Validation and verification of continuum ions and REs in NIMROD*

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Outline

- Early results of NIMROD/M3D-C1 benchmark using DIII-D discharge #141216.
- Phase space tests of NIMROD's continuum RE implementation.
- Formulation of continuum REs for linear (2,1) tearing mode in cylinder.

Ideal internal kink benchmark between NIMROD and M3D-C1.

- Brochard *et al.* (Nucl. Fusion 62 (2022) 036021) benchmarks several codes on an ideal internal kink in DIII-D discharge #141216.

Nucl. Fusion **62** (2022) 036021

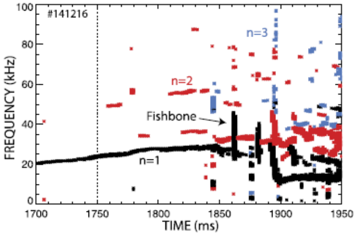


Figure 1. Experimental spectrogram in ms measured on the DIII-D discharge #141216. The first toroidal harmonics from $n = 1$ to 3 are respectively displayed in black, red and blue. A clear $n = 1$ internal kink mode appears around $t = 1750$ ms, while a $n = 1$ fishbone mode emerges at $t \sim 1890$ ms.

Nucl. Fusion 62 (2022) 036021 G. Brochard *et al*

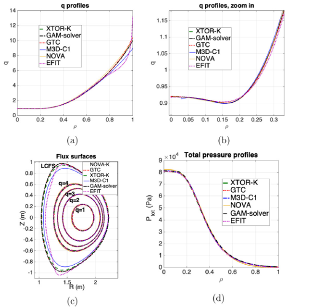


Figure 2. Features of the numerical equilibria used in all codes for the kink benchmark, reconstructed from the DIII-D shot #141216 at $t =$ 1750 ms. (*a*) Safety factor profile (*b*) safety factor profile in the core. (*c*) Flux surfaces in the poloidal plane. (*d*) Total plasma pressure in Pa.

Okay agreement in ideal growth rates.

Nucl. Fusion 62 (2022) 036021

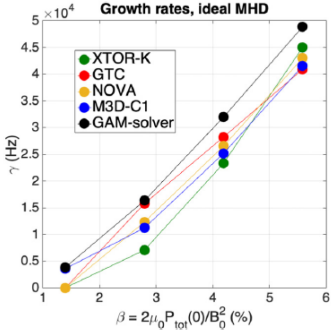
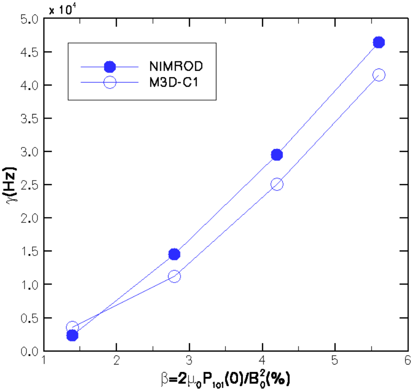
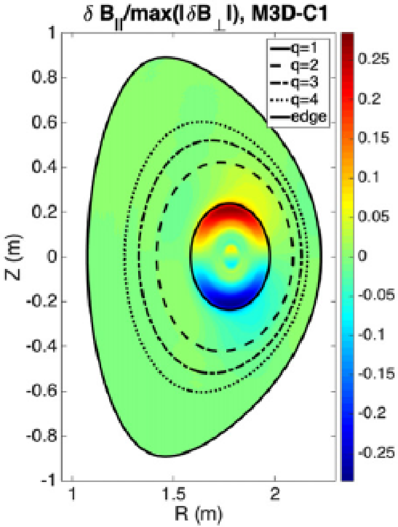


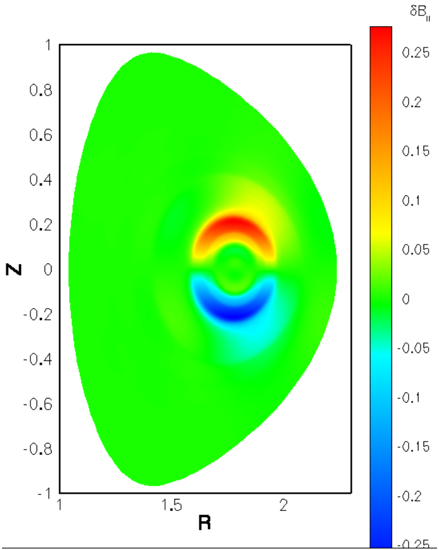
Figure 3. On-axis beta-scan of the internal kink growth rate obtained from the different codes, in the ideal MHD limit. An excellent quantitative agreement is obtained between codes using MHD and gyrokinetic formalisms.



Good agreement for $\delta B_{\|}$ eigenfunction.

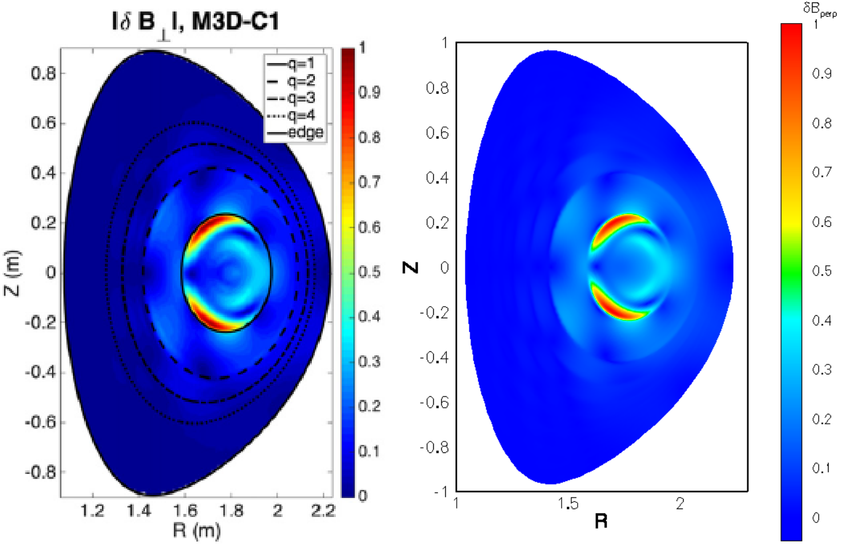
- $\delta B_{\|}$ (finite-β effects) important in these computations*.





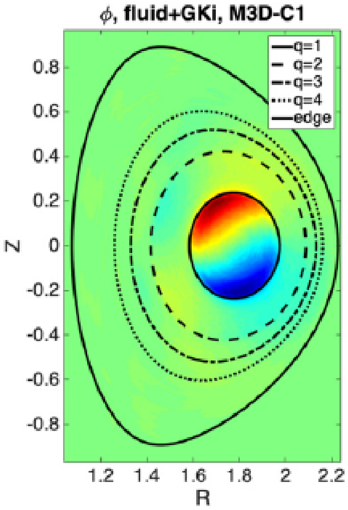
*Graves *et al.*, Plasma Phys. Control. Fusion 61 (2019) 104003

Good agreement for $\delta B_{\perp}$ eigenfunction.



Future work on DIII-D discharge #141216

- Obtain better agreement between NIMROD and M3D-C1 on ideal kink growth rates.
- Follow up on M3D-C1 efforts with thermal ion kinetic effects using NIMROD's ion CEL-DKE continuum kinetic capability.
- Trevor Taylor successfully defended his thesis, “Serendipity shape functions for hybrid fluid/kinetic-PIC simulations" in September.



Continuum kinetics for REs in NIMROD

- Relativistic electron kinetic equation implemented in NIMROD:

$$\text{streaming}: \frac{\partial f}{\partial t} + \frac{cp}{\gamma}\left[\xi\nabla_\parallel - \frac{1}{2}\left(1-\xi^2\right)\nabla_\parallel \ln B \frac{\partial}{\partial \xi}\right] f + \mathbf{v}_D \cdot \nabla f$$

$$\text{acceleration}: -\frac{eE_\parallel}{mc}\left[\xi\frac{\partial f}{\partial p} + \frac{1-\xi^2}{p}\frac{\partial f}{\partial \xi}\right]$$

$$\text{synchrotron}: -\frac{1}{\tau_r}\frac{1-\xi^2}{\gamma}\left[\gamma^2 p\frac{\partial f}{\partial p} - \xi\frac{\partial f}{\partial \xi} + (4p^2 + \frac{2}{1-\xi^2})f\right]$$

$$\text{collisions, avalanche source}: = C_c\left(f\right) + S_A,$$

where $p = \gamma mv/(mc)$, $\gamma = \sqrt{1+p^2}$, $\xi = p_\parallel/p$, and $\tau_r = 6\pi\epsilon_0(mc)^3/(e^4B^2)$ is the radiation time-scale*.

- Phase space discretization uses finite elements in ξ and a colocation method in p.

*A. Stahl, *et al.*, CPC **212** (2017) 269-279

Continuum kinetics for REs in NIMROD

- Completed: phase space tests of acceleration, synchrotron radiation and linearized relativistic collision operator terms.
  1. Free acceleration test used to determine required phase space resolution.
  2. Partial verification on phase space vortex problem: steady-state balance of acceleration, synchrotron and collision terms.
  3. Tested collisional relaxation of shifted Maxwell-Jüttner distribution.

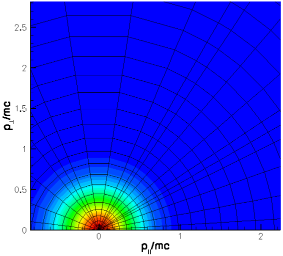
- Underway:
  1. Implementing fully nonlinear, BK formulation of relativistic collision operator in NIMROD (Tyler Markham defends PhD thesis this coming December).
  2. Verifying nonlinear BK operator with results from NORSE code.
  3. Implementing coupling of RE current into NIIMROD's Ohms Law for linear tearing mode studies for comparison with growth rates and real frequencies predicted by NIMROD and M3D-C1 RE fluid models.

Phase space free-acceleration test

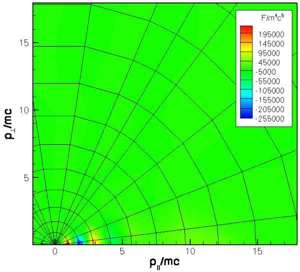
- Unshifted Maxwell-Jüttner distribution:

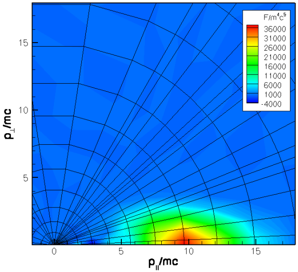
$$f_{MJ}(z) = \frac{zn_0}{4\pi K_2(z)} e^{-\gamma z}$$

with $z = mc^2/(k_B T)$, $\gamma = \sqrt{1+p^2}$ and $p = \gamma v/c$.



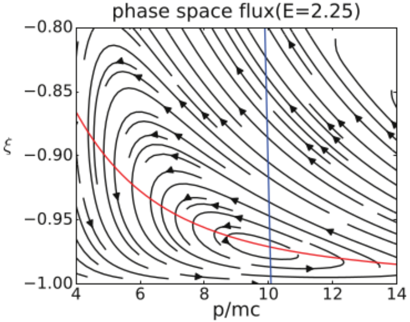
- Free acceleration test shows importance of phase space resolution.

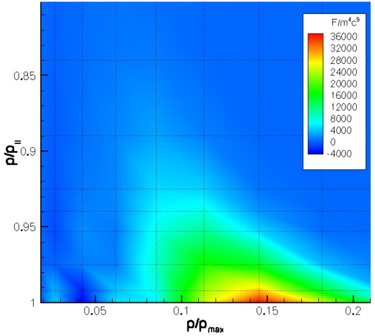




Phase space vortex problem

- Balance of acceleration, synchrotron radiation and linearized collision terms in steady state. *





*Z. Guo, C. J. McDevitt, X.-Z. Tang, PPCF **59** (4) (2017) 044003

Shifted $f_{\mathrm{M}J}$ used to test linearized collision operator

- Shifted Maxwell-Jüttner distribution:

$$f_{\mathrm{M}J}(z, p_b) = \frac{z n_0}{4\pi K_2(z)} e^{-z(\gamma\gamma_b - p_b p \xi)}$$

where $\xi = p_{\|}/p$, $\gamma_b = \sqrt{1 + p_b^2}$, with $p_b = \gamma_b v_b / c$.

- The moment

$$\frac{n_0}{\gamma_b} = 4\pi \int_{-1}^{1} d\xi \int_0^\infty dp p^2 f_{\mathrm{M}J}(z, p_b)$$

returns the density in the moving frame.

- The momentum moment

$$p_b = \frac{2\pi\gamma_b}{n_0} \int_{-1}^{1} d\xi \xi \int_0^\infty \frac{dp}{\gamma} p^3 f_{\mathrm{M}J}(z, p_b)$$

returns the momentum shift, p_b, of the distribution.

Collisional relaxation of shifted f_{MJ} distribution.

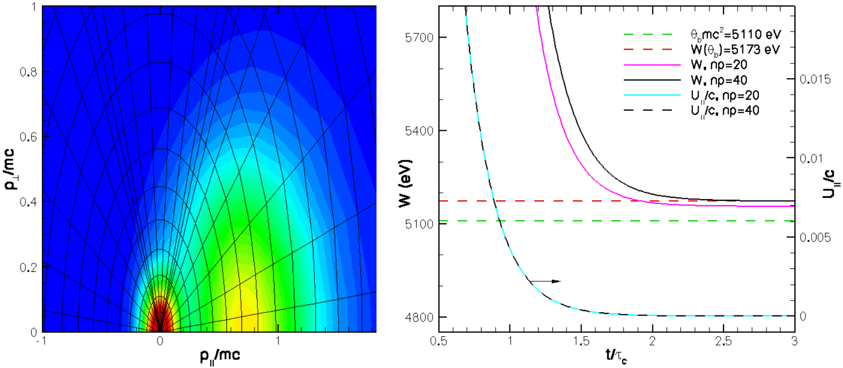
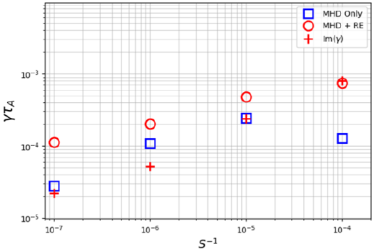
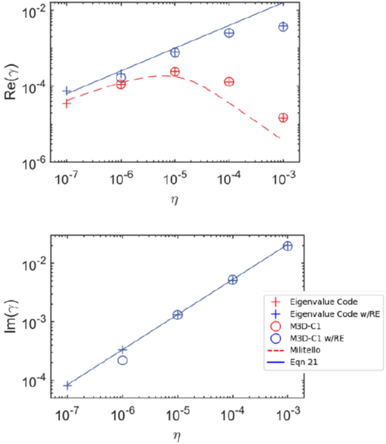


Figure: Shifted f_{MJ} after $t = 0.01\tau_c$ has elapsed. Gridlines depict 7th-order polynomials in 3 FE cells in ξ and np=40 points in p. Right plot shows convergence in the energy moment (W, left axis) and parallel flow ($U_{\|}/c$, right axis), as functions of time for np =20 and 40.

RE fluid models useful for simulating MHD-RE coupling.

- Several codes have recently investigated RE effects on tearing:
  1. JOREK, V. Bandaru et al., PRE **99**, 1-11 (2019)
  2. M3D-C1, C. Liu et al, PoP **27**, 10.1063/5.0018559 (2020)
  3. NIMROD, A. Santerme and C. Sovinec (this meeting, 2022)
- NIMROD (left) and M3D-C1 (right) 2/1 tearing mode growth rates.





Tearing benchmark of fluid and continuum RE formulations in NIMROD

- NIMROD's linearized fluid formulation without perturbed drift terms:

$$\frac{\partial n_r}{\partial t} + c_r \nabla \cdot n_r \mathbf{b}_0 = -c_r \nabla \cdot N_r \delta \mathbf{b}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\delta \mathbf{E}_{\mathrm{ideal}} + \eta \left(\frac{1}{\mu_0} \nabla \times \delta \mathbf{B} - q_e c n_r \mathbf{b}_0 - q_e c N_r \delta \mathbf{b} \right) \right]$$

- Analogous kinetic formulation:

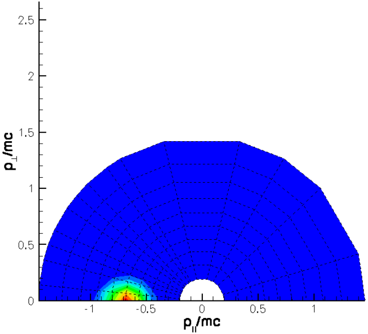
$$\frac{\partial f_r}{\partial t} + \frac{cp}{\gamma} \left[\xi \mathbf{b}_0 \cdot \nabla - \frac{1}{2} \left(1 - \xi^2 \right) \nabla_{\parallel} \ln B_0 \frac{\partial}{\partial \xi} \right] f_r =$$

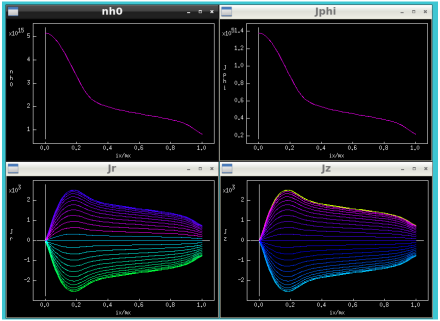
$$-\frac{cp}{\gamma} \left[\xi \delta \mathbf{b} \cdot \nabla - \frac{1}{2} \left(1 - \xi^2 \right) \delta (\nabla_{\parallel} \ln B) \frac{\partial}{\partial \xi} \right] F_r + \frac{e \delta E_{\parallel}}{mc} \left[\xi \frac{\partial}{\partial p} + \frac{1 - \xi^2}{p} \frac{\partial}{\partial \xi} \right] F_r$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\delta \mathbf{E}_{\mathrm{ideal}} + \eta \left(\frac{1}{\mu_0} \nabla \times \delta \mathbf{B} - j_r \mathbf{b}_0 - J_r \delta \mathbf{b} \right) \right]$$

Use shifted f_{MJ} for tearing problem

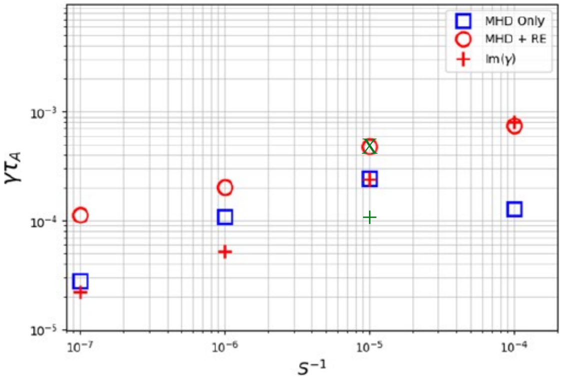
- $F_r(z, p_b) = f_{MJ}$ is normalized so that the RE current, $J_r = q_e \frac{n_0}{\gamma_b} c p_b$, returns a fraction of the initial current. Here, $z = 100$, $p_b = 0.67$, $v_b/c = 0.56$, $v_A/c = 0.0033$.



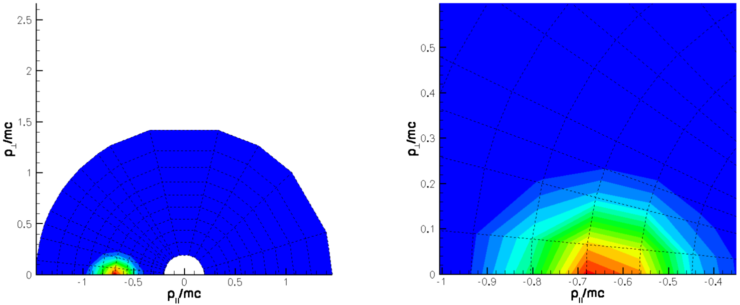


S=1e5 case looked promising (green symbols).

- Details of continuum kinetic RE setup:
  1. 10 speed grid points, 5 cells with 5th-order GLL polynomials in ξ.
  2. fully implicit advance of f_r with $dt \sim \tau_A$.
  3. f_r centered in time with $n, \mathbf{B}$ and T.
  4. converged eigenfunction takes 10 minutes on 100 Cori cores

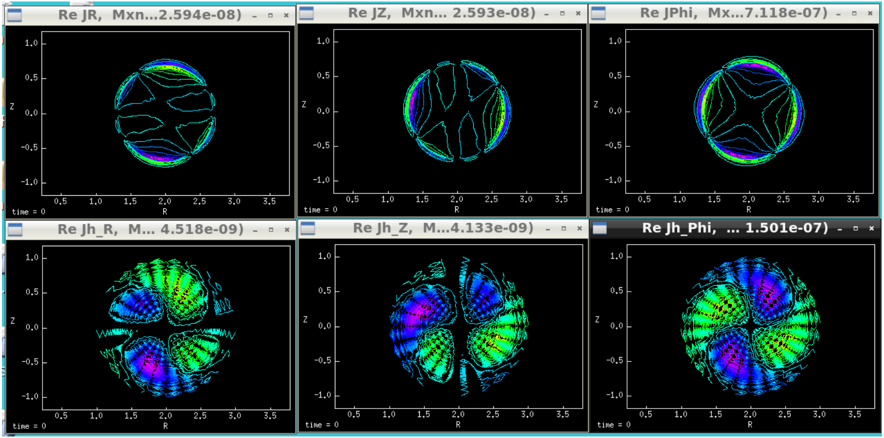


Converged eigenfunction for perturbed f_r similar to $F_r(t=0)$.



But, RE current looks sketchy.

- Need a little spatial diffusion?
- Need to go higher in $p_b = 0.67$ ($v_b/c = 0.56$, $v_A/c = 0.0033$)?
- Implement simultaneous $(f_r, \mathbf{B})$ advance using Picard iteration?



Future Work.

- Obtain better agreement between NIMROD and M3D-C1 on ideal kink growth rates for DIII-D discharge #141216.
- Follow up on M3D-C1 efforts with thermal ion kinetic effects using NIMROD's ion CEL-DKE continuum kinetic capability.
- Modify linear $\mathbf{V}$ and Fhot routine for a simultaneous $\mathbf{B}$ and Fhot_RE advance and carry out benchmark of RE effects on linear tearing with NIMROD's RE fluid model.