Update on RWM Simulations and Analyses

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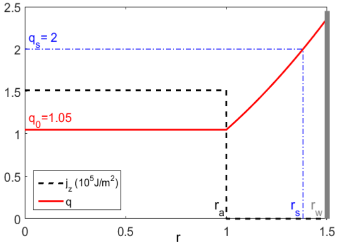
Outline

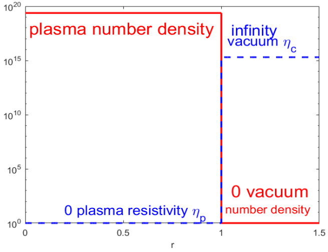
1. Model equilibrium for benchmark and study
2. External kink mode in a single wall grid
3. Resistive wall mode in a double wall grid
4. Summary and discussions

Background and motivation

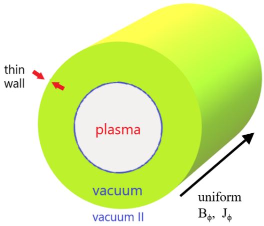
- The goal is to conduct the nonlinear simulation of RWM process leading towards disruptions in toroidal configurations of tokamaks.
- Previous linear NIMROD calculation of RWM based on analytical BCs was benchmarked with J. Finn theory [Montgomery et al 2010].
- Later double-wall grid approach has been developed and tested in NIMROD for the purpose of modeling toroidal nonlinear RWMs [Sovinec and Bunkers 2014, 2019; Wang and Sovinec 2018].
- Here the double-wall grid calculation of cylindrical RWM is further benchmarked with J. Finn theory and applied to the calculation of toroidal RWM.

Model equilibrium model for RWM benchmark





[Finn, 1995]



- Cylindrical ideal plasma with a uniform $B_\phi = 1T$ and $J_\phi = 1.5158 \times 10^5 J/m^2$.
- The $q_s = 2$ resonant surface is in vacuum region.

$$q(r) = \begin{cases} q_0 = 2B_0/(\mu_0 R J_0), & r < r_a \\ q_0 \dfrac{r^2}{r_a^2}, & r > r_a \end{cases}$$

Analytical theory on external kink mode growth rate in such a model equilibrium [Finn, 1995]

1. Ideal wall mode growth rate with a perfectly conducting wall at r_c

$$\gamma_{xk}^2 = \frac{2B_0^2}{\mu_0\rho_0 R^2 q_0^2}\frac{(m-nq_0)}{(1-r_a/r_w)^{2m}}(1-(\frac{r_a}{r_w})^{2m}-(m-nq_0))$$

2. No wall growth rate

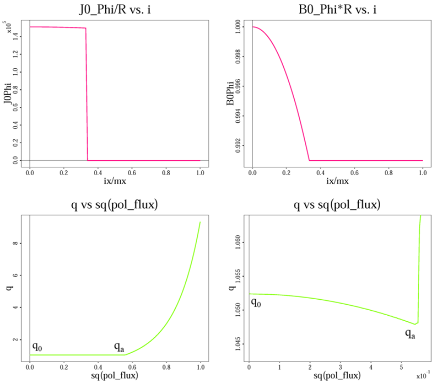
$$\gamma_\infty^2 \equiv \gamma_{xk}^2(r_w=\infty) = \frac{2B_0^2}{\mu_0\rho_0 R^2 q_0^2}(m-nq_0)(1-(m-nq_0))$$

3. RWM growth rate with a thin resistive wall at r_c

$$\gamma_{RW} = \frac{2m}{\tau_w(1-r_a/r_w)^{2m}}(-\frac{\gamma_\infty^2}{\gamma_{xk}^2})$$

Model equilibrium generated in cylindrical geometry numerically for NIMROD calculations with slight modifications

Actual q_0 and q_a are slightly different from the original model. Thus the growth rate calculation is modified.



$$\gamma_{xk}^2 = \frac{2B_0^2}{\mu_0\rho_0 R^2 q_0^2}\frac{(m-nq_0)}{(1-r_a/r_w)^{2m}}$$

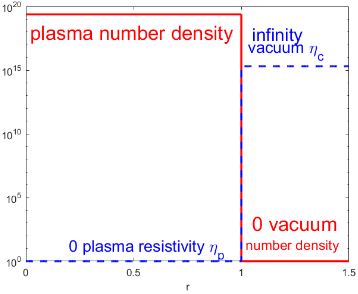
$$\times\left(1-\left(\frac{r_a}{r_w}\right)^{2m}-(m-nq_0)\right)$$

$$\Downarrow \quad \Downarrow \quad \Downarrow$$

$$\gamma_{xk}^2 = \frac{2B_0^2}{\mu_0\rho_0 R^2 q_0^2}\frac{(m-nq_0)^2}{((1-r_a/r_w)^{2m})(m-nq_a)}$$

$$\times\left(1-\left(\frac{r_a}{r_w}\right)^{2m}-(m-nq_a)\right)$$

Hyperbolic tangent profiles adopted to model plasma and vacuum regions



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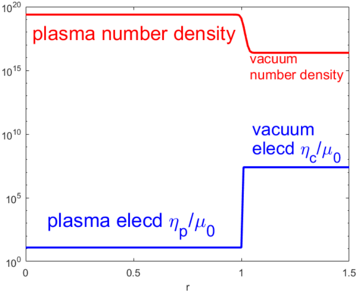
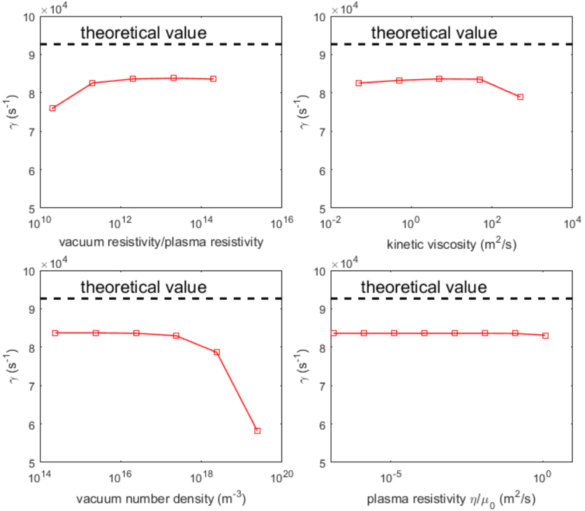
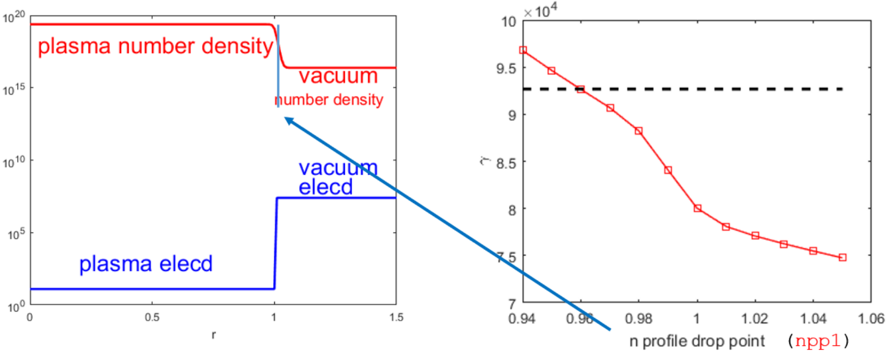


Weak dependence regime of plasma and vacuum parameters identified for ideal wall external kink growth

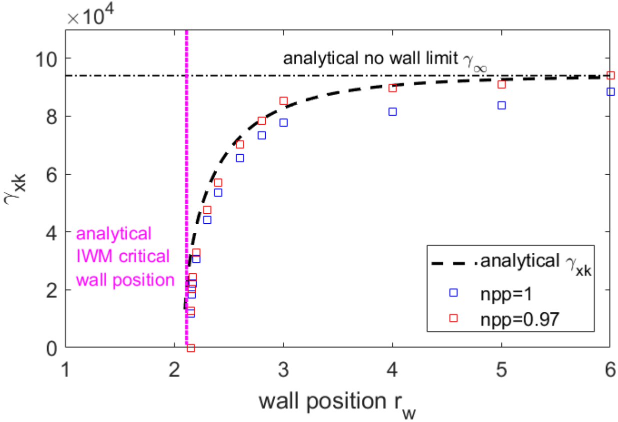


External kink mode growth rate sensitive to the location of plasma-vacuum interface in presence of ideal wall



nq=(ndens-nedge)*(tanh((npp1-r)*npp2)+1)/2+nedge

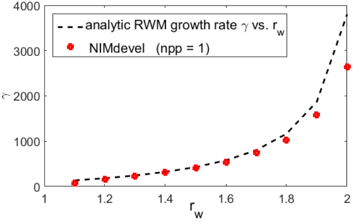
Ideal wall external kink growth agrees with theory with proper choice of plasma-vacuum interface location

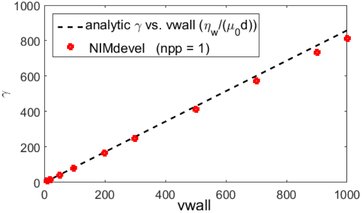


With npp=0.97 ideal wall mode growth rates γ_{xk} from NIMROD match best to analytical results.

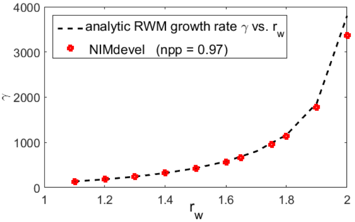
Resistive wall mode growth rates agree with theory in cylindrical geometry with less sensitive dependence on plasma-vacuum interface location

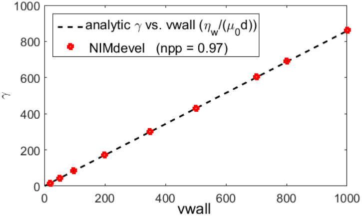
npp=1:



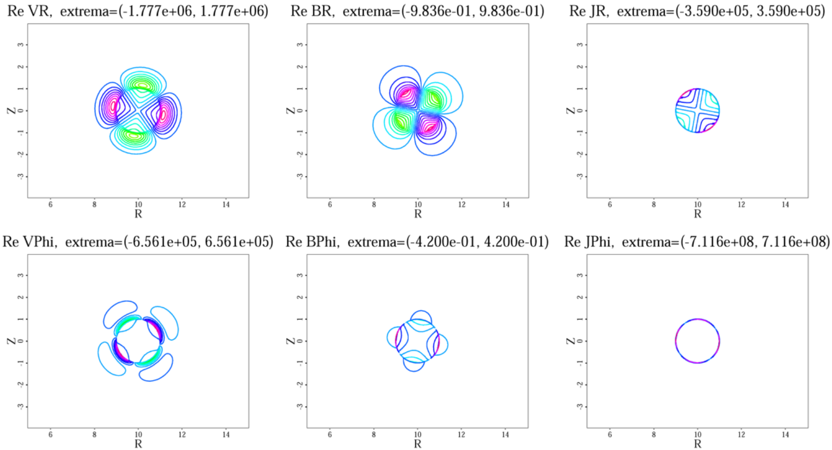


npp=0.97:



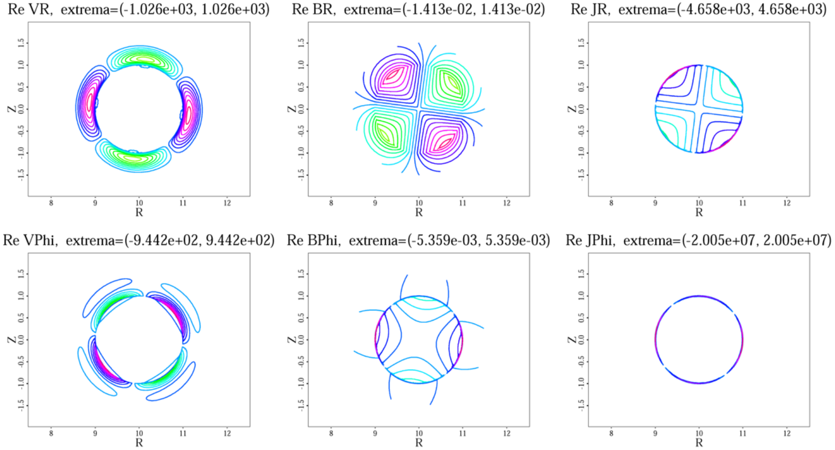


Ideal wall mode structure in cylindrical geometry (NIMdevel)



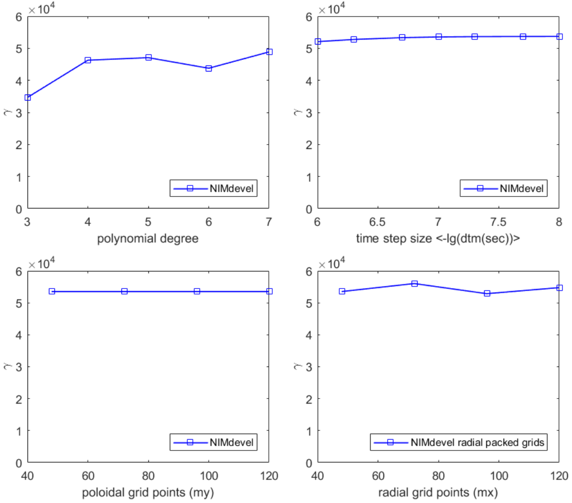
$R_0 = 10, r_w = 3$

Resistive wall mode structure in cylindrical geometry (NIMdevel)

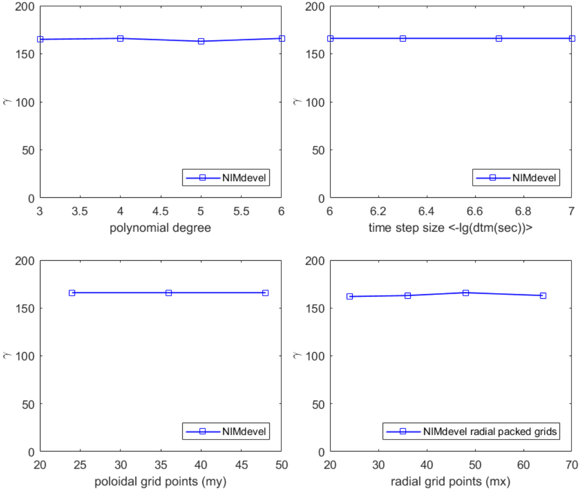


$R_0 = 10, r_w = 1.5$

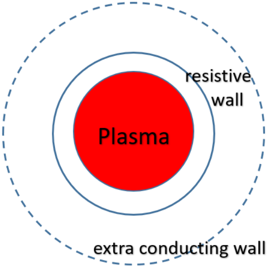
Convergence tests for ideal wall mode in cylindrical geometry (NIMdevel)

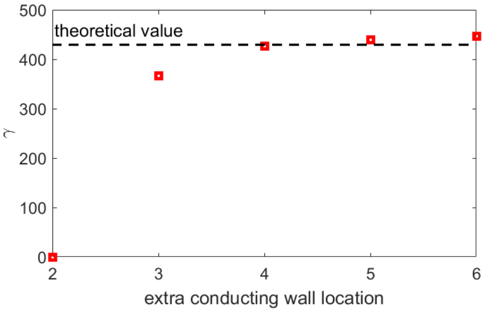


Convergence test for resistive wall mode in cylindrical geometry (NIMdevel)



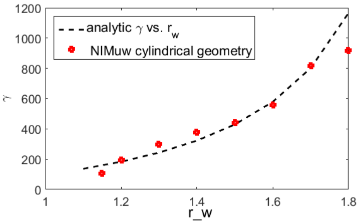
Resistive wall mode in cylindrical geometry with double wall grid

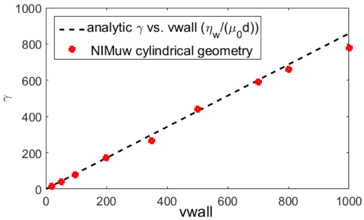




When outer perfect conducting wall is sufficiently far away, RWM growth rate approaches theory value.

Resistive wall mode growth rates in cylindrical geometry with double wall grid agree with theory

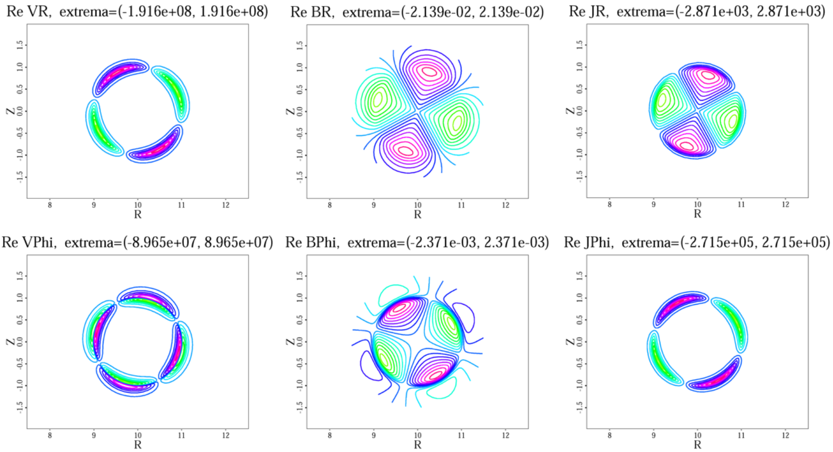




RWM growth rate dependence on wall position ($vwall = 500$) and wall resistivity ($r_w = 1.5$).

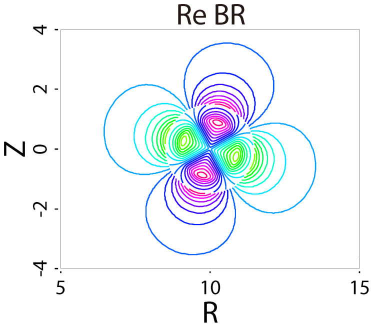
The growth rates match well with analytical results for $r_w = 1.2a \sim 1.8a$.

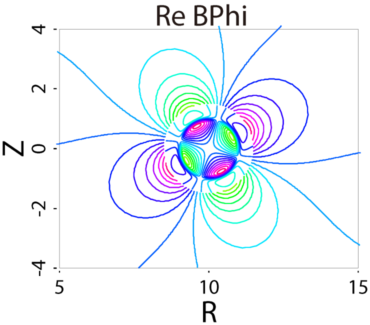
Resistive wall mode structure in cylindrical geometry inside inner wall (physical)



$R_0 = 10, r_w = 1.5$, outer wall locates at $r_{extr} = 6$.

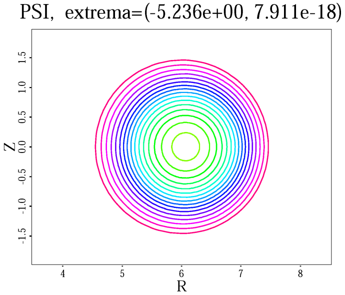
RWM field structure along with vacuum field between inner (physical) and outer (computational) walls

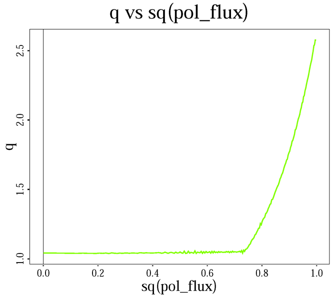




Magnetic filed is continuous across inner resistive wall.

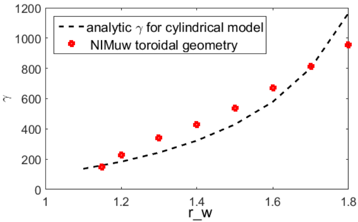
NIMEQ generates toroidal equilibrium with same model tokamak profiles as in cylindrical geometry

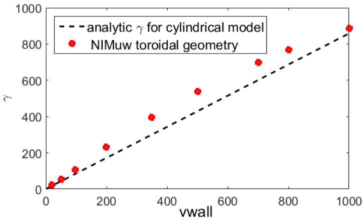




$R_0 = 6, r_w = 1.5$

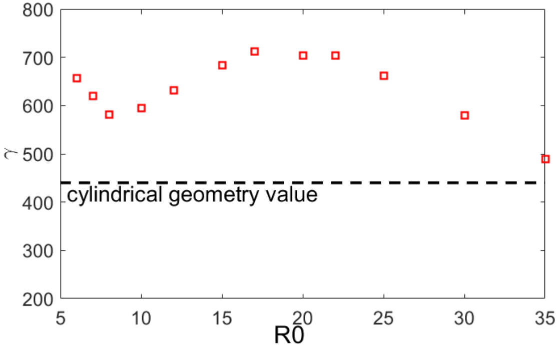
RWM growth rates in toroidal equilibrium deviate from theory value for cylindrical geometry





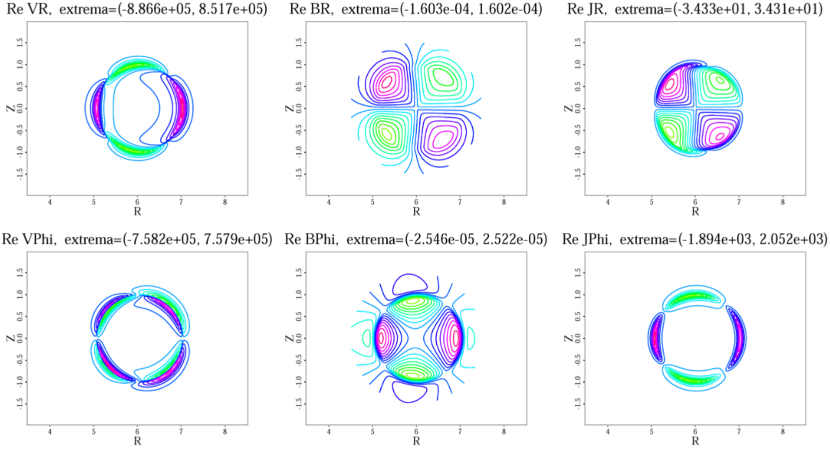
RWM growth rates versus wall position ($vwall = 500$) and wall resistivity ($r_w = 1.5$), $R_0 = 10$.
The growth rates agree well with analytical results for $r_w = 1.2a \sim 1.8a$.

Toroidal effect on RWM growth rate approaches cylindrical value at large aspect ratio



RWM growth rates calculated in toroidal geometry higher than cylindrical geometry.

Resistive wall mode structure in toroidal geometry inside inner (physical) wall shows slight inboard-outboard asymmetry



$R_0 = 6$, $r_w = 1.5$, outer (computational) wall locates at $r_{extr} = 5$.

Summary and discussions

1. Linear growth of external kink mode of a model equilibrium in presence of both ideal and resistive wall calculated using NIMROD are in good agreement with analytical theory.
2. External kink mode growth from NIMROD calculation sensitive to location of plasma-vacuum interface.
3. Resistive wall mode growth becomes enhanced with in- and out-board asymmetric mode structure in toroidal configuration.
4. Next step is to extend the calculation for nonlinear RWM in cylindrical and toroidal configurations.

References I

Finn, J. M. (1995).
Resistive wall stabilization of kink and tearing modes.
Physics of Plasmas, 2(1):355 – 386.