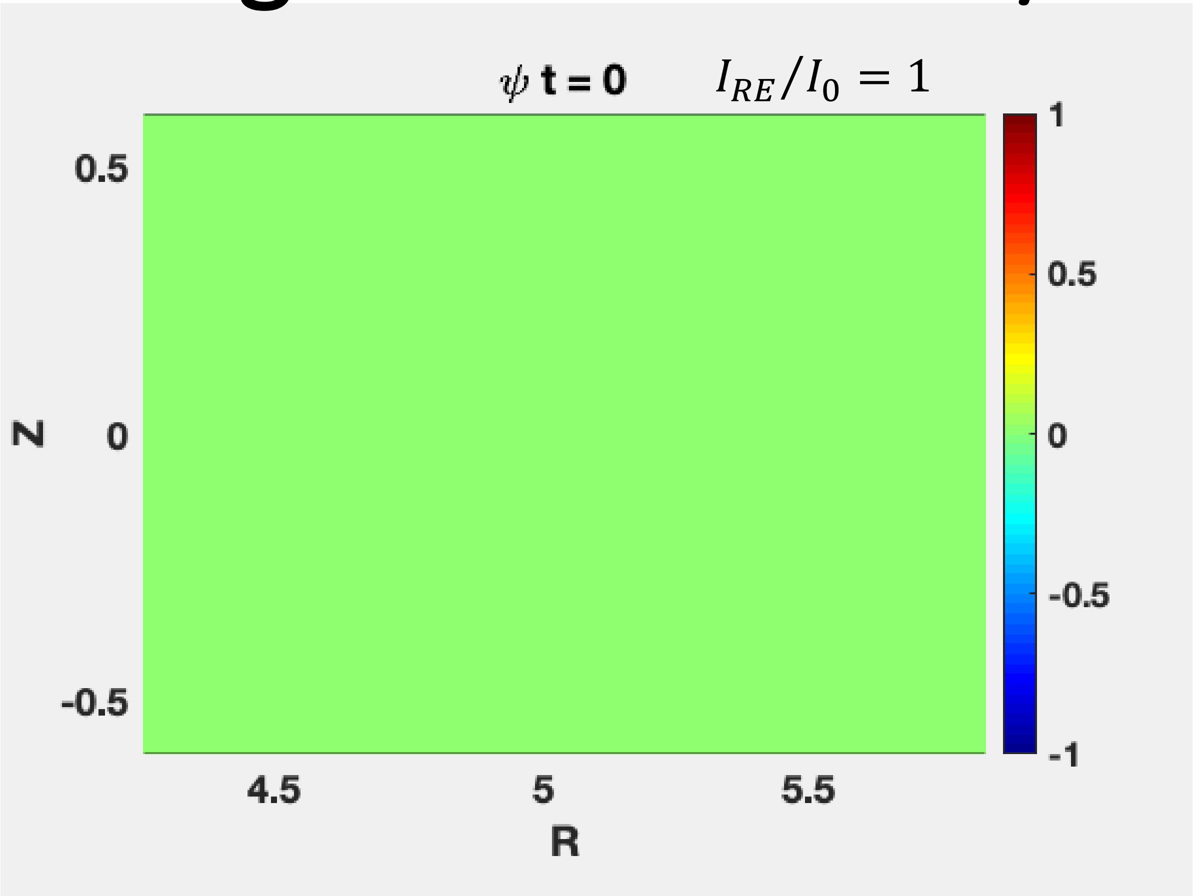
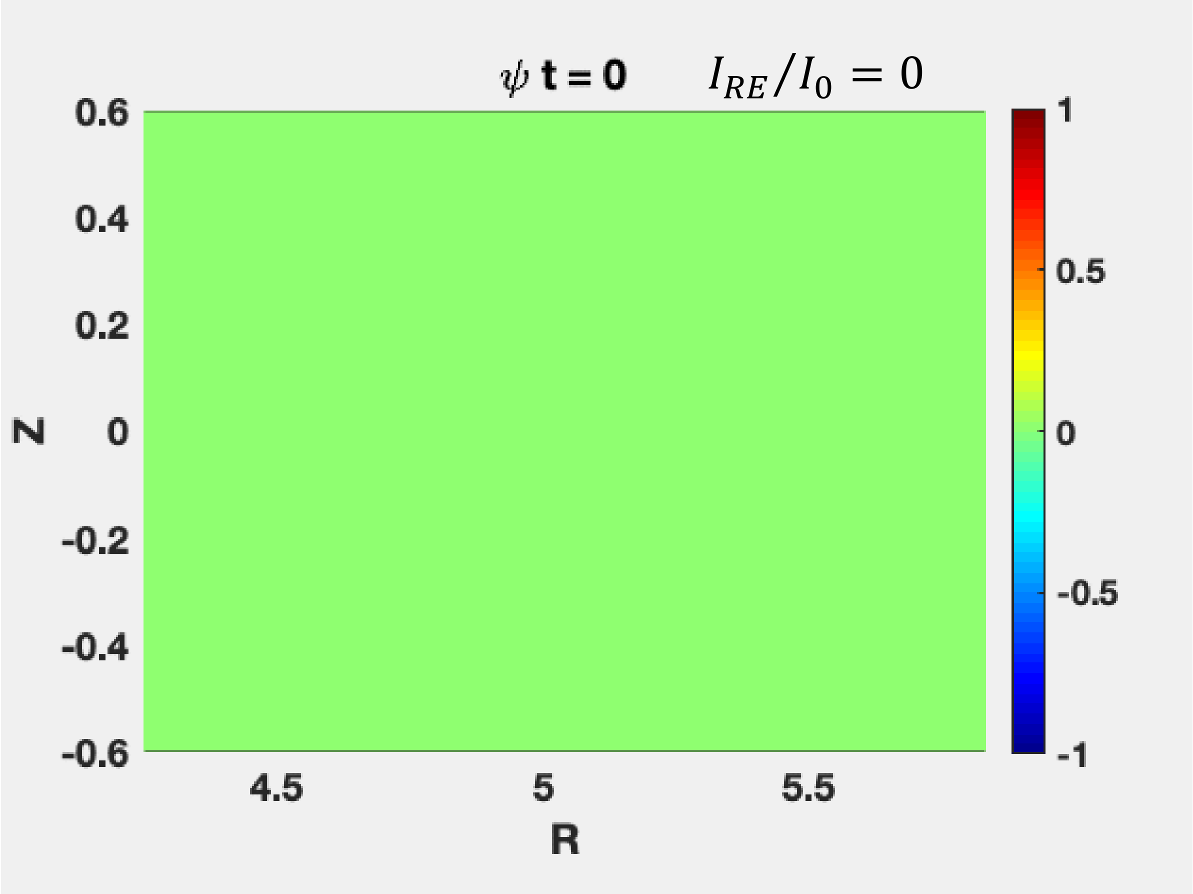
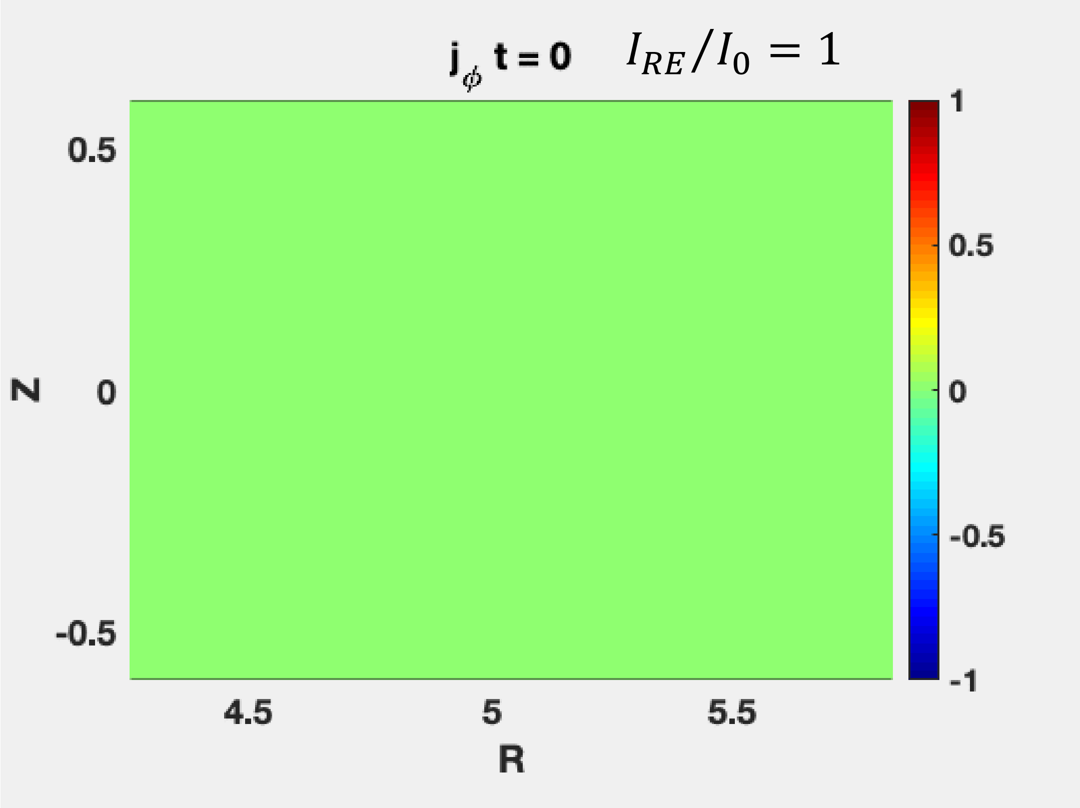
Magnetic island of 1/1 kink mode

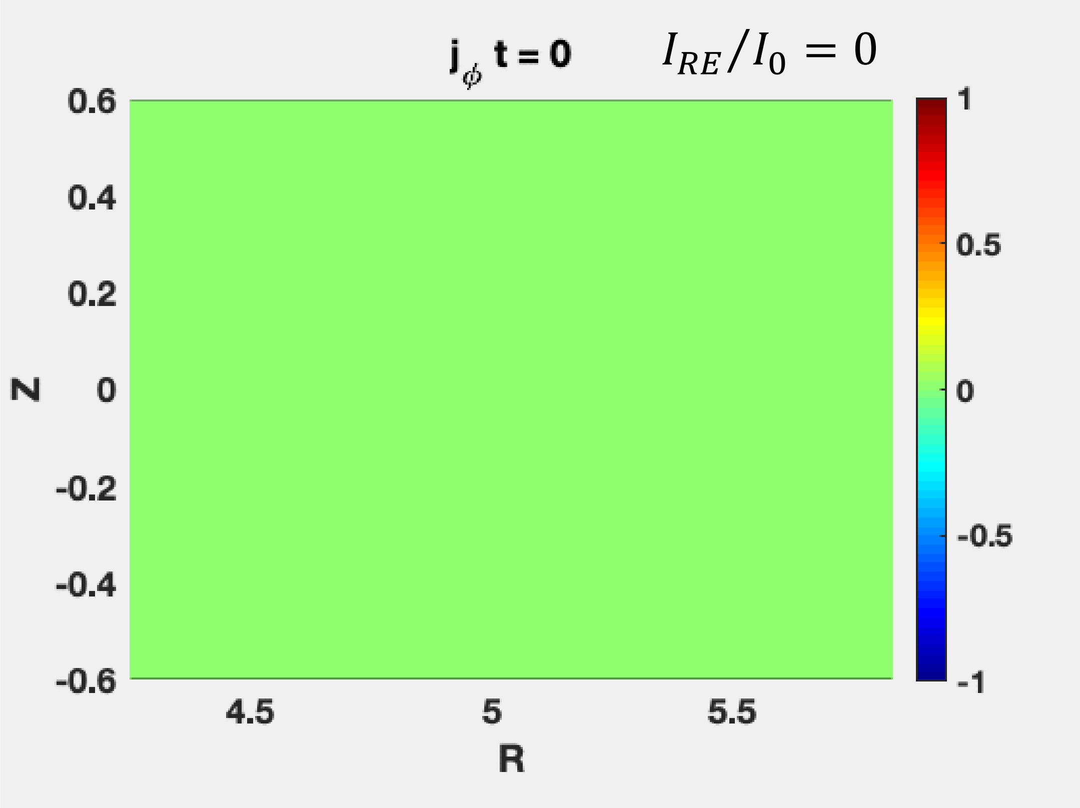




- The mode structure of 1/1 kink mode with RE is similar with 1/1 kink mode with out RE.
- The runaways drive the 1/1 kink mode islands rotate with a constant frequency.

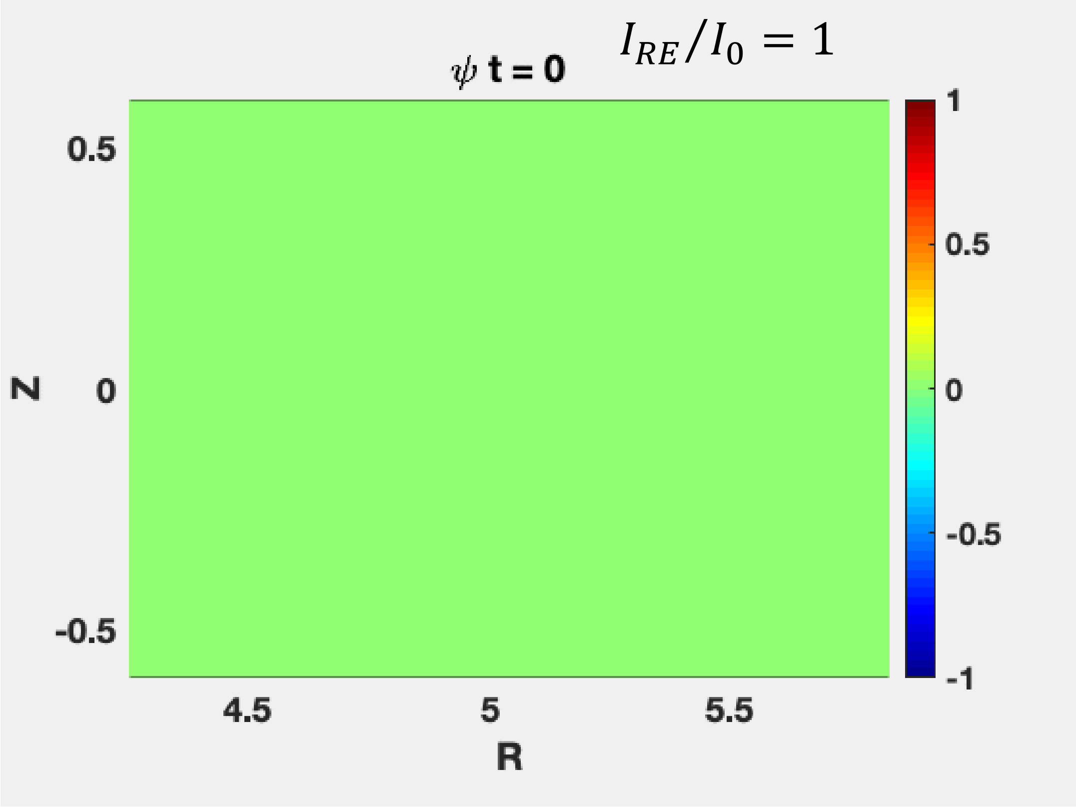
Toroidal current perturbation of 1/1 kink mode

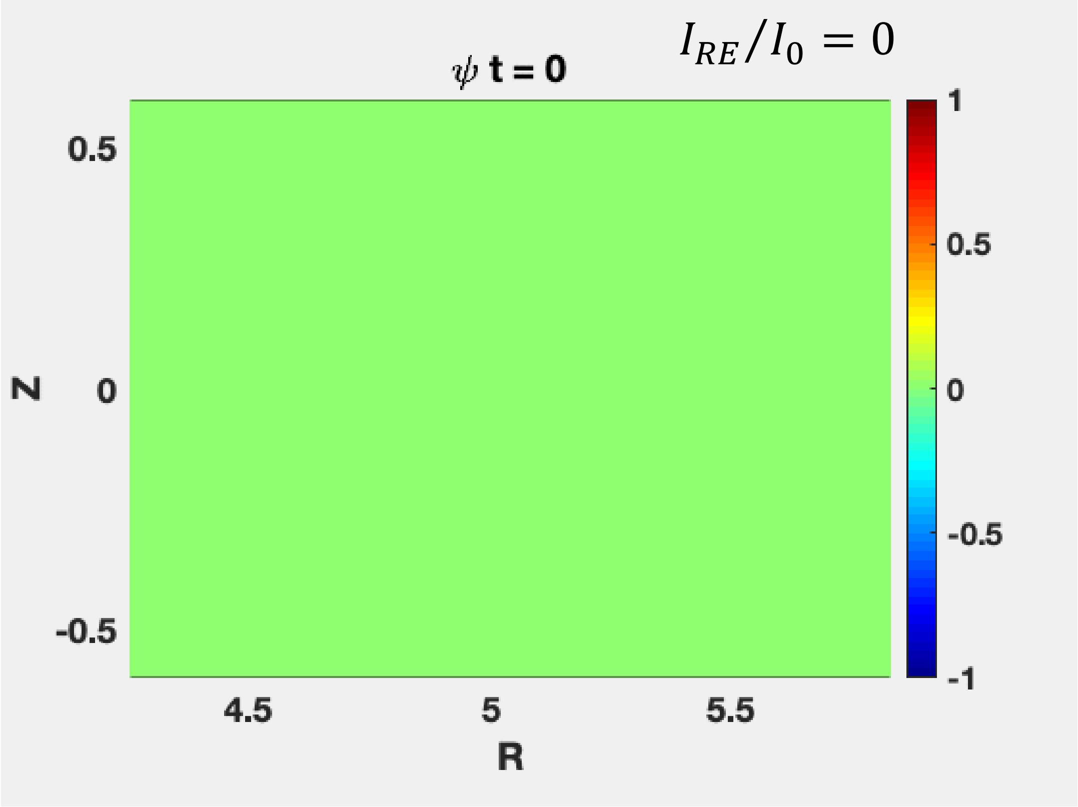




- The RE current perturbation is more peaked around the rational surface than without runaways, and also has a rotation.

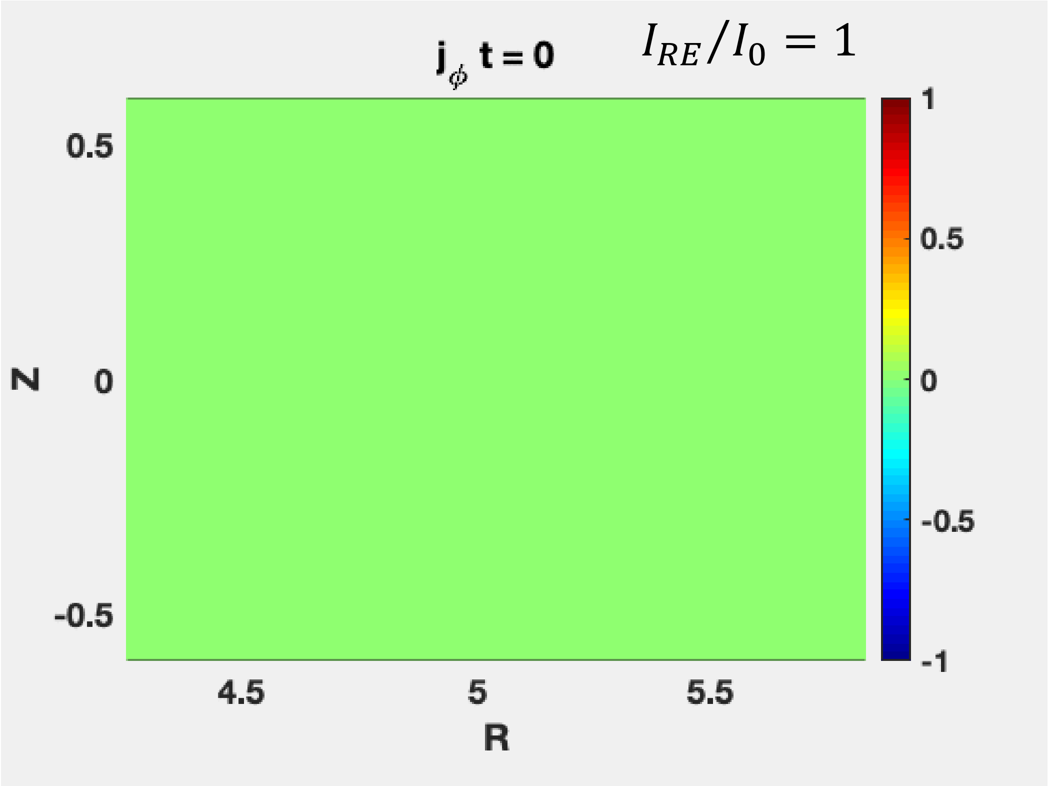
Magnetic island of 2/1 kink mode

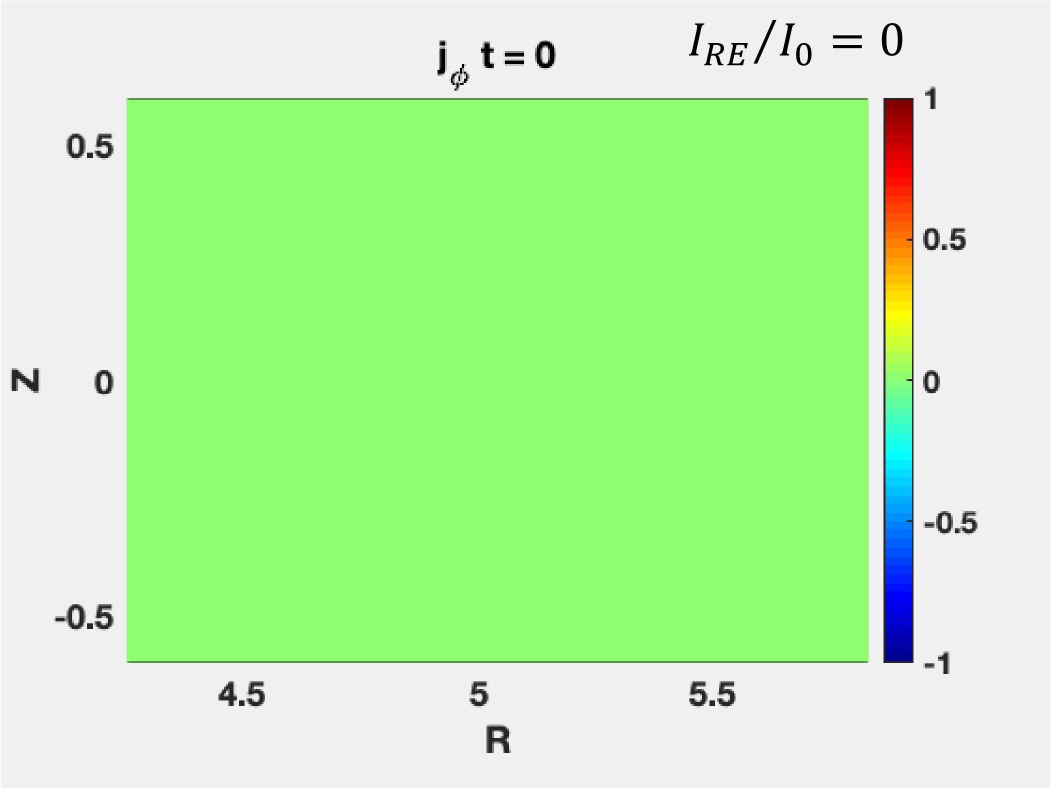




- The mode structure of 2/1 kink mode with RE is similar with 2/1 kink mode with out RE.
- The runaways drive the 2/1 kink mode islands rotate with a constant frequency.

Toroidal current perturbation of 2/1 kink mode





- The RE current perturbation is more peaked around the rational surface than without runaways, and also has a rotation.

Simulation of MHD instabilities with fluid runaway electron model in M3D-C^1

C. Zhao[1], C. Liu[1], S. C. Jardin[1], N. Ferraro[1]

[1]Princeton Plasma Physics Laboratory, Princeton, NJ

PPPL PRINCETON PLASMA PHYSICS LABORATORY

Outline

- Introduction to M3D-C^1
- Basic equations of runaway electrons (RE) in M3D-C^1
- Eigen values solved from reduced MHD equations with RE
- Linear simulation of kink mode with RE
- Nonlinear simulations with RE
- Summary and future work

1.Introduction to M3D-C^1

3D Extended MHD Equations in M3D-C^1

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\mathbf{V}) = \nabla \bullet D_n \nabla n + S_n$$

Density equation

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla\Phi, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla_\perp \bullet \frac{1}{R^2}\nabla\Phi = -\nabla_\perp \bullet \frac{1}{R^2}\mathbf{E}$$

Field equation

$$nM_i\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \bullet \nabla\mathbf{V}\right) + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \bullet \mathbf{\Pi}_i + \mathbf{S}_m$$

Momentum equation

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne}\left(\mathbf{R}_c + \mathbf{J} \times \mathbf{B} - \nabla p_e - \nabla \bullet \mathbf{\Pi}_e\right) - \frac{m_e}{e}\left(\frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \bullet \nabla \mathbf{V}_e\right) + \mathbf{S}_{CD}$$

Generalized Ohm's law

$$\frac{3}{2}\left[\frac{\partial p_e}{\partial t} + \nabla \bullet \left(p_e \mathbf{V}\right)\right] = -p_e \nabla \bullet \mathbf{V} + \frac{\mathbf{J}}{ne} \bullet \left[\frac{3}{2}\nabla p_e - \frac{5}{2}\frac{p_e}{n}\nabla n + \mathbf{R}_c\right] + \nabla\left(\frac{\mathbf{J}}{ne}\right) : \mathbf{\Pi}_e - \nabla \bullet \mathbf{q}_e + Q_\Delta + S_{eE}$$

$$\frac{3}{2}\left[\frac{\partial p_i}{\partial t} + \nabla \bullet \left(p_i \mathbf{V}\right)\right] = -p_i \nabla \bullet \mathbf{V} - \mathbf{\Pi}_i : \nabla\mathbf{V} - \nabla \bullet \mathbf{q}_i - Q_\Delta + S_{iE}$$

Pressure equations

$$\mathbf{R}_c = \eta ne\mathbf{J}, \qquad \mathbf{\Pi}_i = -\mu\left[\nabla\mathbf{V} + \nabla\mathbf{V}^\dagger\right] - 2(\mu_c - \mu)(\nabla \bullet \mathbf{V})\mathbf{I} + \mathbf{\Pi}_i^{GV} \qquad \mathbf{q}_{e,i} = -\kappa_{e,i}\nabla T_{e,i} - \kappa_\parallel \nabla_\parallel T_{e,i}$$

$$\mathbf{\Pi}_e = (\mathbf{B}/B^2)\nabla \bullet \left[\lambda_h \nabla\left(\mathbf{J} \bullet \mathbf{B}/B^2\right)\right], \qquad Q_\Delta = 3m_e(p_i - p_e)/\left(M_i \tau_e\right)$$

Blue terms are 2-fluid terms. Also, now have impurity and pellet models for disruption mitigation. **NOT reduced MHD.**

3D finite elements method in M3D-C^1

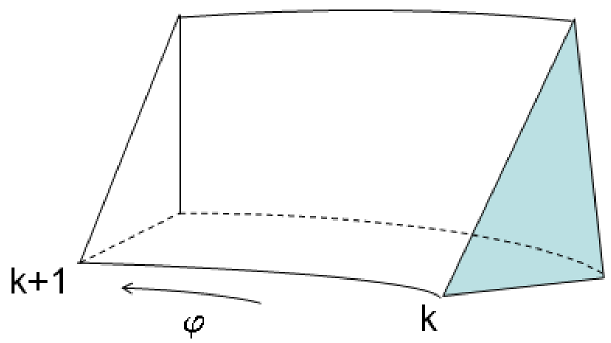
- M3D-C^1 uses high-order curved triangular prism elements with C^1 *continuity.*

- Within each triangular prism, there is a polynomial in (R,φ,Z) with 72 coefficients.

- The solution *and 1st derivatives* are constrained to be continuous from one element to the next.

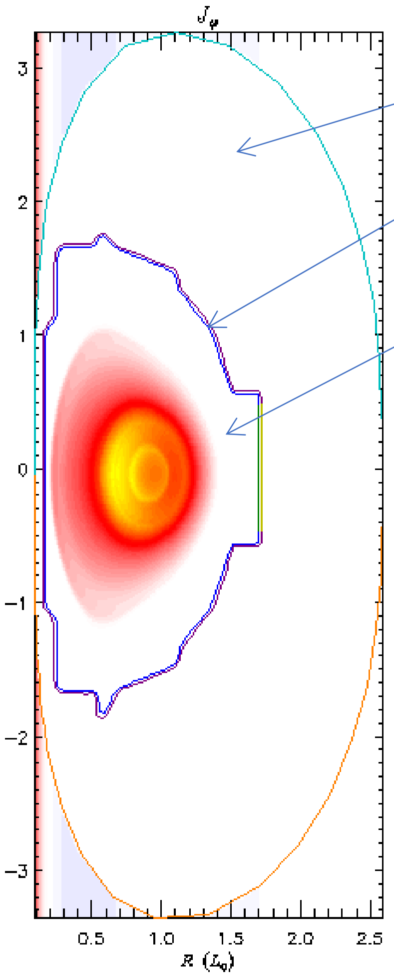
- Thus, there is much more resolution than for the same number of linear elements.

- Error ~ h^5



Also, implicit time-stepping allows for very long time simulations

Adapted mesh in M3D-C^1



Vacuum (**J**=0)

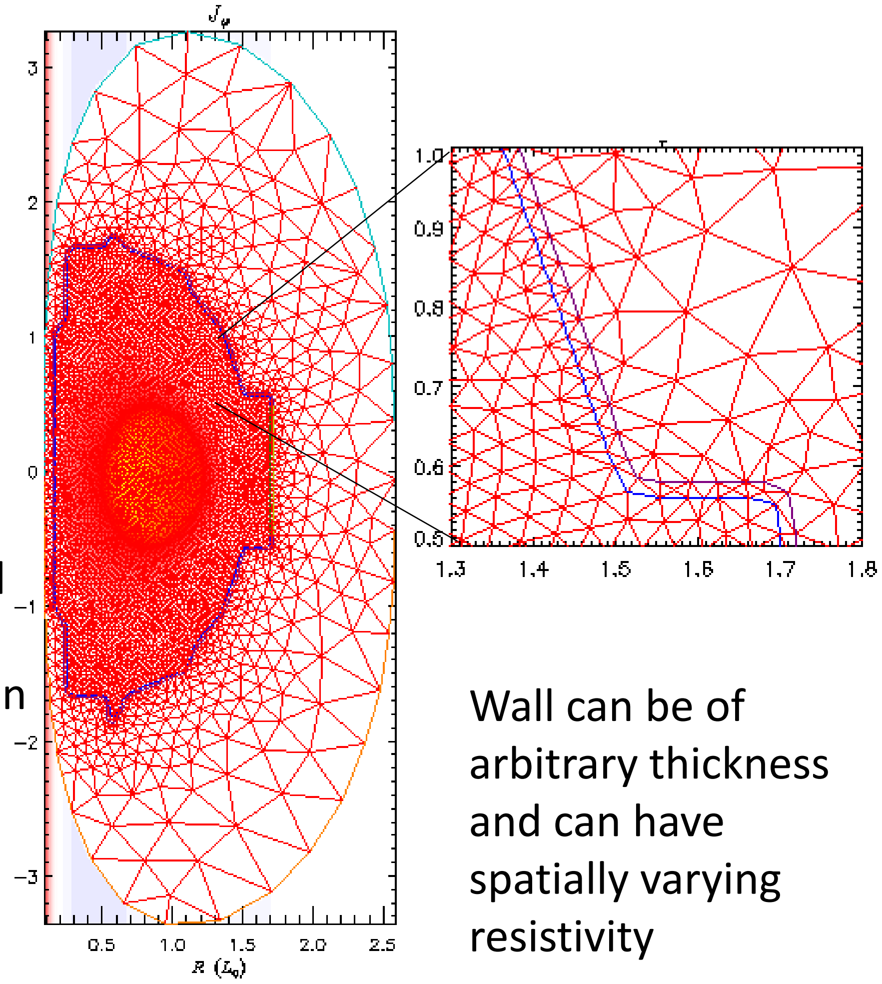
RW (**E** = η_W **J**)

Plasma (X-MHD)

BC:

- **v**, p, n set at inner wall
- **B** set at outer (ideal) wall

- No boundary conditions on **B** or **J** at the resistive wall
- (halo) Current can flow into and out of the wall



Wall can be of arbitrary thickness and can have spatially varying resistivity

*Ferraro, et al. ,Phys Plasma **23** 056114 (2015)

2. Basic equations of runaway electrons (RE)

Fluid Runaway Electron Model

- In our model, the runaways move practically at a very large speed c (is much higher than Alfven speed) and parallel to the magnetic field line.
- Runaway electron is coupled to bulk plasmas through the runaway current in generalized Ohm's law.
- We also add parallel diffusion term to RE on 3D nonlinear simulation to stabilize the numerical instabilities.

$$\frac{\partial n_{RE}}{\partial t} + \nabla \bullet \left(n_{RE} c \frac{\mathbf{B}}{B}\right) = S_{RE}$$

RE density equation

$$\mathbf{J}_{RE} = -e n_{RE} c \frac{\mathbf{B}}{B}$$

RE current assumption

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne}\left(\mathbf{R}_c - \mathbf{R}_{RE} - \nabla \bullet \Pi_e\right) + \mathbf{S}_{CD}$$

Single fluid Ohm's law with RE

$\mathbf{R}_{RE} = \hbar ne \mathbf{J}_{RE}, D_{RE}$ is the parallel diffusivity of runaway electrons.

Red terms are additional runaway electron terms.

3. Eigen values solved from reduced MHD equations with RE

Reduced MHD equations with RE

$$\omega\psi - k_{||}\phi = i\eta\left(\nabla_\perp^2\psi + j\right)$$

$$\omega\nabla_\perp^2\phi - k_{||}\nabla_\perp^2\psi = \frac{mj_0'}{r}\psi$$

$$\left(k_{||} + \omega v_A/c\right) j = \frac{mj_0'}{r}\left(\psi + v_A\phi/c\right)$$

P. Helander, 2007

Where $$\nabla_\perp^2 = \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) - \frac{m^2}{r^2}$$

$$k_{||} = \frac{nq(r) - m}{r}$$

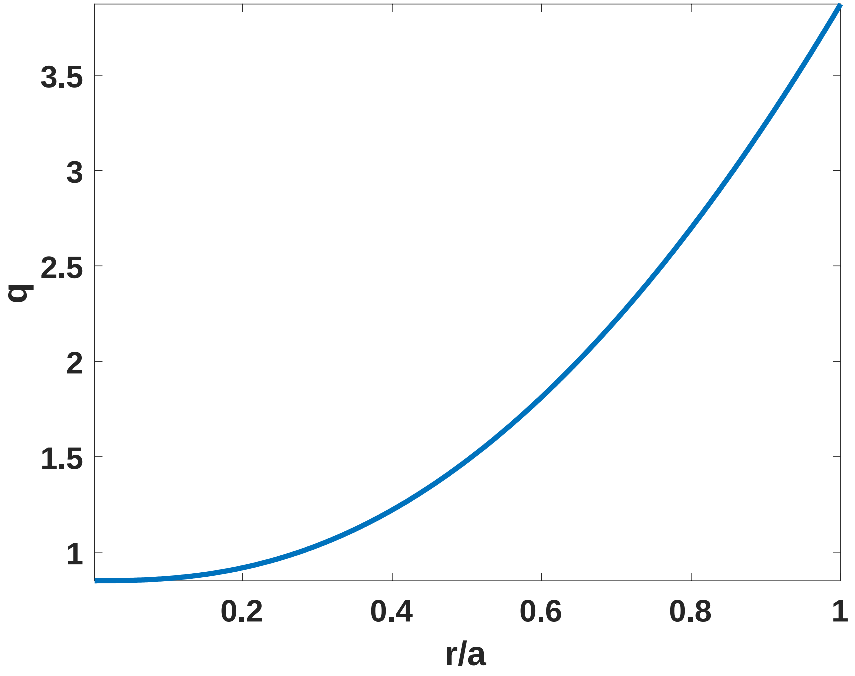
$j_0' = \frac{d}{dr}j_0$, and j is RE current.

- We transform the equations to the matrix and use Matlab eigenvalue solver to get the eigenvalue ω (real frequency and growth rate) and eigenvectors Ψ, ϕ, j (mode structure) of the mode.

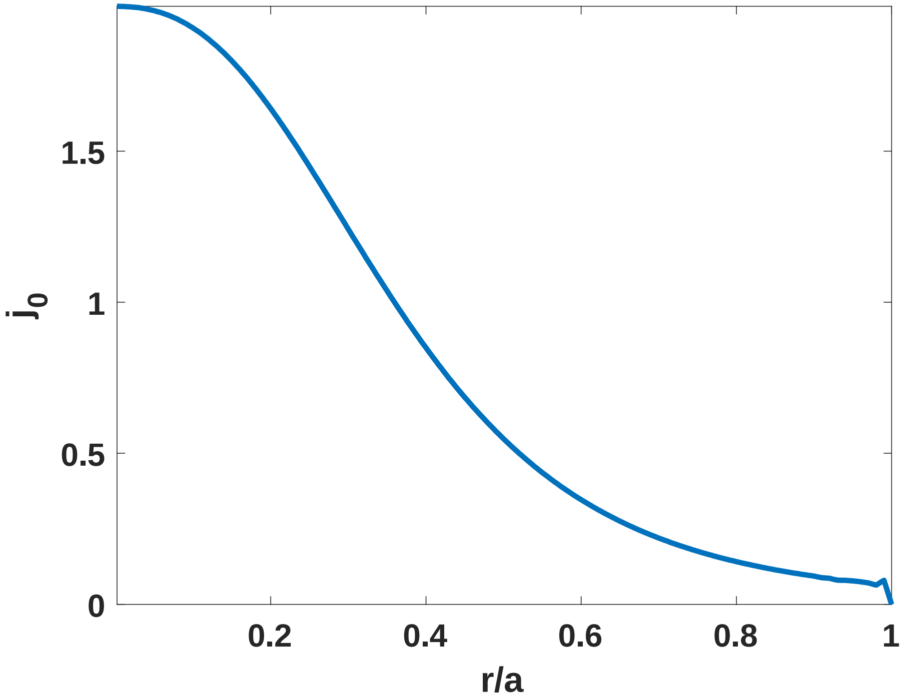
$$\omega\begin{bmatrix} I & 0 & 0 \\ 0 & \nabla_\perp^2 & 0 \\ 0 & 0 & v_A/c \end{bmatrix}\begin{bmatrix} \psi \\ \phi \\ j \end{bmatrix} = \begin{bmatrix} i\eta\nabla_\perp^2 & k_{||} & i\eta \\ k_{||}\nabla_\perp^2 + mj_0'/r & 0 & 0 \\ mj_0'/r & mj_0'v_A/rc & -k_{||} \end{bmatrix}\begin{bmatrix} \psi \\ \phi \\ j \end{bmatrix}$$

Equilibrium for 1/1 kink mode eigen function calculation

- q - profile



- Current - profile

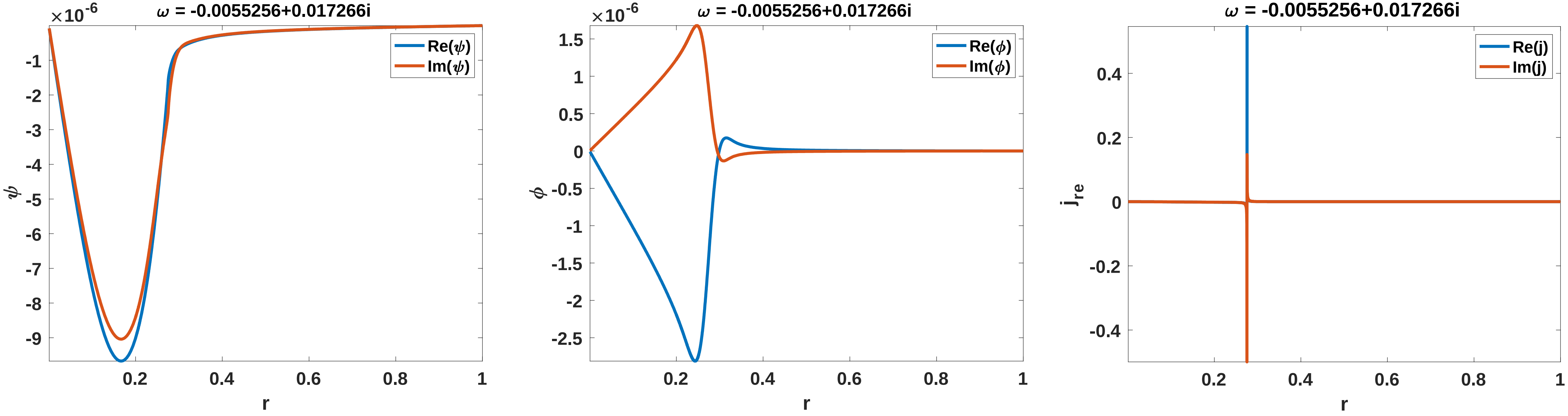


- Other parameters of equilibrium

$\eta = 1.0 \times 10^{-5} \mu_0 v_A$ ·1m, m = 1, n = 1,
$c = 240 v_A$, matrix size = 2000 × 2000

- We use these global profiles to calculate the global properties of 1/1 kink mode

Eigenvalues and eigen vectors of 1/1 kink mode with RE

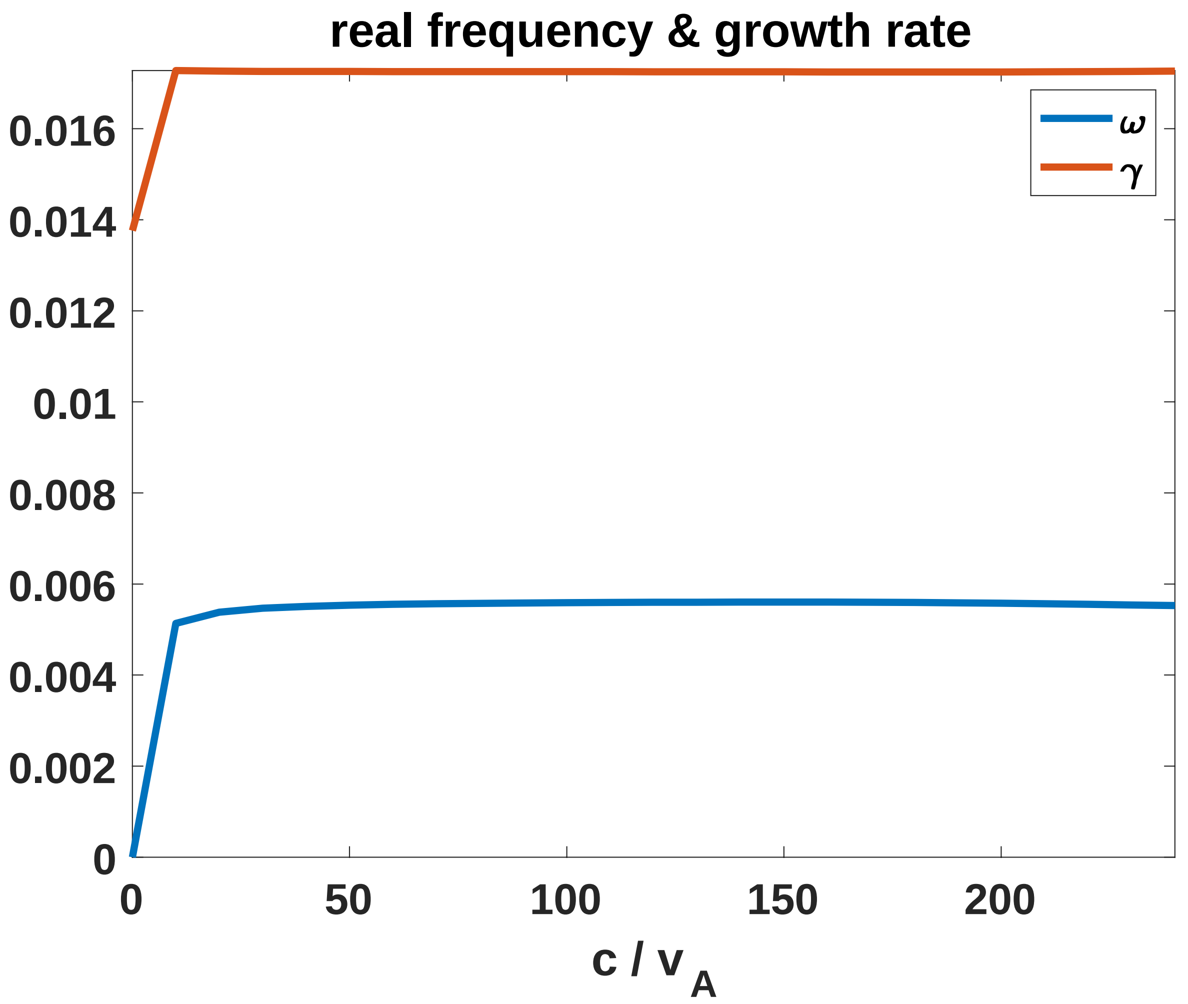


- The radial structures of magnetic flux and electric potential are the conventional 1/1 kink mode structures.

- The RE current has a very steep peak around q = 1 surface.

- The real frequency is about 3 times lower than growth rate.

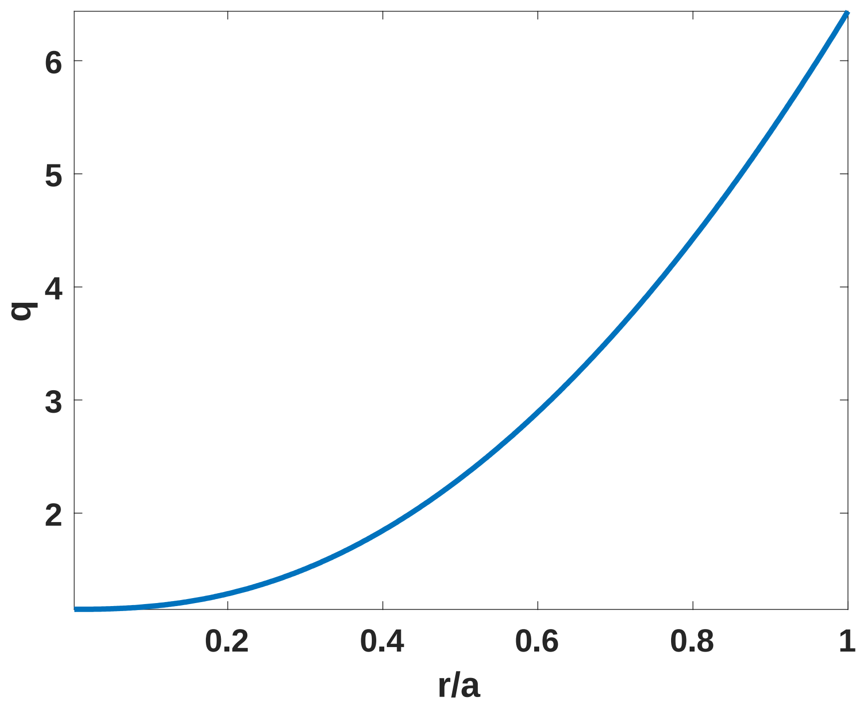
Linear growth rate and real frequency of kink mode with different runaway velocity



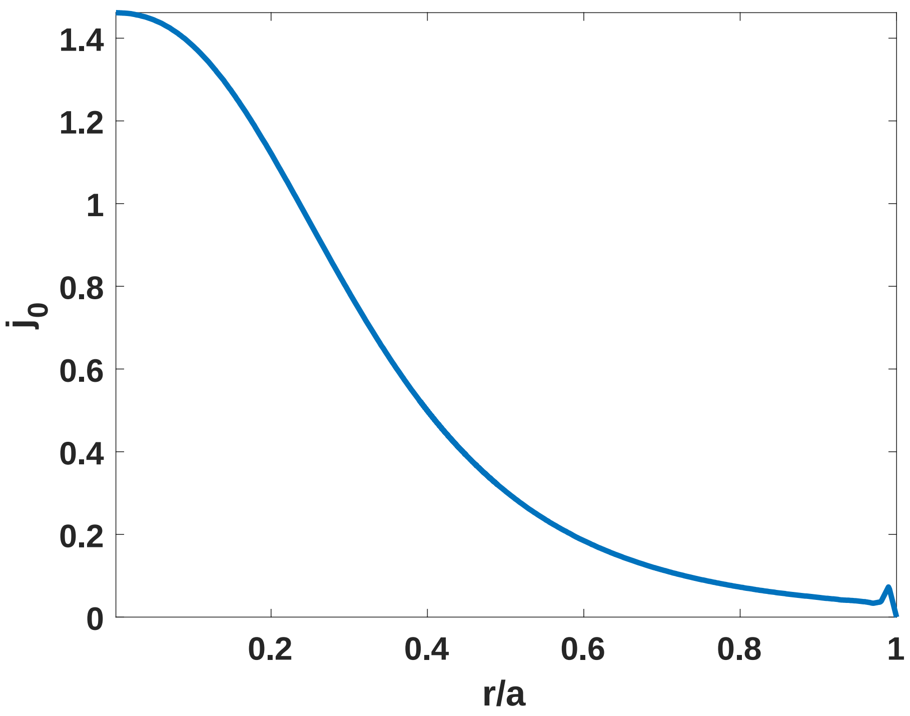
- The 1/1 kink mode has a nearly constant real frequency when the RE speed is large enough.
- The growth rate is also becomes a lager and constant when RE speed lager than a certain value.
- It indicate that the 1/1 kink mode jump from one eigen state to another by the affect of a strong enough RE current.
- The growth rate is about 3 times larger than real frequency.

Equilibrium for 2/1 kink mode eigen function calculation

- q - profile



- Current - profile

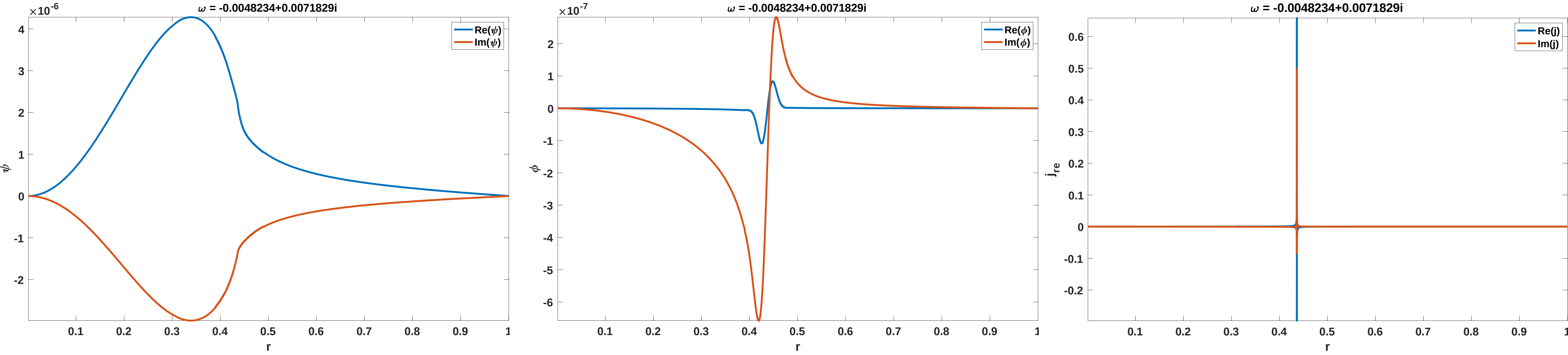


- Other parameters of equilibrium

r/a = (0,1), $\eta = 1.0 \times 10^{-5} \mu_0 v_A \cdot 1$m, m = 1, n = 1, $c = 240 v_A$, matrix size = 2000 × 2000

- We use these global profiles to calculate the global properties of 2/1 kink mode

Eigenvalues and eigen vectors of 2/1 kink mode with RE



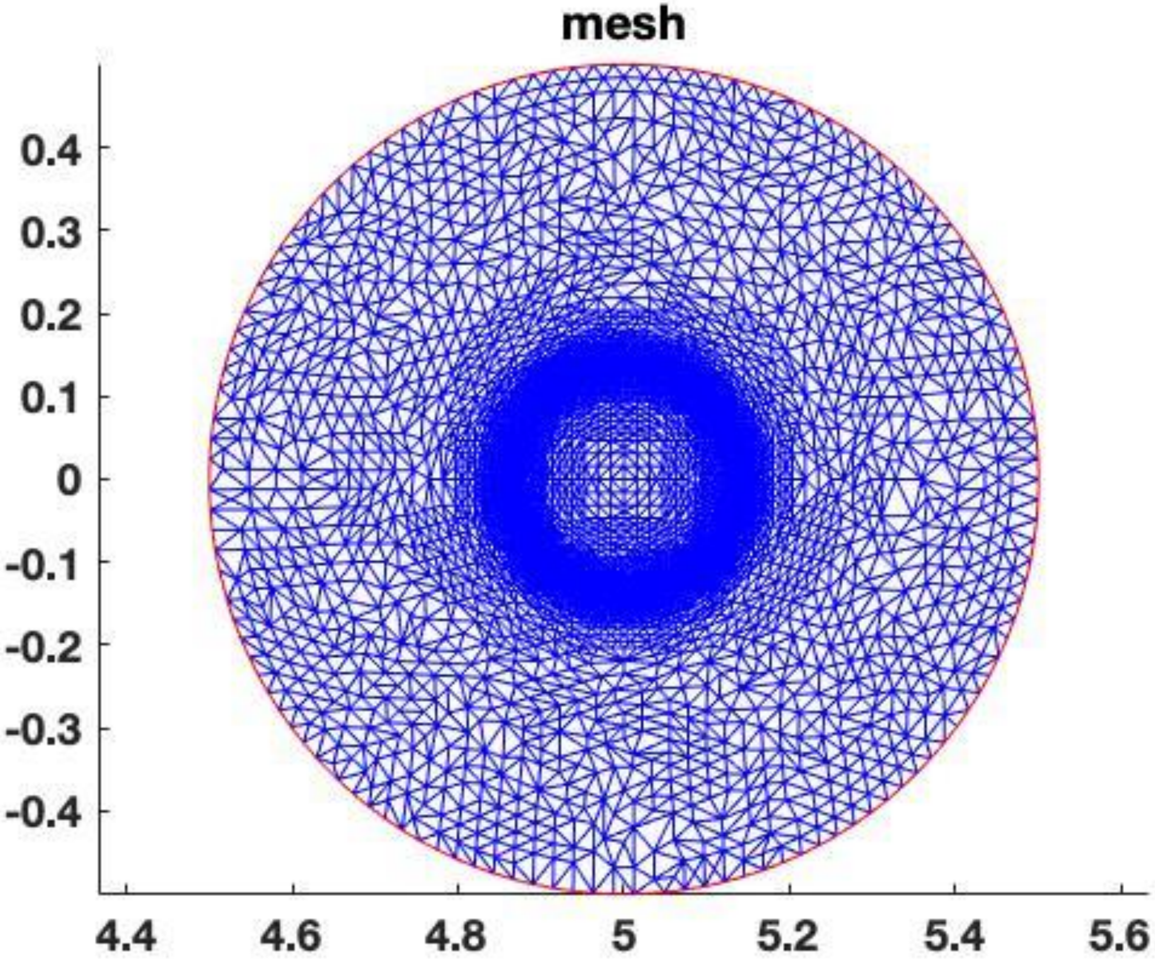
- The radial structures of magnetic flux and electric potential are the conventional 2/1 kink mode structures.

- The RE current has a very steep peak around q = 2 surface.

- The real frequency do not change very much from 1/1 kink mode while the growth rate is much smaller. It indicate that the rotation is mainly driven by RE current.

4. Linear simulation of kink mode with RE

Mesh and basic parameters for 1/1 kink mode simulation

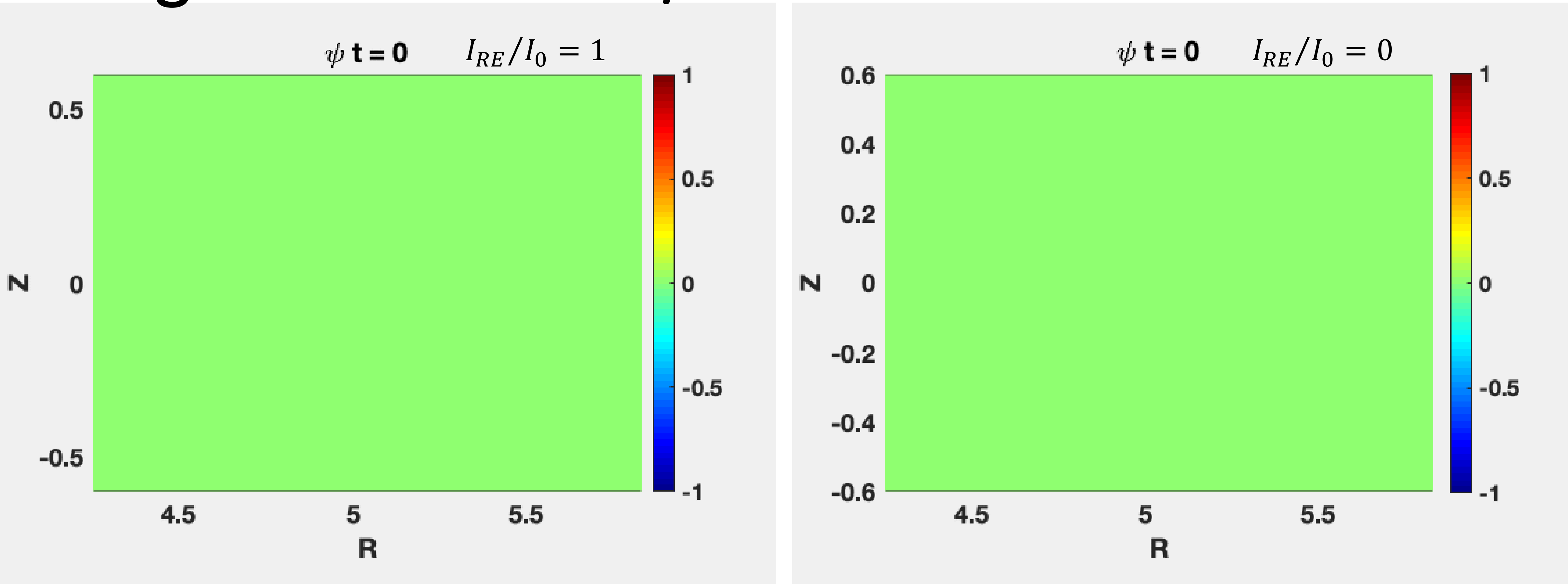


- Parameters of equilibrium

$$\beta_0 = 1.0 \times 10^{-6}$$
$$q_0 = 0.85$$
$$q = q_0 \left[1.0 + \left(\frac{r_{norm}^2}{4}\right)\right]^{1.25}, r_{norm} = \frac{r}{a}$$
$$a = 0.5m$$
$$B_0 = 4.2T$$
$$\eta = 2.0 \times 10^{-5}\ \Omega\mathrm{m}$$
$$n_0 = 1.0 \times 10^{20} m^{-3}$$
$$c = 240 v_A$$
$$N_{elements} = 1 \times 10^4$$
$$\Delta t = \tau_A$$

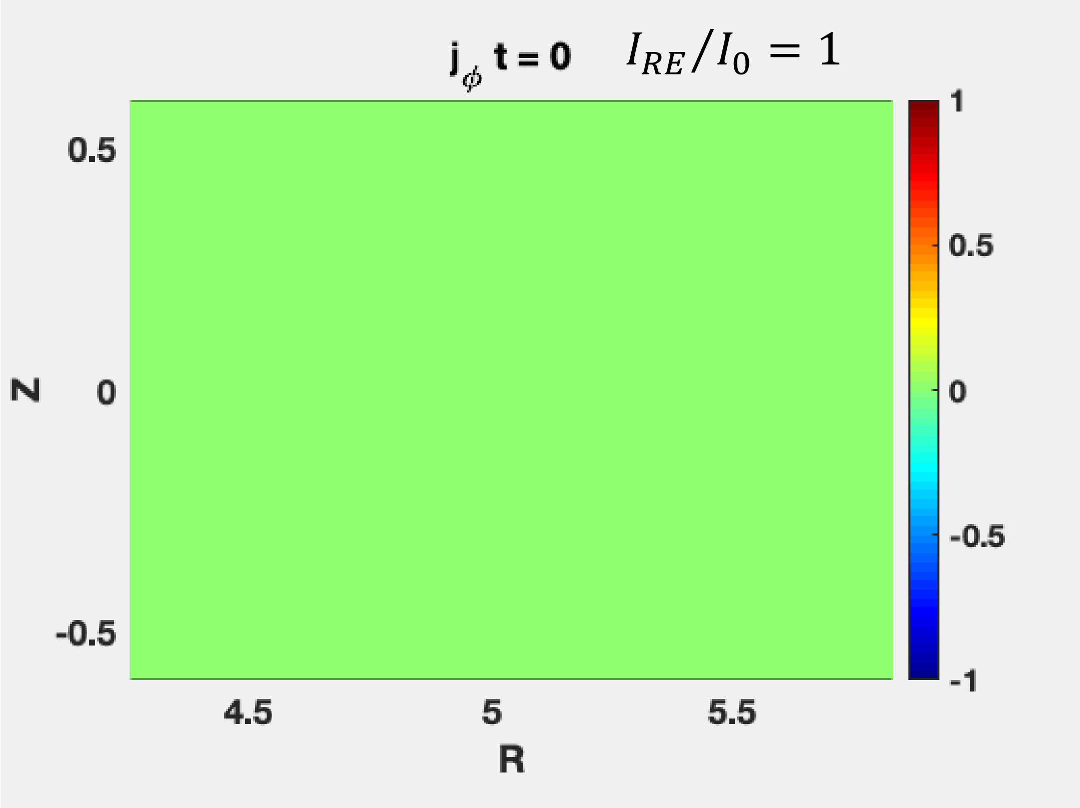
- In our simulations, we use an adaptive mesh which has increased resolution near the q = 1 rational surface.

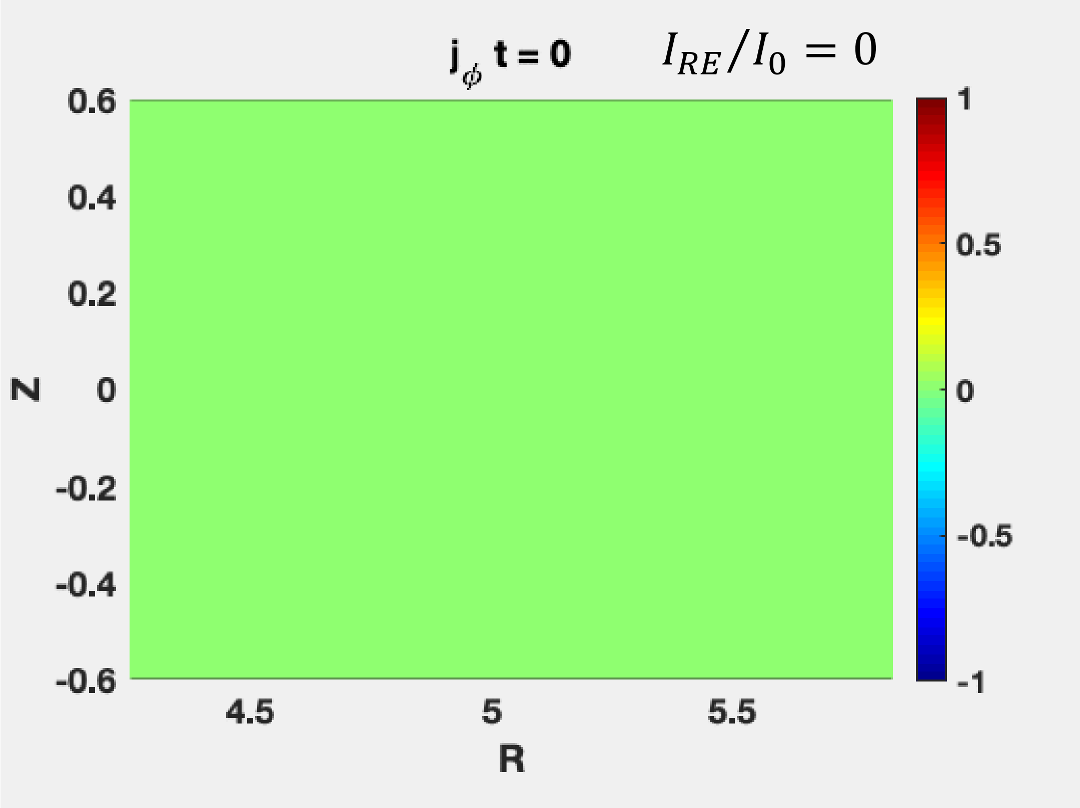
Magnetic island of 1/1 kink mode



- The mode structure of 1/1 kink mode with RE is similar with 1/1 kink mode with out RE.
- The runaways drive the 1/1 kink mode islands rotate with a constant frequency.

Toroidal current perturbation of 1/1 kink mode

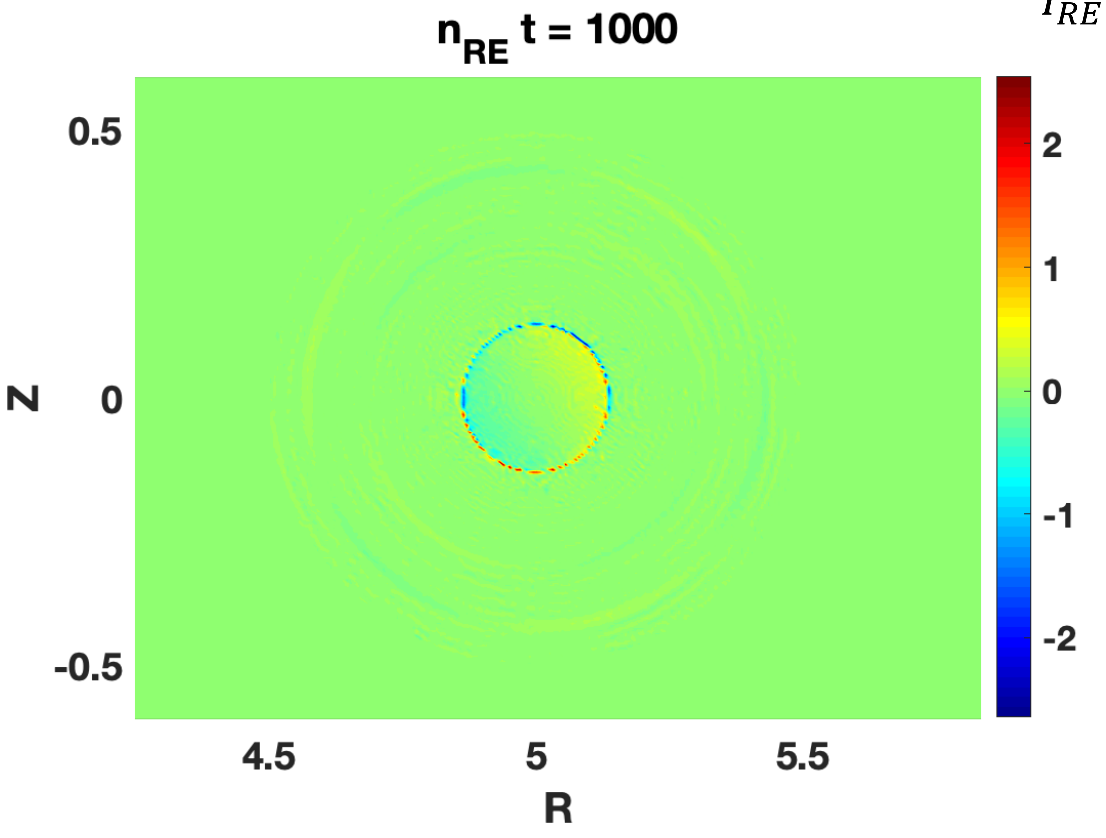


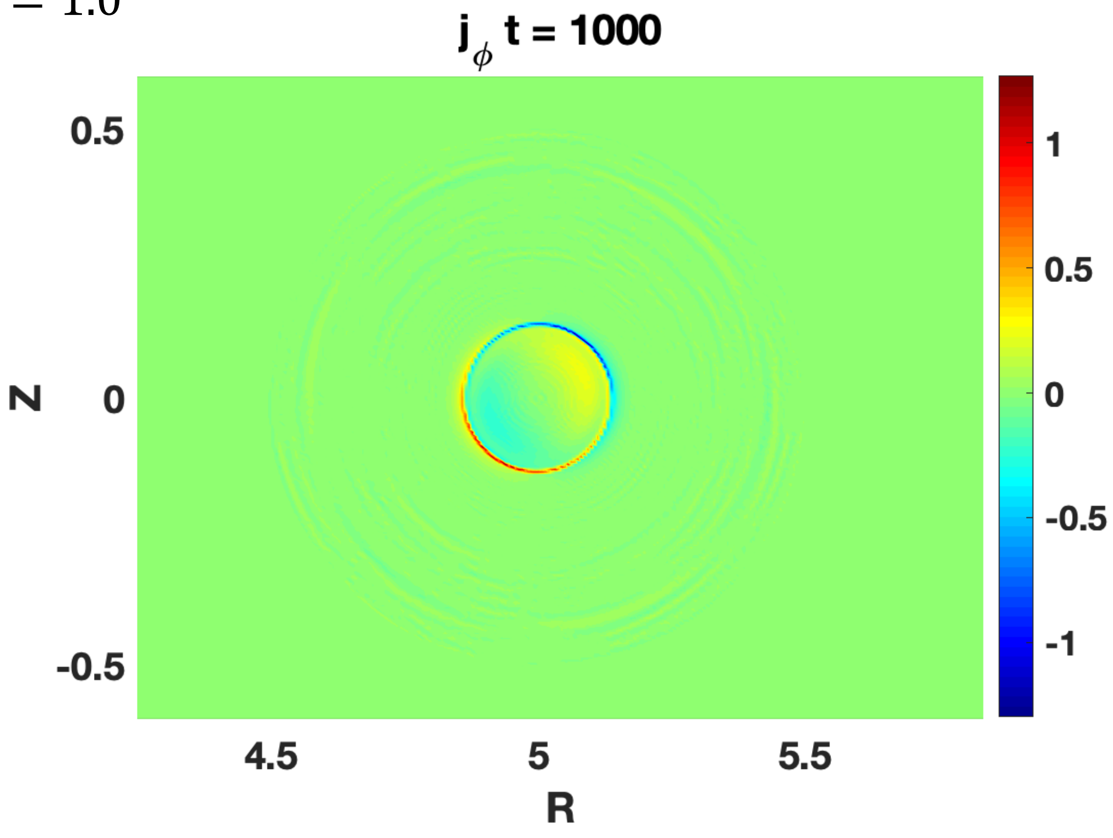


- The RE current perturbation is more peaked around the rational surface than without runaways, and also has a rotation.

RE density and current perturbation of 1/1 kink mode

$I_{RE}/I_0 = 1.0$

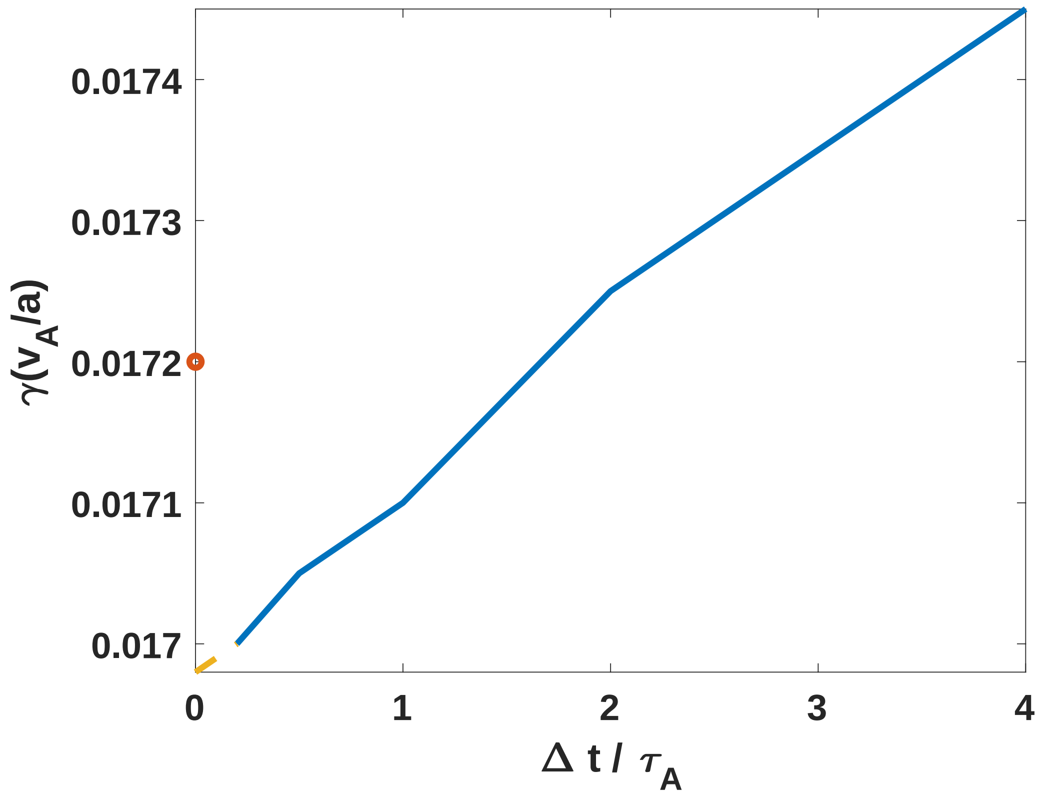




- The perturbed RE density is peaked around the q=1 surface and drive a toroidal current peaked around the rational surface. $dJ_f \sim dJ_{RE} = -edn_{RE}c$
- The peaked RE current affect the magnetic reconnection of 1/1 kink mode and make it has a different linear properties.

1/1 kink mode numerical convergence

$N_{elements} = 1 \times 10^4$

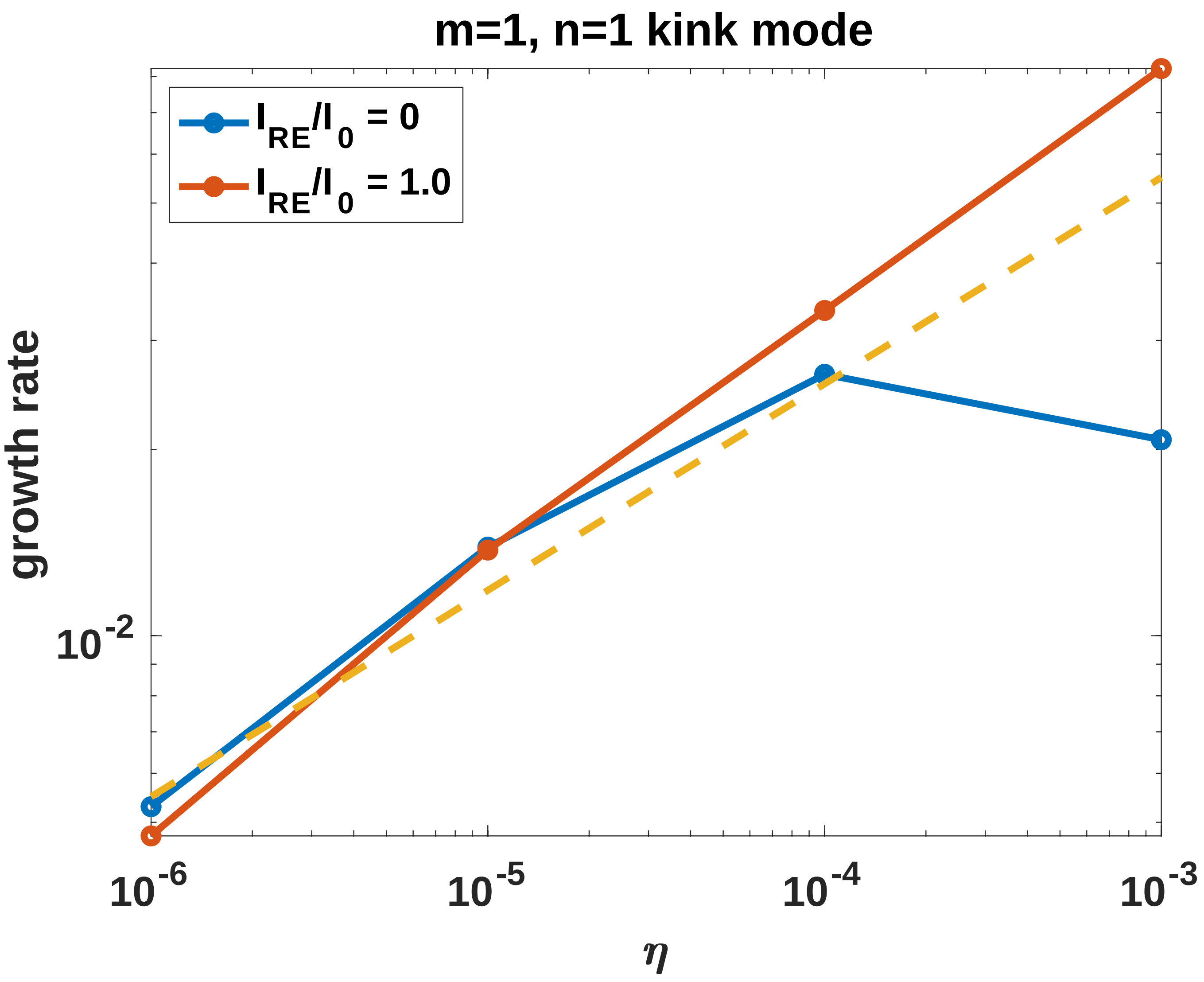


- We use the eigen value calculated from reduced MHD equations instead of the growth rate at Δt = 0 point. (red point)

- The growth rate calculate by M3D-C1 is converged to 0.017 when Δt = 0, and it has a ~1% deviation with the eigenvalue.

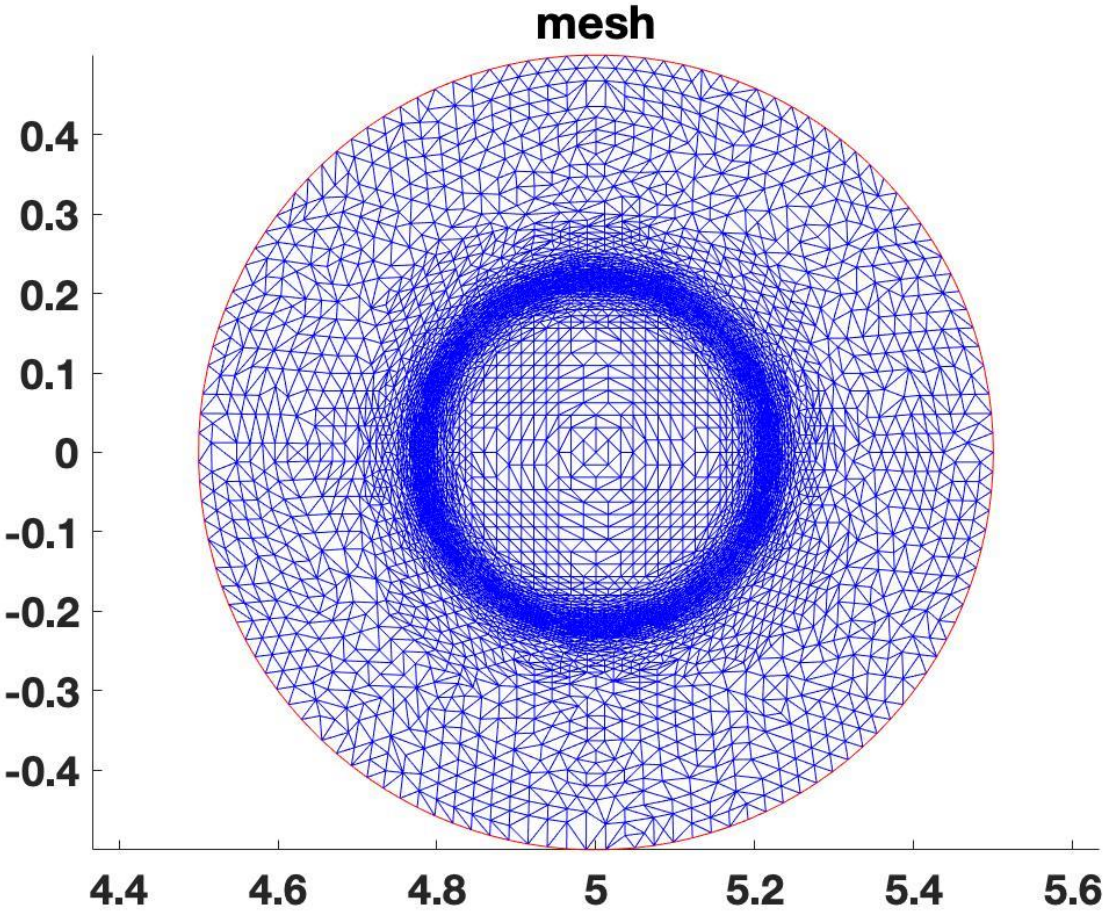
- It indicate that the M3D-C1 simulation is consistent with the eigenvalue calculation.

Linear growth rate of kink mode with different resistivity



- For low resistivity cases, the growth rate of 1/1 kink mode with and without RE obeys the 1/3 low of resistivity.
- For higher resistivity cases, the resistivity correction is more clearly for no RE 1/1 kink mode but do not affect the 1/1 kink mode with RE.
- It means that runaway current have restrained the resistivity correction of kink mode.

Mesh and basic parameters for 2/1 kink mode simulation

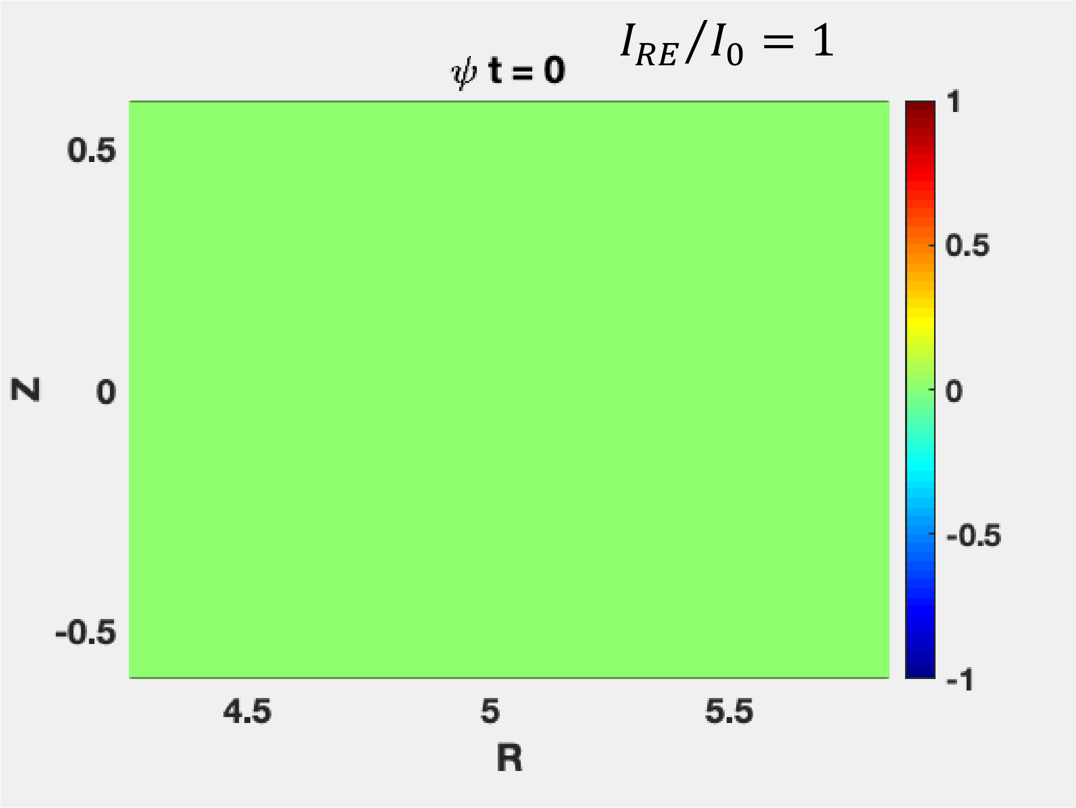


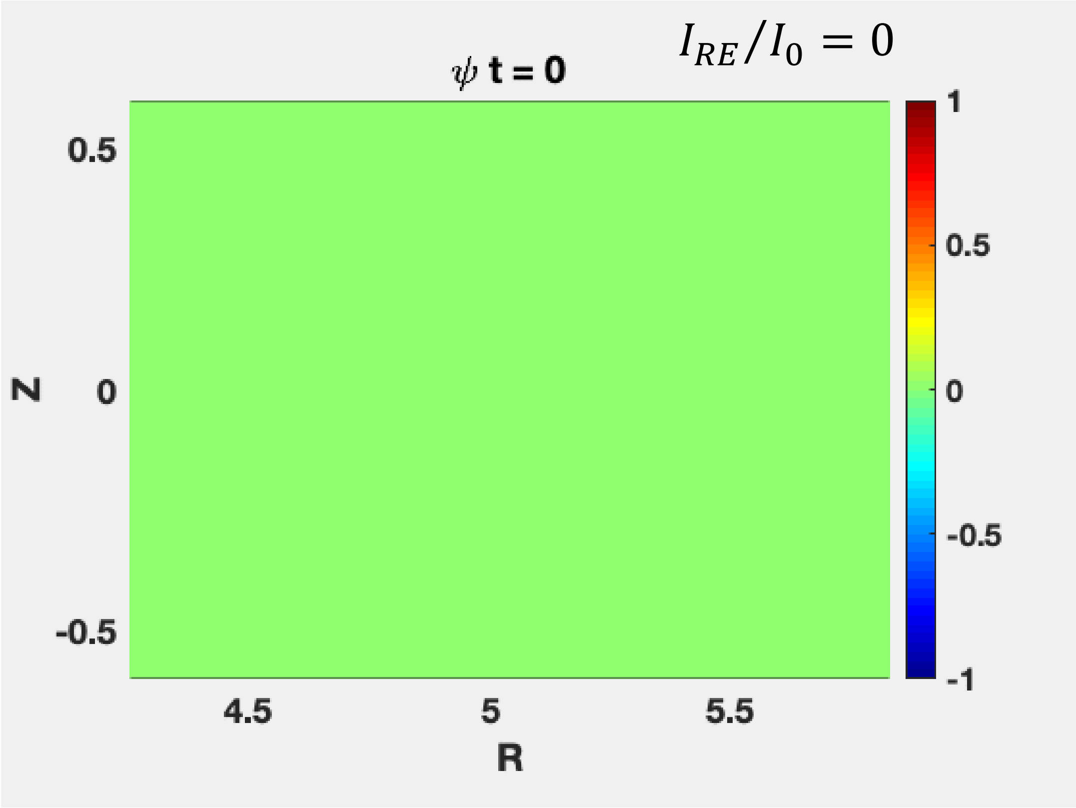
- Parameters of equilibrium

$$\beta_0 = 1.0 \times 10^{-6}$$
$$q_0 = 1.15$$
$$q = q_0 \left[1.0 + \left(\frac{r_{norm}^2}{5}\right)\right]^{1.2}, r_{norm} = \frac{r}{a}$$
$$a = 0.5m$$
$$B_0 = 4.2T$$
$$\eta = 2.0 \times 10^{-5}\ \Omega\mathrm{m}$$
$$n_0 = 1.0 \times 10^{20} m^{-3}$$
$$c = 240 v_A$$
$$N_{elements} = 1 \times 10^4$$
$$\Delta t = \tau_A$$

- In our simulations, we use an adaptive mesh which has increased resolution near the q = 2 rational surface.

Magnetic island of 2/1 kink mode





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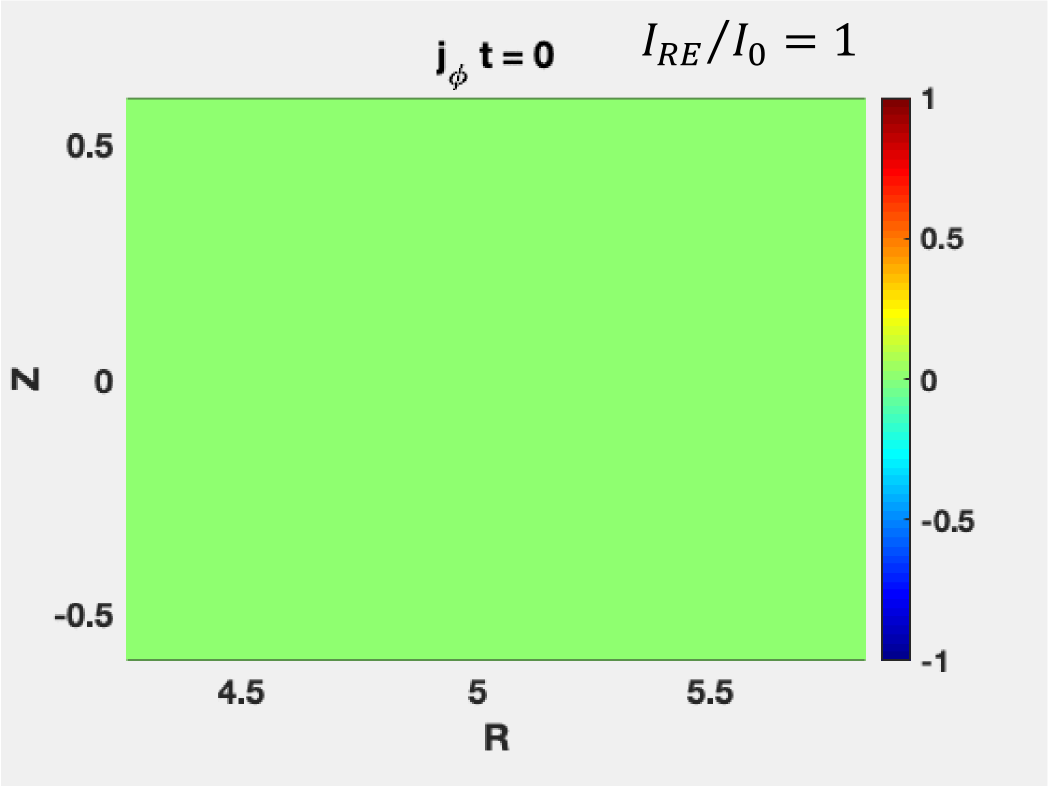
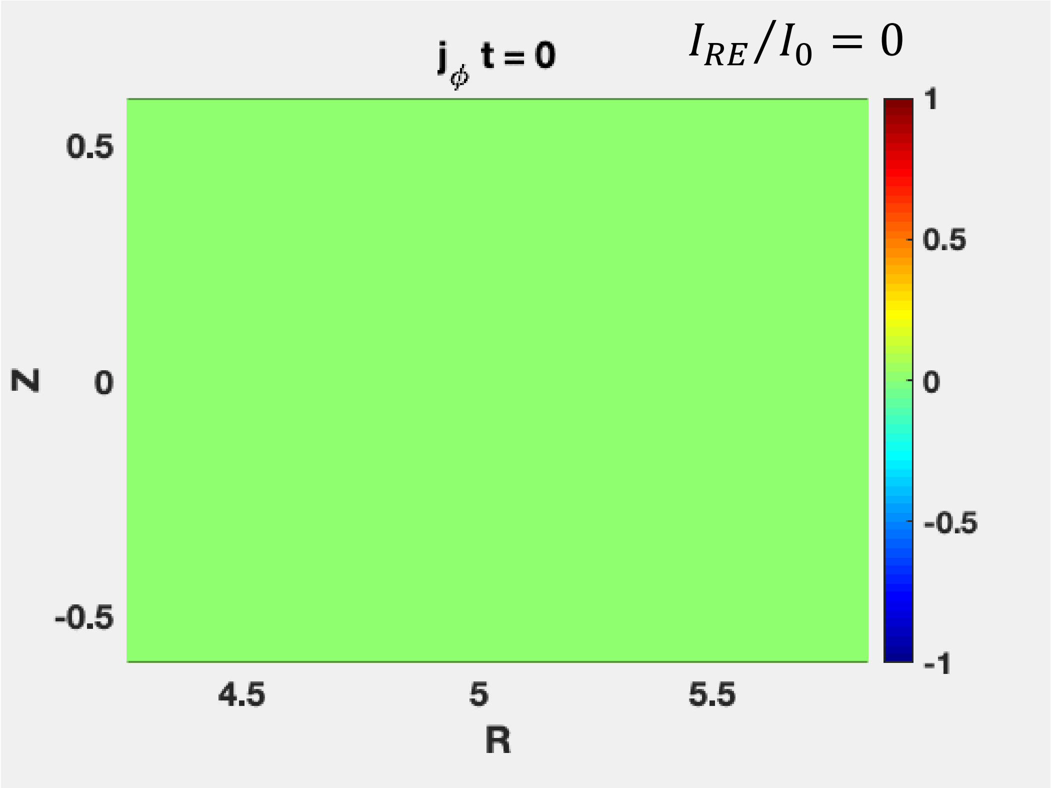
Toroidal current perturbation of 2/1 kink mode

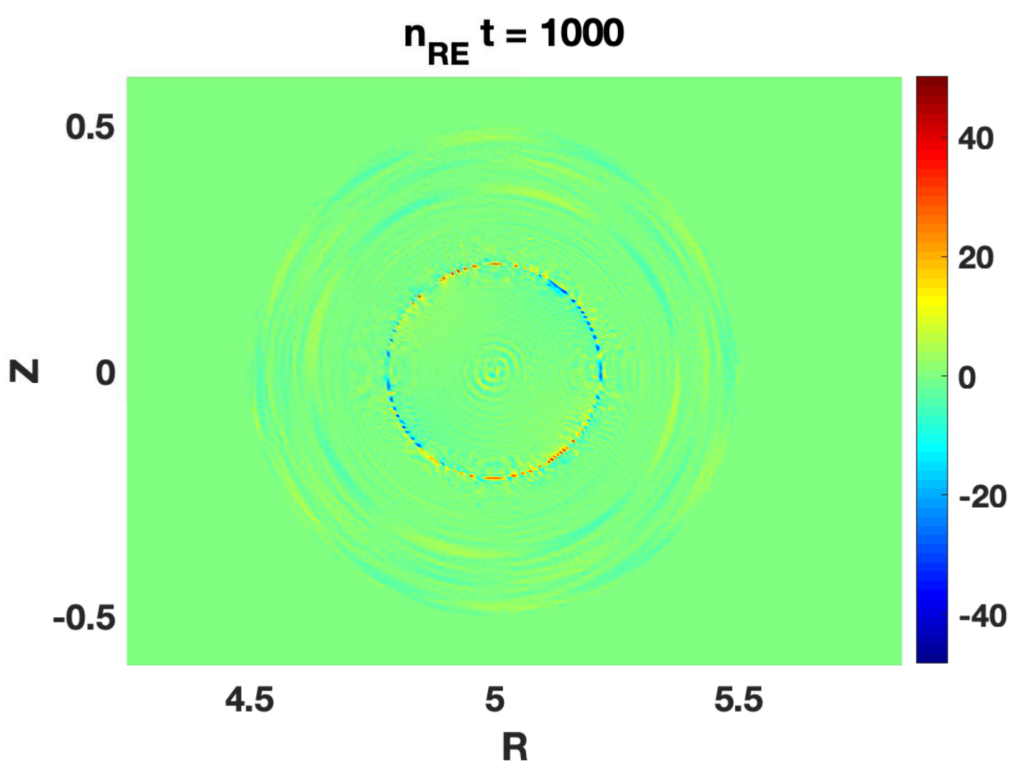


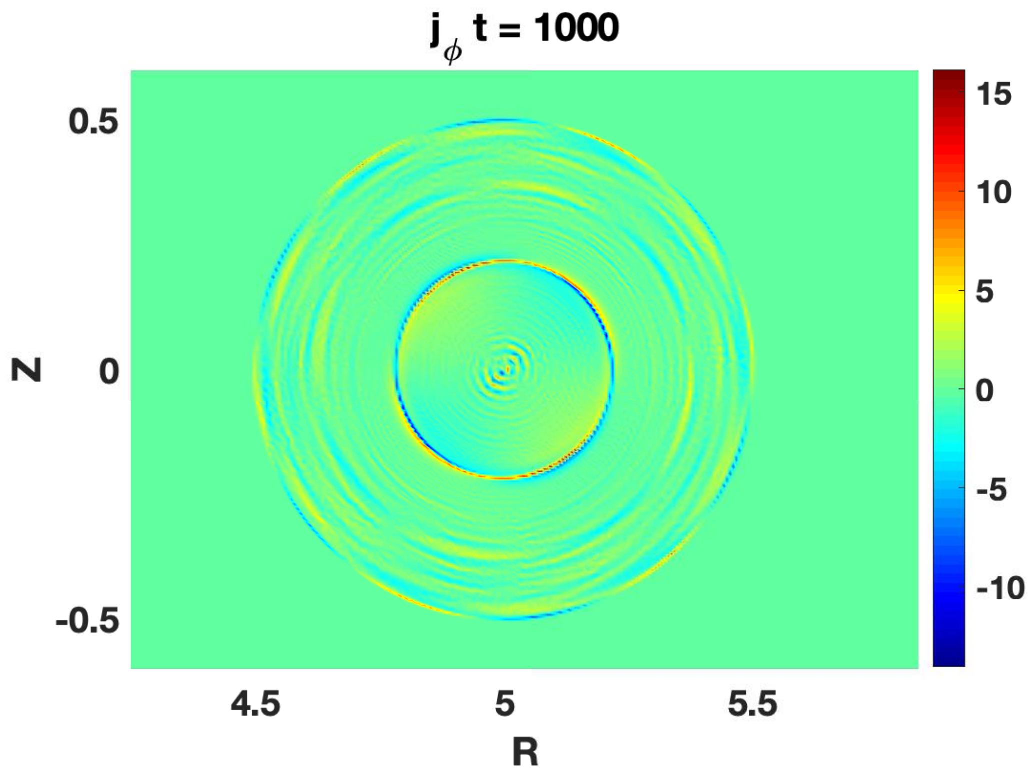


- The RE current perturbation is more peaked around the rational surface than without runaways, and also has a rotation.

RE density and current perturbation of 2/1 kink mode

$I_{RE}/I_0 = 1$

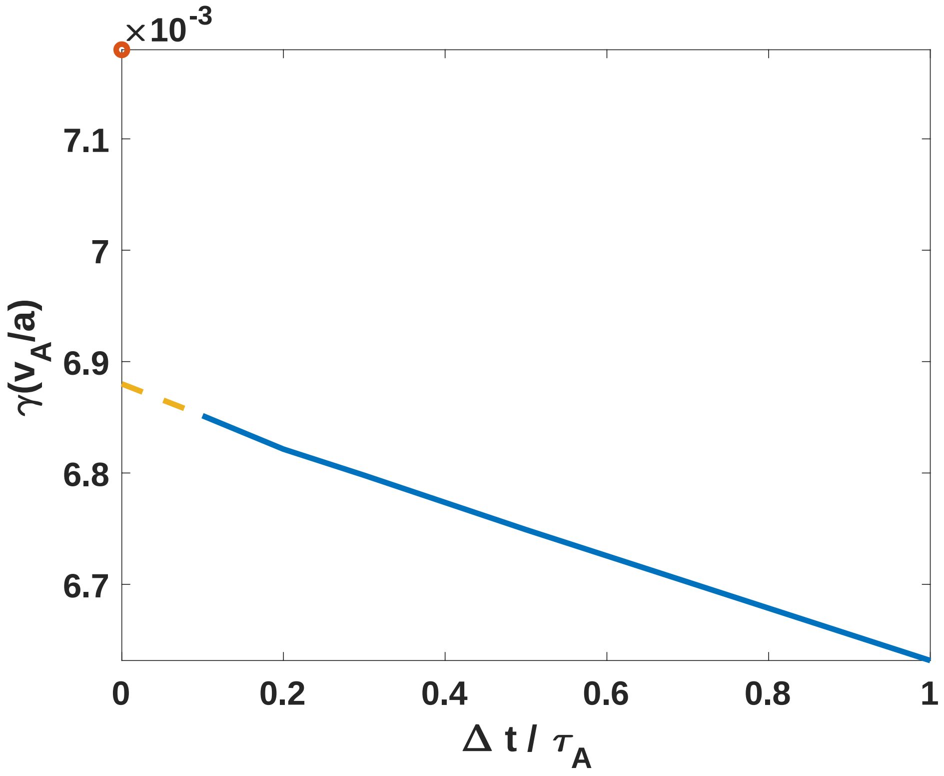




- The perturbed RE density is peaked around the q=1 surface and drive a toroidal current peaked around the rational surface. $dJ_f \sim dJ_{RE} = -edn_{RE}c$
- The peaked RE current affect the magnetic reconnection of 2/1 kink mode and make it has a different linear properties.

2/1 kink mode numerical convergence

$N_{elements} = 1 \times 10^4$

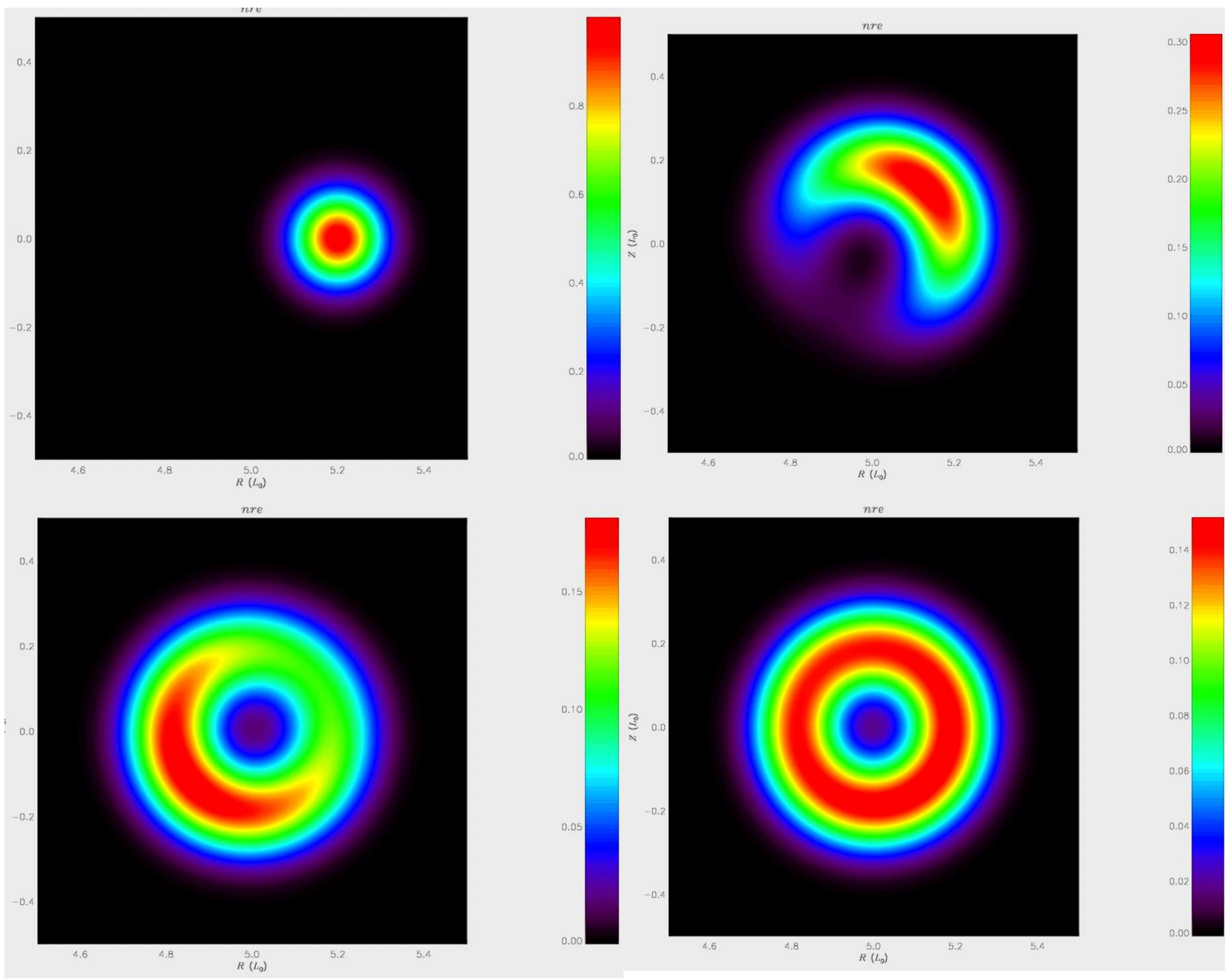


- We use the eigen value calculated from reduced MHD equations instead of the growth rate at Δt = 0 point. (red point)
- The growth rate calculate by M3D-C1 is converged to 0.0069 when Δt = 0, and it has a ~4% deviation with the eigenvalue. The mistake is lager than 1/1 kink mode.
- It indicate that the simulation is consistent with the eigenvalue calculation.

5. Nonlinear simulations with RE

Nonlinear poloidal convection of RE

- Circular cross section of ϕ=0
- q=1.1 everywhere
- RE density is initialized like a gaussian localized in one side.
- Fully implicit time advance (backward Euler).
- The runaway electron density moves along the magnetic field and also has some parallel diffusion, so that it become uniform at every magnetic surfaces.



6. Summary and future works

- The perturbed toroidal current of 1/1 and 2/1 kink mode will be peaked around the rational surface by the RE current effects.

- The RE current affect the 1/1 and 2/1 kink mode rotate with time, and if the runaway speed is large enough, it do not affect the growth rate and real frequency when it increase.

- The growth rate of 1/1 and 2/1 kink mode with RE is converged to the eigenvalue calculated from reduced MHD equations when $\Delta t \sim 0$.

- We still working on the nonlinear sawtooth cases with RE and with finish it in future.