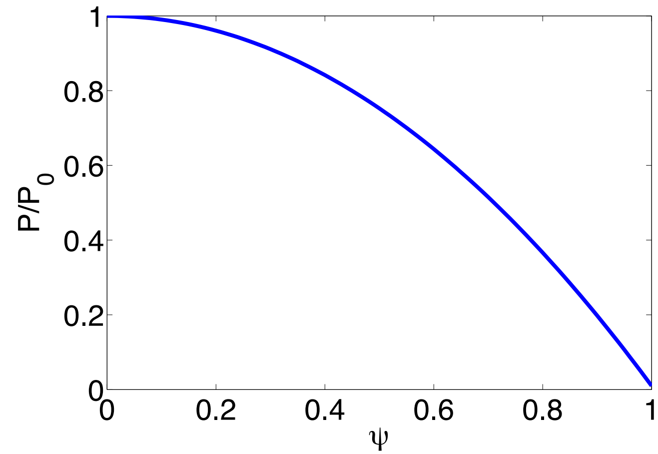
$f(E_{\|}) > 0$

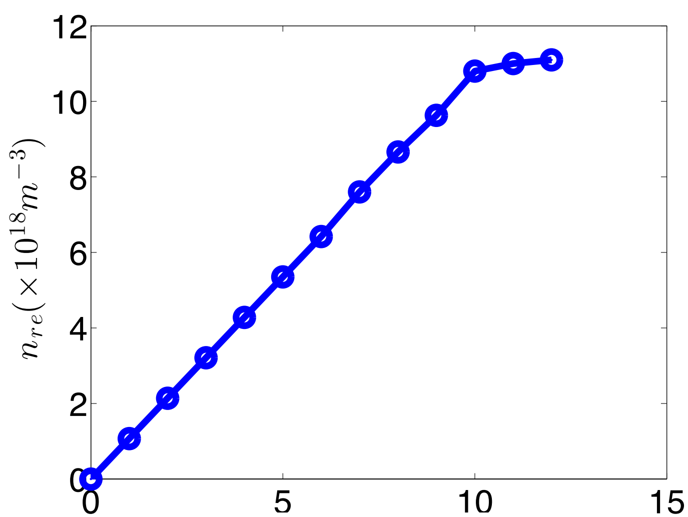
Iteration scheme is numerically stable even with a large time step.

Simulation results

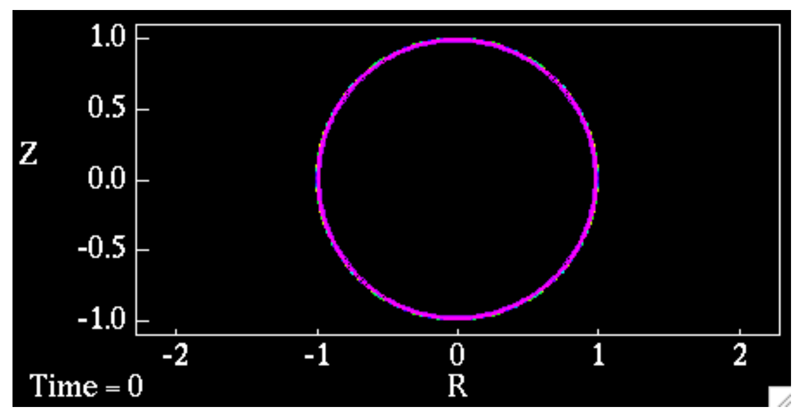


low density side





high density side



Contours of runaway electron density

Conclusion

- We have summarized two types of derivation of the reduced model of runaways in the interaction with the low frequency resistive MHD mode;

- The tests demonstrate the implicit solve of the RE density equation, which release the CFL time constraint. But we haven't yet demonstrated the full implicit evolution.

- The implementation of the nonlinear source term is needed to make a self-consistent initial value simulation.

Reduced RE kinetic equation with whistlers

- Because of the cyclotron resonance, the magnetic momentum is not a constant of motion.

- Instead of the magnetic momentum, $E - \omega\mu$ becomes a constant of motion of runaway electrons, which leads to a 3D kinetic equation in the interaction with the whistlers.

$$\frac{\partial f}{\partial t} + c\hat{b} \cdot \nabla f + \frac{d\mu}{dt}\frac{\partial f}{\partial \mu} + \omega_c \frac{\partial f}{\partial \psi} = 0$$

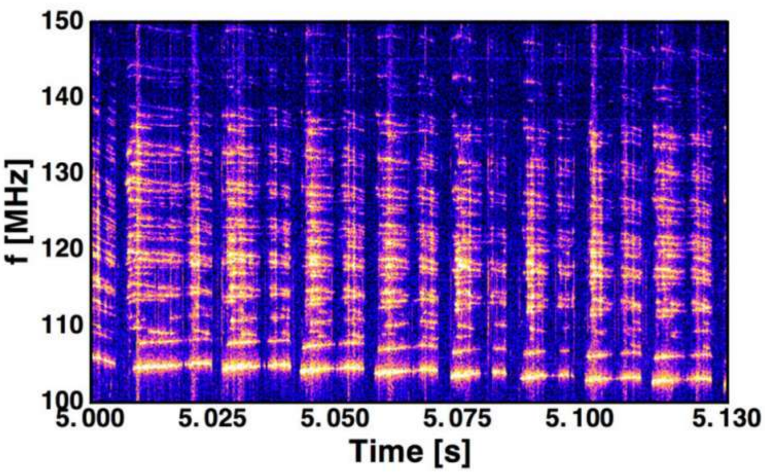
Previous work has investigated kinetic instabilities with runaway electrons

- Due to the highly relativistic speed, the runaway electrons are observed to drive the whistler wave unstable with the kinetic resonance condition,

$$\omega - k_{\|}v_{\|} - k_{\perp}v_d - l\omega_{ce}/\gamma = 0$$

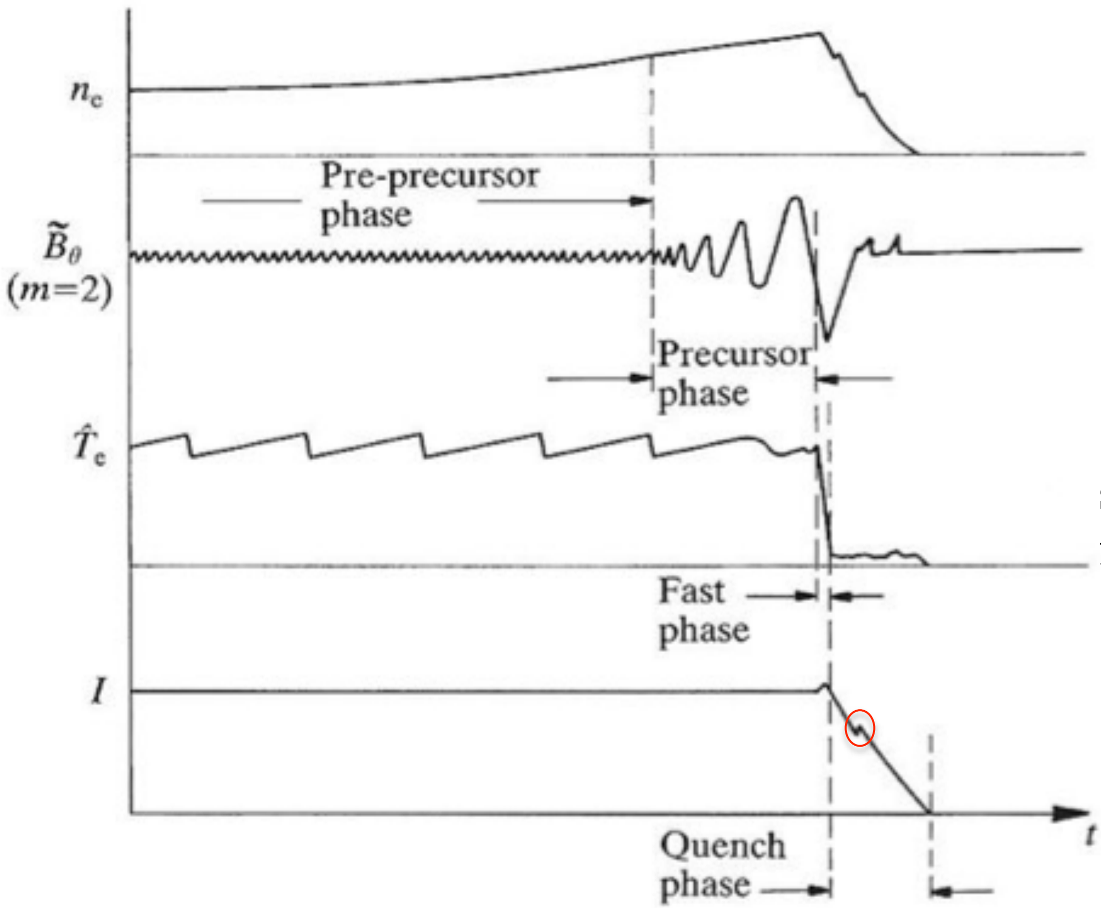
(D.A.Spong, et.al. Phys.Rev.Lett. 120, 155002(2018)

C.Liu, et.al. Phys.Rev.Lett.120, 265001(2018))



Spectrogram of whistler wave activity measured in DIII-D.

Major disruptions in tokamaks often have similar characteristics



plasma density increases

growth of m=2 NTM

sawtooth instability followed by a fast temperature quench

slow current quench due to the generation of runaway electrons

A typical time dependence of the plasma density, m=2 magnetic fluctuations, central temperature and plasma current during a period of major disruption.
(F.C. Schuller 1995 Plasma Phys. Control. Fusion 37 A135)