Multispecies impact on tokamak edge modeling

by

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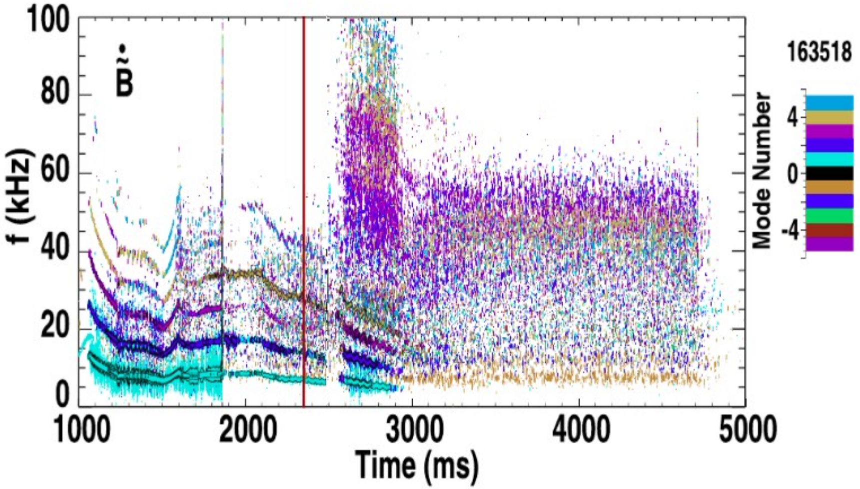
QH-mode is a naturally occurring ELM-free state[1]

- **Hypothesis: saturated 3D fluctuations drive particle and thermal transport to maintain steady state pedestal profiles[2]**
- **Low n implies MHD**
- **How well can MHD modeling characterize the low-n perturbations observed during QH-mode?**



Edge harmonic oscillation during *standard QH-mode*

Broadband fluctuations during *wide-pedestal QH-mode*

[1] Burrell et al., PoP (2016)
Chen et al., NF (2016)
[2] Snyder et al., NF (2007)

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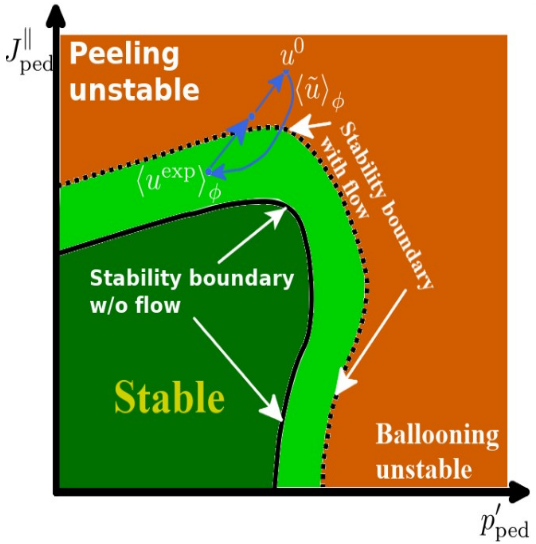
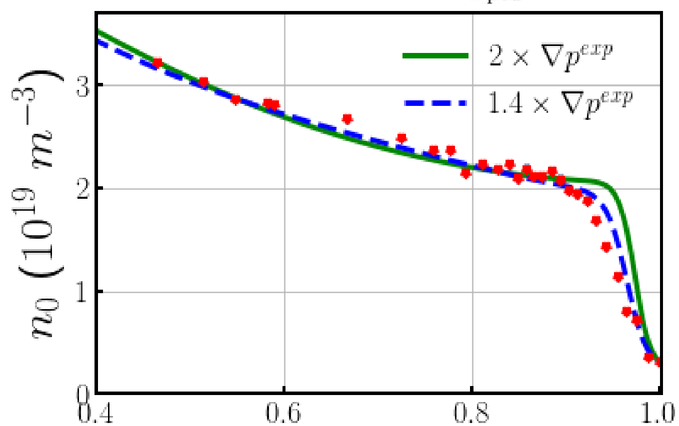
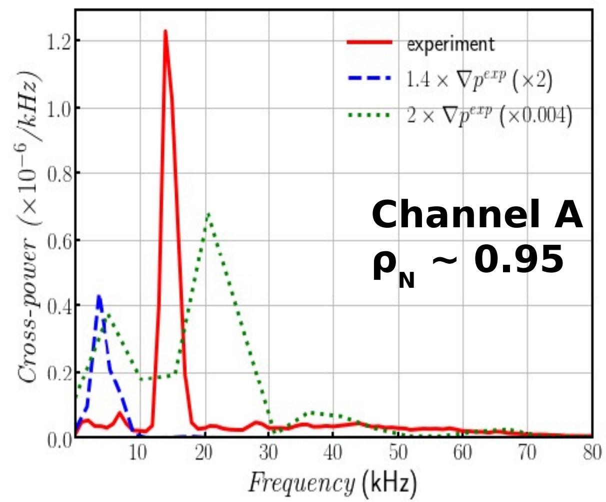
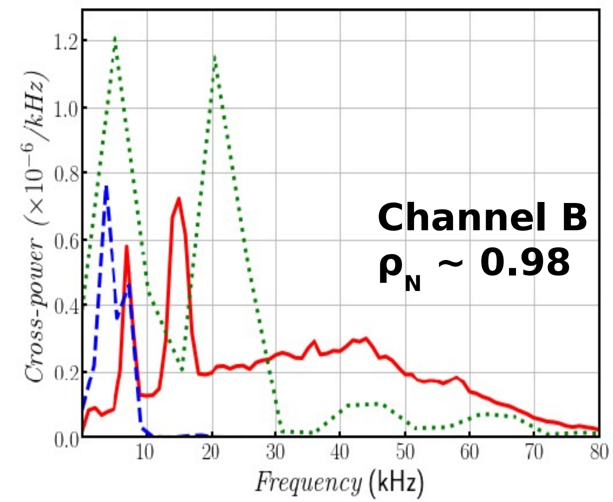
Prior modeling shows simulations bracket measured beam emission spectroscopy (BES) fluctuation amplitude[1]

- **EFIT based on “best fit” to experimental data is stable when ExB flow is included with single fluid**
- **To destabilize: density and temperature gradients increased in the pedestal region**
- **Nonlinear relaxation relaxes plasma profiles back towards measured state**





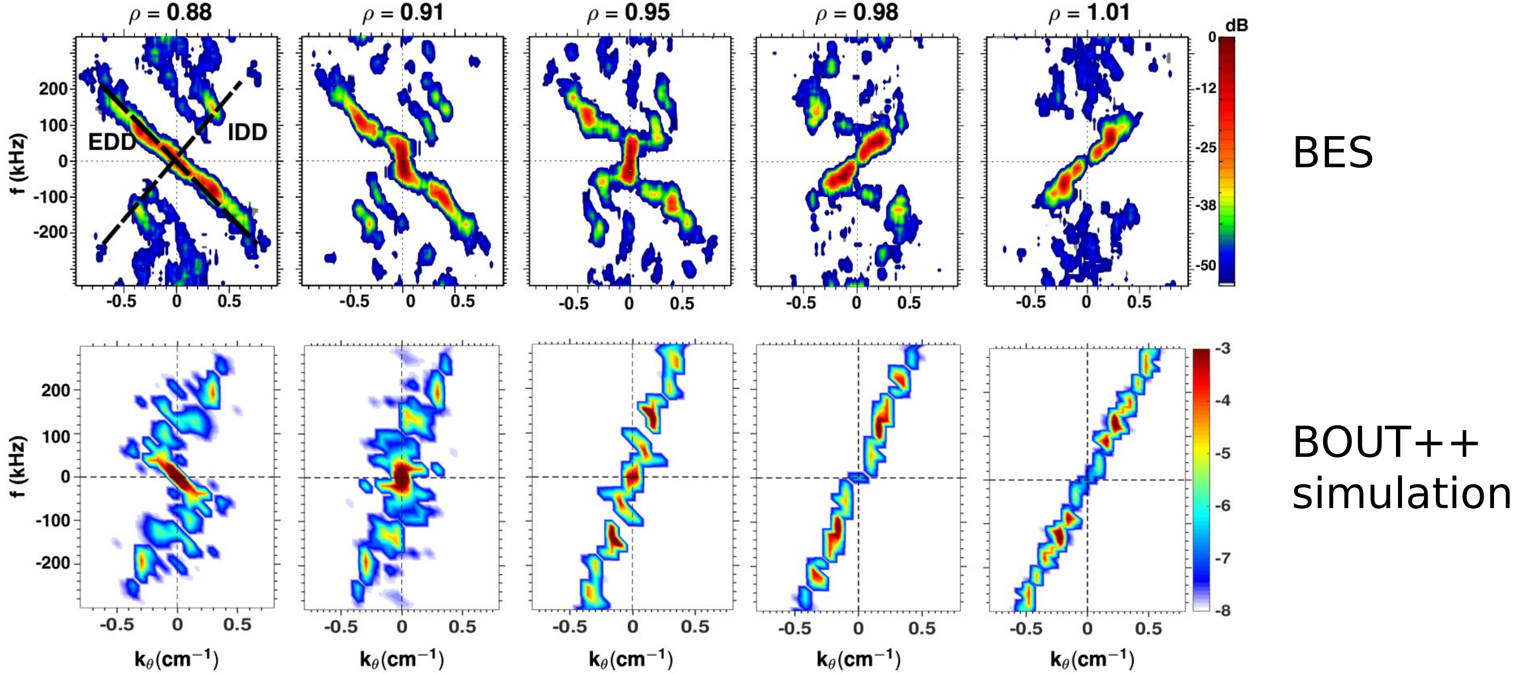




[1] Pankin et al., NF (2020)

Reduced, two-fluid nonlinear modeling reproduces electron/ion dynamics observed by BES[1]

Indicates two-fluid effects are critical to capture fluctuation dynamics



Contour plot of power spectral density of low-k turbulence versus frequency and wave number

[1] Li et al., NF (2022)

Major improvements to model fidelity

- First NIMROD[1] simulations with unreduced MHD, full shaping and
- Two-fluid, FLR terms (Hall, ion gyroviscosity, cross heat flux)
- Multispecies collisionalities that incorporate impact of carbon
- Heuristic neoclassical closures
- Relaxation sources (scan drive within same computation / reconstruction)

[1] Sovinec et al., JCP (2004)

Ohm's law includes two-fluid terms, heuristic non-axisymmetric bootstrap current drive[1] and multispecies resistivity

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\mathbf{v} \times \mathbf{B} - \frac{\mathbf{J} \times \mathbf{B} - \nabla p_e}{n_e e} + \frac{\nabla \cdot \Pi_e}{n_e e} - \eta \mathbf{J} \right] - \mathbf{S}_B$$

$$\nabla \cdot \Pi_e = \frac{\langle \rho_e \mu_e \rangle_\phi}{\langle \rho_{qe} \rangle_\phi} \left\langle B_0^2 \right\rangle \frac{\mathbf{J}_{pol,n_\phi>0} \cdot \langle \mathbf{B}_{pol} \rangle_\phi}{\left\langle \mathbf{B}_{pol}^4 \right\rangle_\phi} \langle \mathbf{B}_{pol} \rangle_\phi$$

$$\eta = \left\langle \frac{m_e \nu_e}{n_e e^2} \right\rangle_\phi \qquad \nu_e = \tau_e^{-1} = \sum_{\beta \neq e} \tau_{e\beta}^{-1} \simeq \sum_{\beta \neq e} \frac{16\sqrt{\pi}}{3} n_\beta \left(\frac{m_e}{2 k_B T_e} \right)^{3/2} \left(\frac{q_\beta e}{4 \pi \epsilon_0 m_e} \right)^2 \ln \Lambda_{e\beta}$$

$$\mathbf{S}_B = M_B(R, Z) \nu_B \left(\langle \mathbf{B} \rangle_\phi - \mathbf{B}\,(t = 0) \right)$$

Mask 1 inside LCFS, 0 elsewhere

Kinetic-EFIT initial state

$$\nu_B = 5 \times 10^{-3} s^{-1}$$

$\langle x \rangle_\phi$ - ϕ average

$\langle x \rangle$ - FSA

[1] Gianakon PoP (2002)
Howell PoP (2022)

Momentum eqn includes magnetized viscosity, heuristic non-axisymmetric poloidal flow damping and multispecies collisionality

$$mn\frac{\partial \mathbf{v}}{\partial t} + mn\mathbf{v}\cdot\nabla\mathbf{v} = \mathbf{J}\times\mathbf{B} - \nabla p - \nabla\cdot\mathbf{\Pi}_{\perp I} - \nabla\cdot\mathbf{\Pi}_{\times I} - \nabla\cdot\mathbf{\Pi}_{\parallel I} - \nabla\cdot\mathbf{\Pi}_{NCI} - \mathbf{S}_v$$

$$\mathbf{\Pi}_{\perp I} \simeq \mu_{\perp I}\mathbf{W} \qquad \mu_{\perp I} \simeq \sum_i m_i \kappa_{\perp i}$$

$$\mathbf{v} = \mathbf{v}_d = \mathbf{v}_c$$

$$\mathbf{\Pi}_{\times I} = \sum_i \frac{p_i}{4\omega_{ci}}[\hat{\mathbf{b}}\times\mathbf{W}\cdot\left(\mathbf{I}+3\hat{\mathbf{b}}\hat{\mathbf{b}}\right) - \left(\mathbf{I}+3\hat{\mathbf{b}}\hat{\mathbf{b}}\right)\cdot\mathbf{W}\times\hat{\mathbf{b}}]$$

$$\mathbf{\Pi}_{\parallel I} = -\frac{3}{2}\mu_{\parallel I}\hat{\mathbf{b}}\cdot\mathbf{W}\cdot\hat{\mathbf{b}}\left(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{1}{3}\mathbf{I}\right) \qquad \mu_{\parallel I} \simeq \sum_i m_i \kappa_{\parallel i}$$

$$\nabla\cdot\Pi_{NCI} = \sum_i \langle\rho_i\mu_i\rangle_\phi \langle B_0^2\rangle \frac{\mathbf{v}_{pol,n_\phi>0}\cdot\langle\mathbf{B}_{pol}\rangle_\phi}{\left\langle\mathbf{B}_{pol}^4\right\rangle_\phi}\langle\mathbf{B}_{pol}\rangle_\phi$$

$$\underbrace{\mathbf{S}_v = M_v(R,Z)}_{\text{Mask 1 inside LCFS, 0 elsewhere}}\nu_v\left(\langle\mathbf{v}\rangle_\phi - \mathbf{v}\,(t=0)\right)$$

Mask 1 inside LCFS, 0 elsewhere

$$\nu_v = 5\times10^{-3}s^{-1}$$

Temperature equation incorporates magnetized heat flux

$$\frac{n_I}{\Gamma-1}\left(\frac{\partial k_B T_i}{\partial t}+\mathbf{v}\cdot\nabla k_B T_i\right)=-n_I k_B T_i \nabla\cdot\mathbf{v}-\nabla\cdot\mathbf{q}_I-n_I D_{hT}\left(\mathbf{e}_\phi\cdot\nabla\right)^2\nabla^2 T_i-S_{Ti}$$

$$\frac{n_e}{\Gamma-1}\left(\frac{\partial k_B T_e}{\partial t}+\mathbf{v}_e\cdot\nabla k_B T_e\right)=-n_e k_B T_e \nabla\cdot\mathbf{v}_e-\nabla\cdot\mathbf{q}_e-n_e D_{hT}\left(\mathbf{e}_\phi\cdot\nabla\right)^2\nabla^2 T_e-S_{Te}$$

$$\kappa_{\perp s}=\left\langle\frac{n_s k_B T_s}{m_s\nu_s}\frac{\gamma'_{1s}\left(\omega_{cs}/\nu_s\right)^2+\gamma'_{0s}}{\left(\omega_{cs}/\nu_s\right)^4+\delta_{1s}\left(\omega_{cs}/\nu_s\right)^2+\delta_{0s}}\right\rangle_\phi\simeq\left\langle\frac{n_s k_B T_s \nu_s}{m_s\omega_{cs}^2}\gamma'_{1s}\right\rangle_\phi$$

$$\kappa_{\times s}=\frac{n_s k_B T_s}{m_s\nu_s}\frac{\left(\omega_{cs}/\nu_s\right)\left[\gamma''_{1s}\left(\omega_{cs}/\nu_s\right)^2+\gamma''_{0s}\right]}{\left(\omega_{cs}/\nu_s\right)^4+\delta_{1s}\left(\omega_{cs}/\nu_s\right)^2+\delta_{0s}}\simeq\frac{5}{2}\frac{n_s k_B T_s}{m_s\omega_{cs}}$$

$$\kappa_{\parallel s}=\left\langle\frac{n_s k_B T_s}{m_s\nu_s}\frac{\gamma'_{0s}}{\delta_{0s}}\right\rangle_\phi$$

$$\mathbf{q}_I=\sum_i\mathbf{q}_i=-\left(\sum_i\kappa_{\parallel i}\mathbf{b}\cdot\nabla k_B T_i+\kappa_{\perp i}\nabla_\perp k_B T_i+\kappa_{\times i}\mathbf{b}\times\nabla k_B T_i\right)$$

$$\min\left(\kappa_{\perp s}\right)=1\ m^2/s$$

$$n_I=\sum n_i$$

$$\mathbf{q}_I=\sum\mathbf{q}_i$$

$$\mathbf{v}_e=\mathbf{v}-\mathbf{J}/n_e e$$

$$D_h=1\ m^4/s$$

(if used)
Equivalent to diffusion of ~ 1 m²/s for n=1; scales as n²

$$S_{T\alpha}=M_T(R,Z)\nu_T\left(\left\langle T_\alpha\right\rangle_\phi-T_\alpha\left(t=0\right)\right)$$

$$\nu_T=5\times10^{-3}s^{-1}$$

Separate density equations are evolved

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{v}) = \nabla \cdot D_n \nabla n_d - D_h \left(\hat{\mathbf{e}}_\phi \cdot \nabla\right)^2 \nabla^2 n_d - S_{nd}$$

$$\frac{\partial n_c}{\partial t} + \nabla \cdot (n_c \mathbf{v}) = \nabla \cdot D_n \nabla n_c - D_h \left(\mathbf{e}_\phi \cdot \nabla\right)^2 \nabla^2 n_d - S_{nc}$$

$$n_e = n_d + Z_c n_c$$

$$D_n = 1\ m^2/s$$

$$S_{n\alpha} = M_n(R,Z)\nu_n \left(\langle n_\alpha \rangle_\phi - n_\alpha\,(t=0)\right)$$

$$\nu_n = 5 \times 10^{-3} s^{-1}$$

Zhadanov[1] formulas used for multispecies collision frequencies

$$r_D = \left(\sum_s \frac{n_s q_s^2}{\epsilon_0 k_B T_s} \right)^{-1/2}$$

$$\Lambda_{\alpha\beta} = \frac{12\pi\epsilon_0}{|q_\alpha q_\beta|} \frac{\mu_{\alpha\beta}}{\gamma_{\alpha\beta}} r_D$$

$$\tau_{\alpha\beta}^{-1} = \frac{16\sqrt{\pi}}{3} n_\beta \left(\frac{\gamma_{\alpha\beta}}{2} \right)^{3/2} \left(\frac{q_\alpha q_\beta}{4\pi\epsilon_0 \mu_{\alpha\beta}} \right)^2 \ln \Lambda_{\alpha\beta}$$

$$\mu_{\alpha\beta} = \frac{m_\alpha m_\beta}{m_\alpha + m_\beta}$$

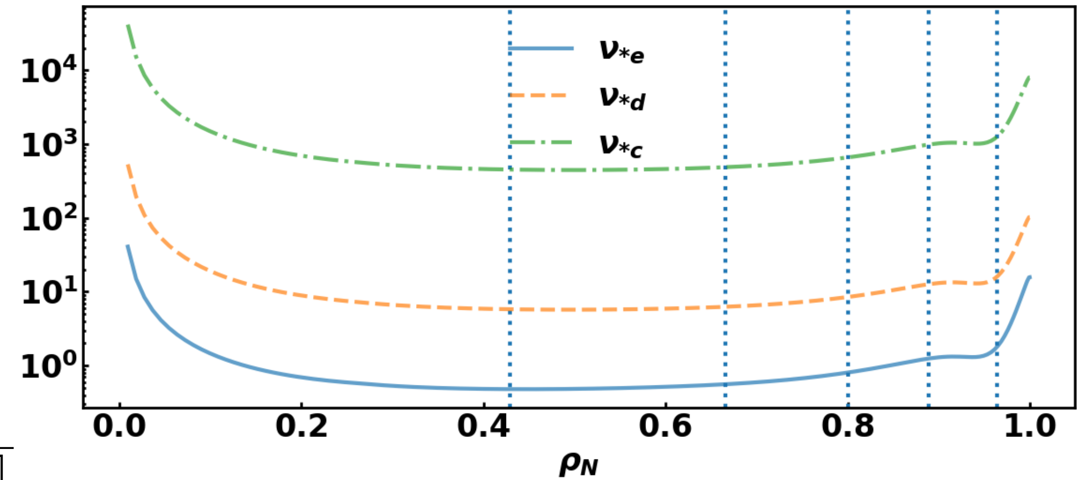
$$\gamma_\alpha = \frac{m_\alpha}{kT_\alpha}$$

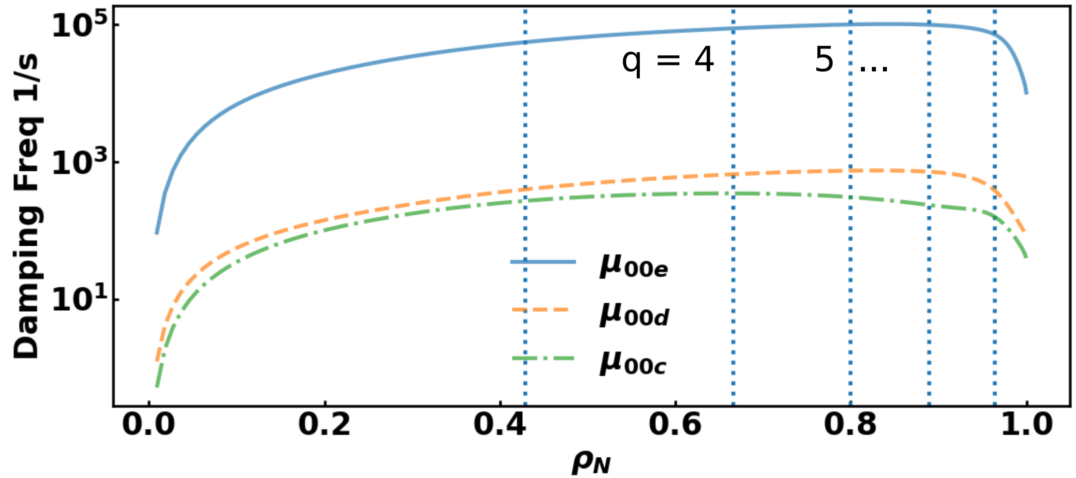
$$\gamma_{\alpha\beta} = \frac{\gamma_\alpha \gamma_\beta}{\gamma_\alpha + \gamma_\beta}$$

[1] Zhadanov "Transport processes in multicomponent plasma"

Neoclassical coefficient computed from extrapolation across collisionality regimes[1,2]

$$\mu_s = \frac{(\nu_s \tau_{ss})\,(f_t/f_c)\,K_s^B}{\left[1+\nu_{*s}^{1/2}+2.92\nu_{*s}\frac{K_s^B}{K_s^P}\right]\left[1+\frac{2K_s^P}{3\omega_{ts}\tau_{ss}K_s^{PS}}\right]}$$

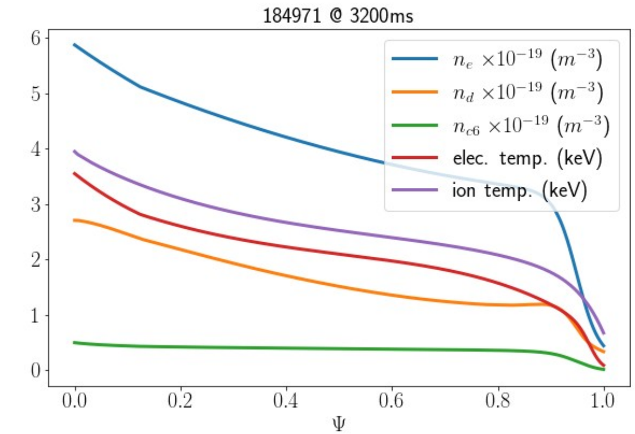


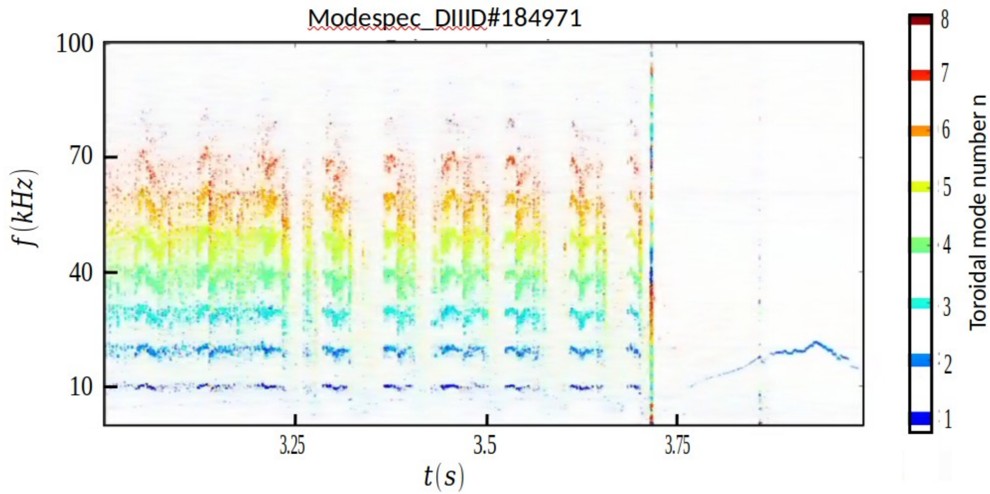


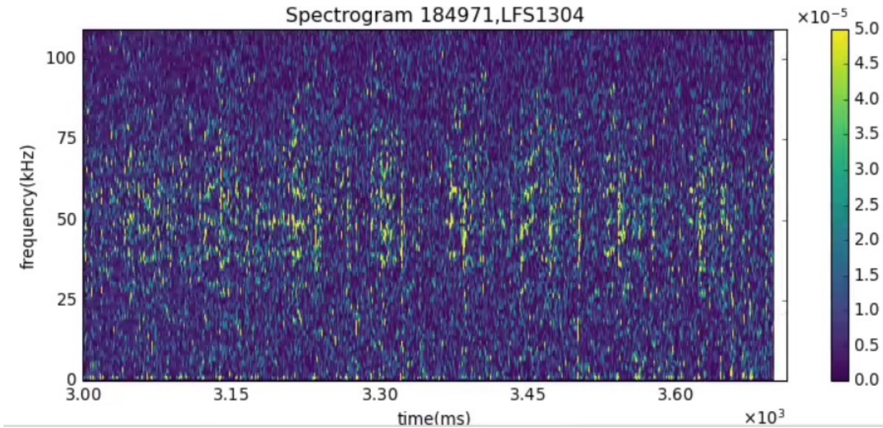
[1] Kim et al., PFB (1991); errata
[2] Callen CPTC report 09 6-rev1 (2010)

Choose DIII-D wide-pedestal QH-mode shot 184971 at 3200ms for local diagnostic availability

- **Both BES and ECEI measurements available for later validation comparison**
- **Novel aspect for QH-mode: normal I_p shot**
  - BES measurements aligned with fieldlines



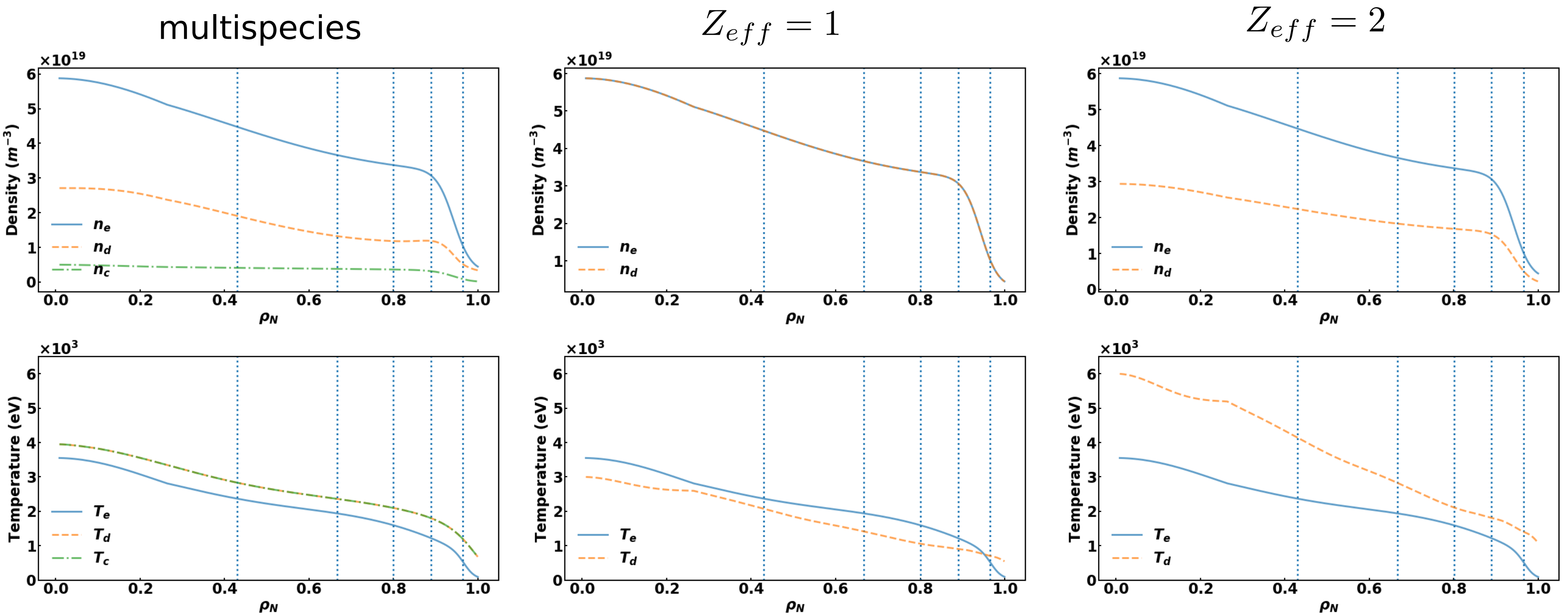




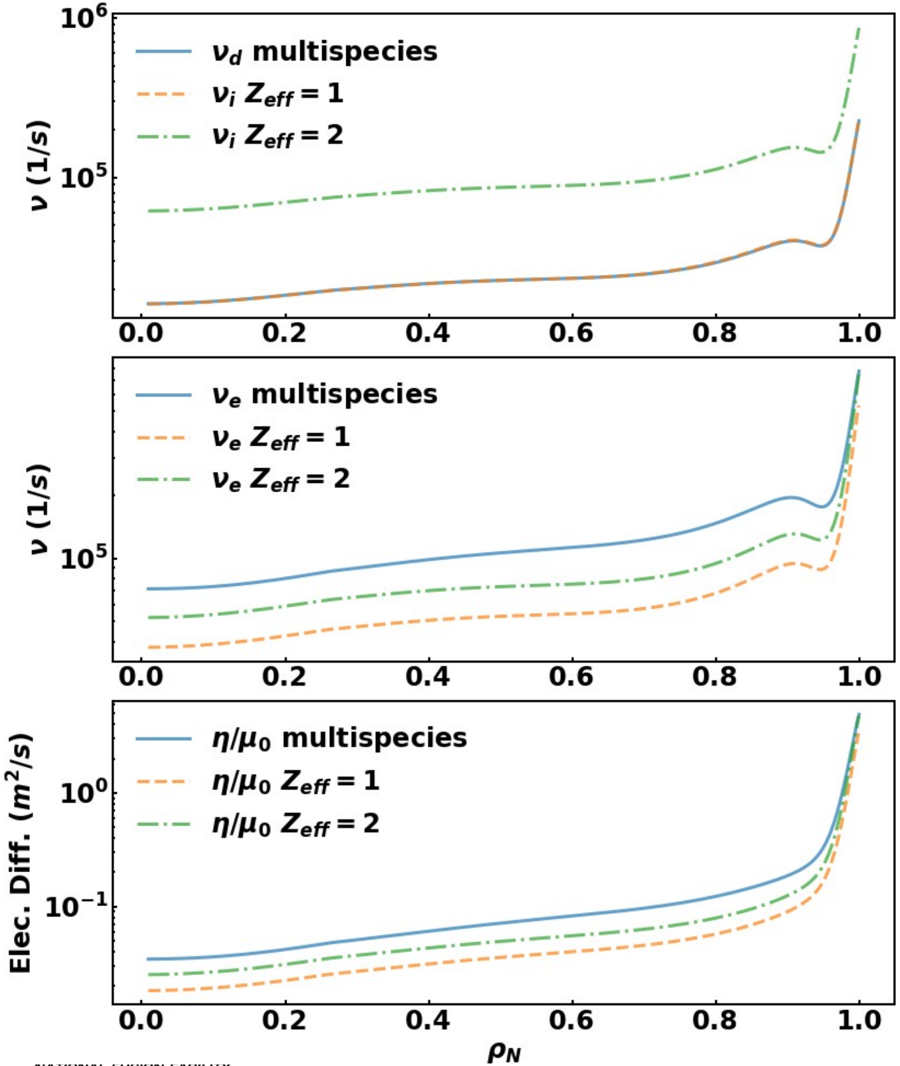
ECEI spectrogram
(see Ref. 1 for more on ECEI in QH)

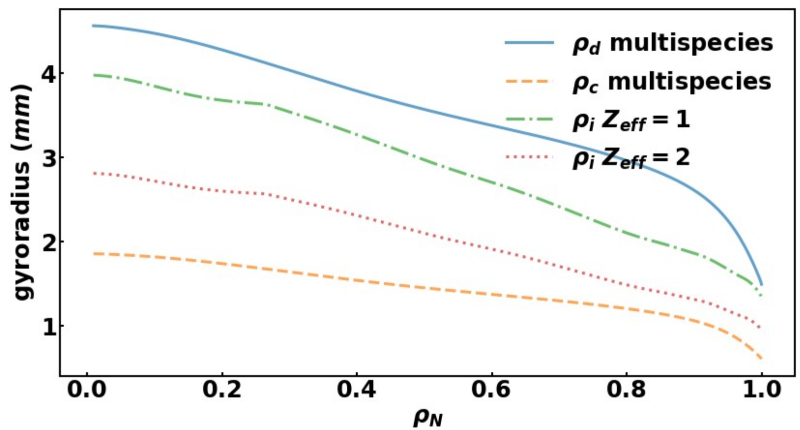
[1]Yu et al., PPCF (2022)

Ion temperature used as free parameter to match pressure without multispecies

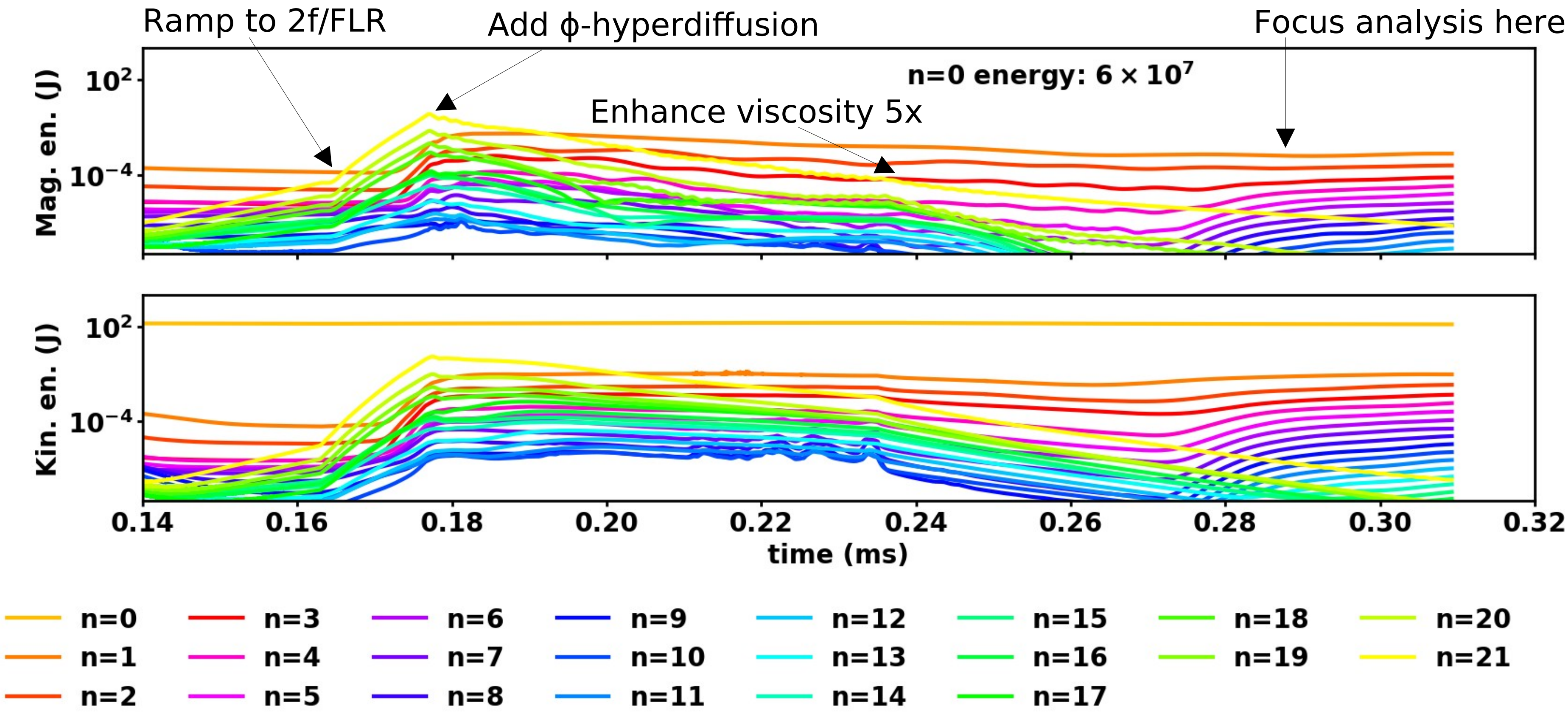


Collisionality and gyroradius vary from multispecies by factors of 2-3x

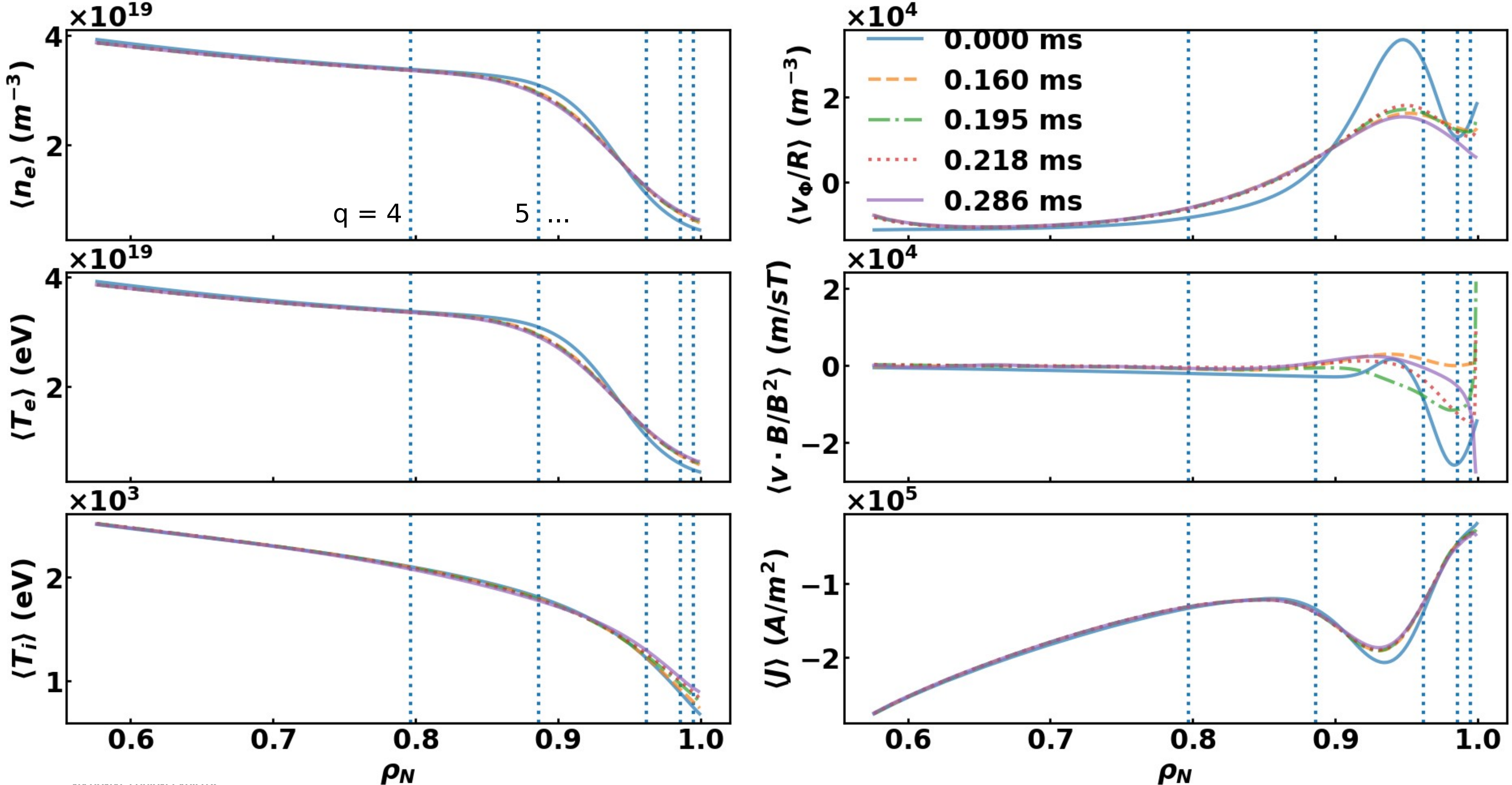




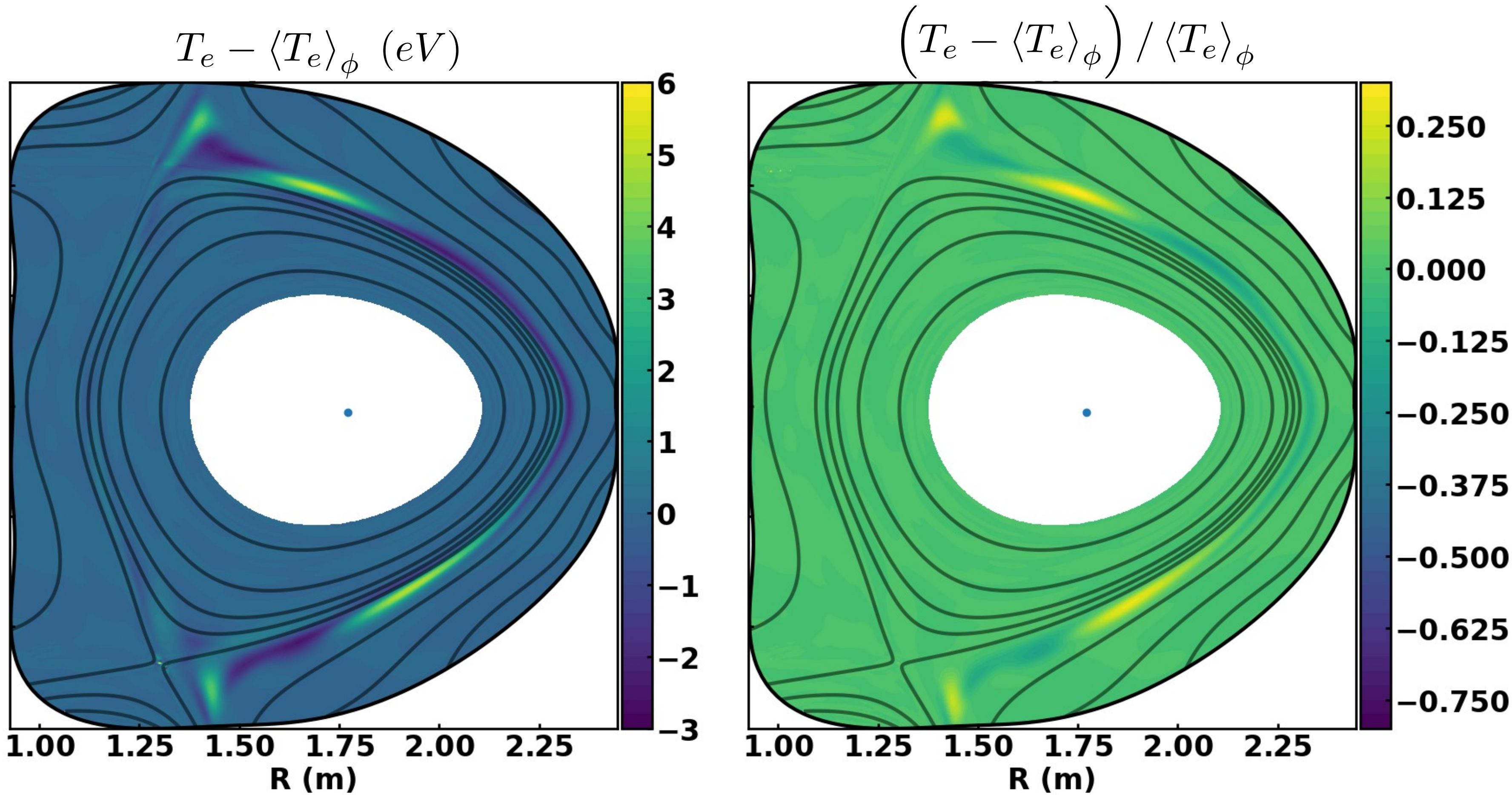
Global energy decomposition shows model tuning used to achieve low-n saturated state



After nonlinear saturation the profiles relax; toroidal flow is most impacted



The temperature perturbation is localized within the edge pedestal and SOL and dominated by n=1



t=0.286ms

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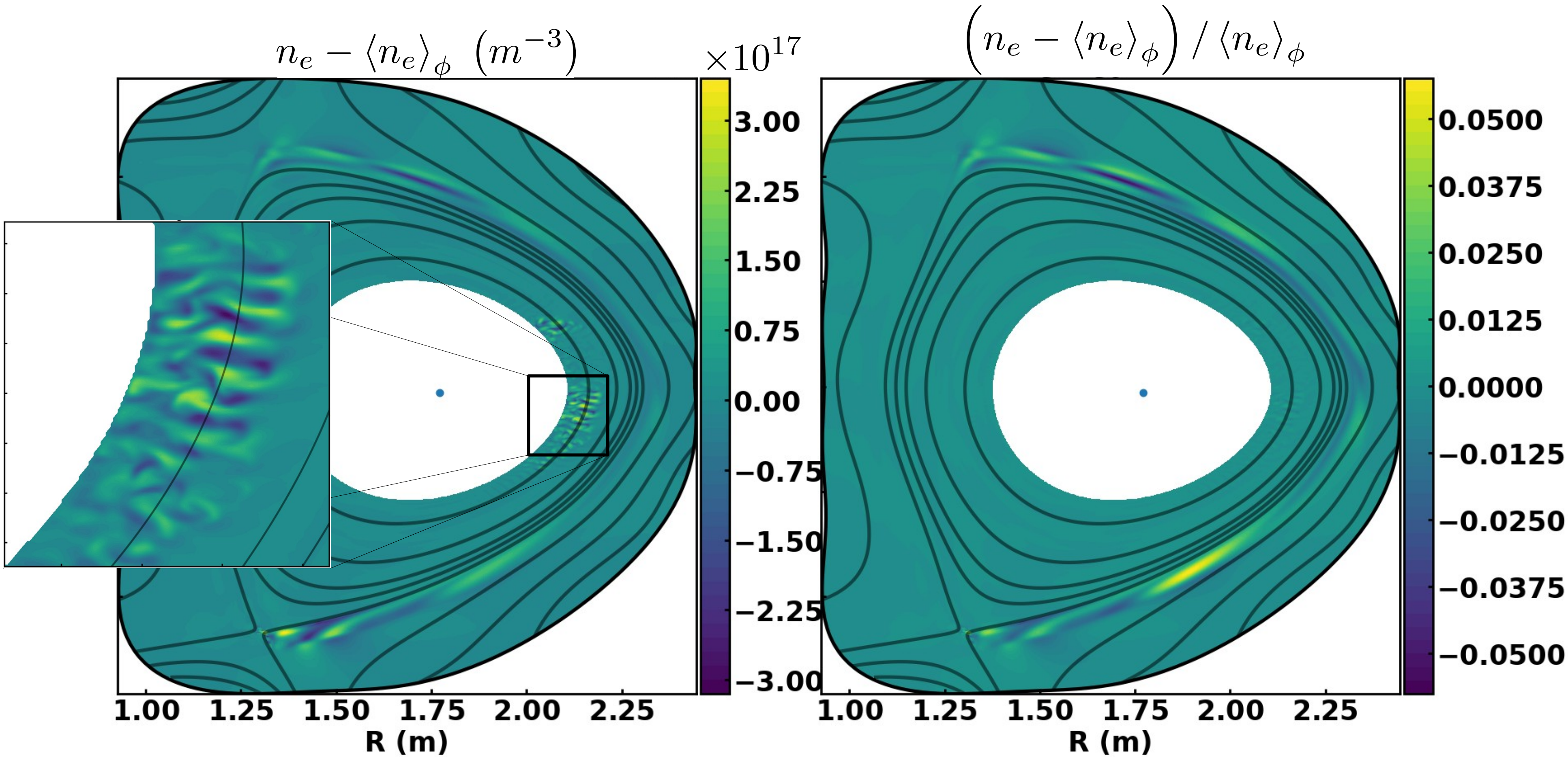
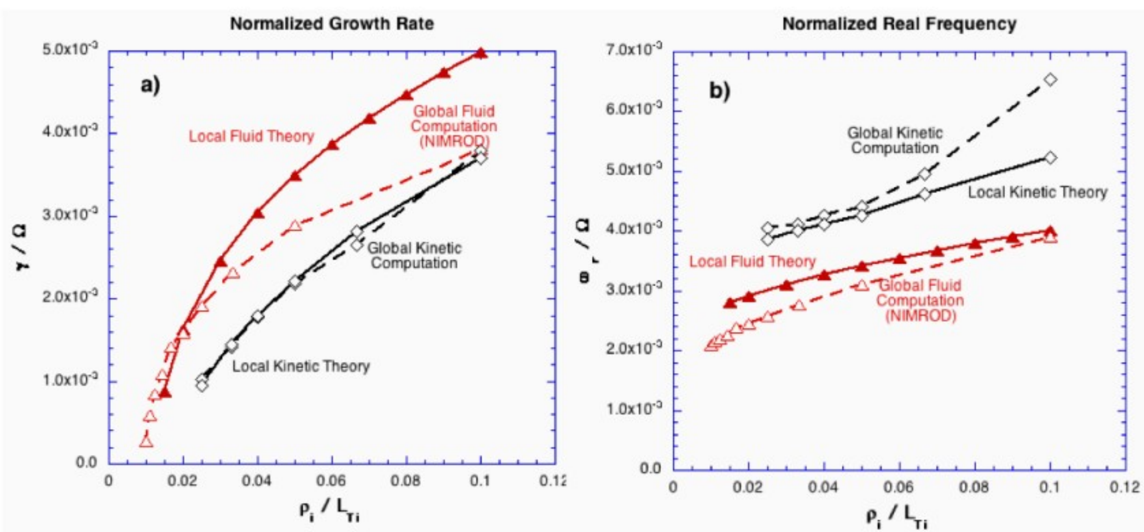
The density perturbation exhibits non-field aligned structure in the core that produces minimal transport

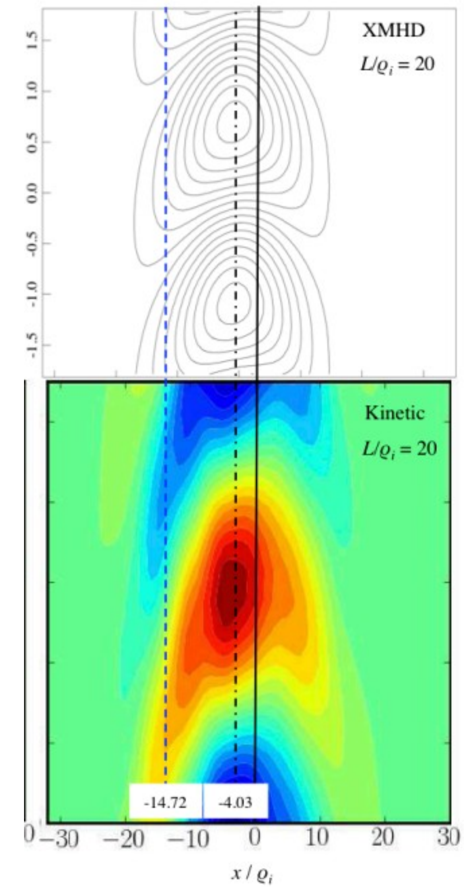


t=0.286ms

Extended-MHD model produces ITG-like modes[1]

- **Candidate to explain non-field aligned density fluctuations**
- **Magnetized thermal conduction likely suppresses temperature perturbation**





[1] Schnack et al., PoP (2013)

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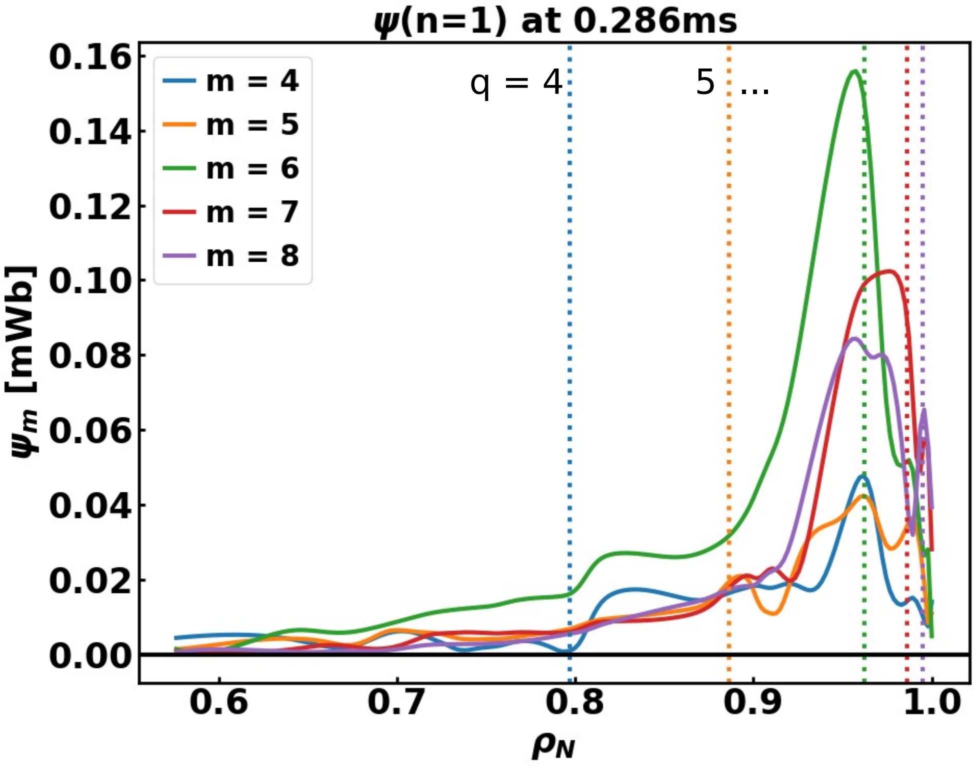
Straight-field line decomposition used to represent magnetic flux

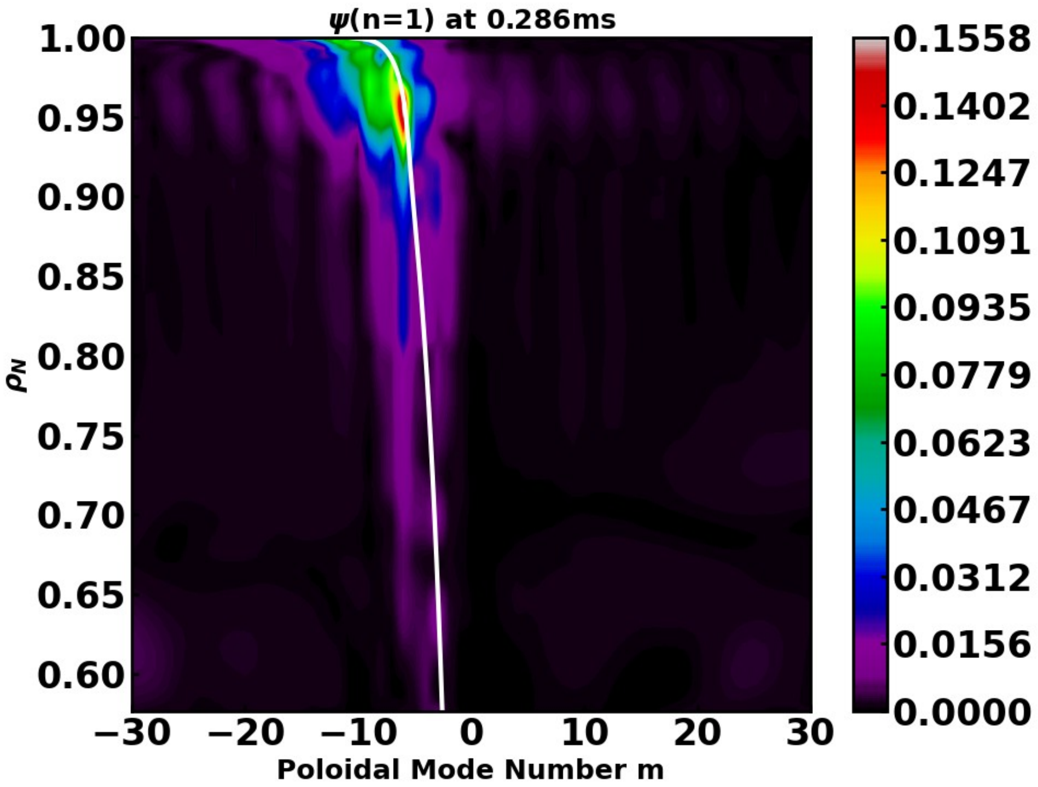
$$\tilde{\psi}_{m,n} = \oint \oint d\Theta d\Phi \frac{\tilde{\mathbf{B}} \cdot \nabla\psi_0 \exp\left(in\Phi - im\Theta\right)}{\nabla\psi_0 \cdot \nabla\Theta \times \Phi}$$

- **Definition is independent of choice of radial coordinate at the resonant surface[1]**
- **Permits examination of poloidal mode structure**

[1] Park et al., PoP (2008)

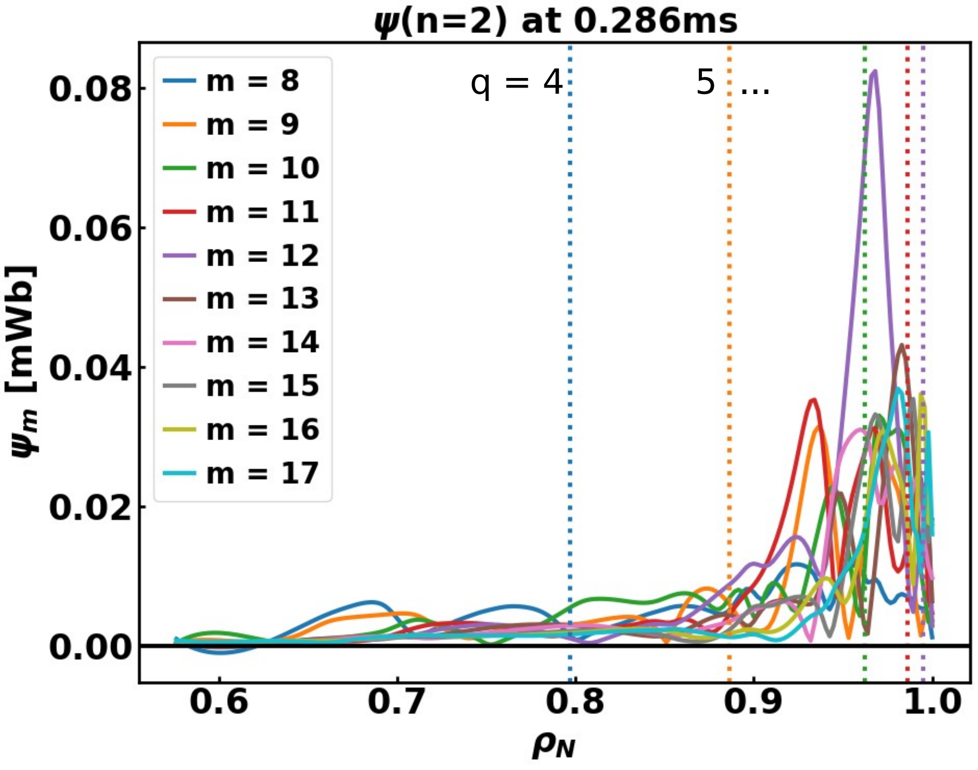
n=1 shows dominant 6/1, 7/1 tearing modes

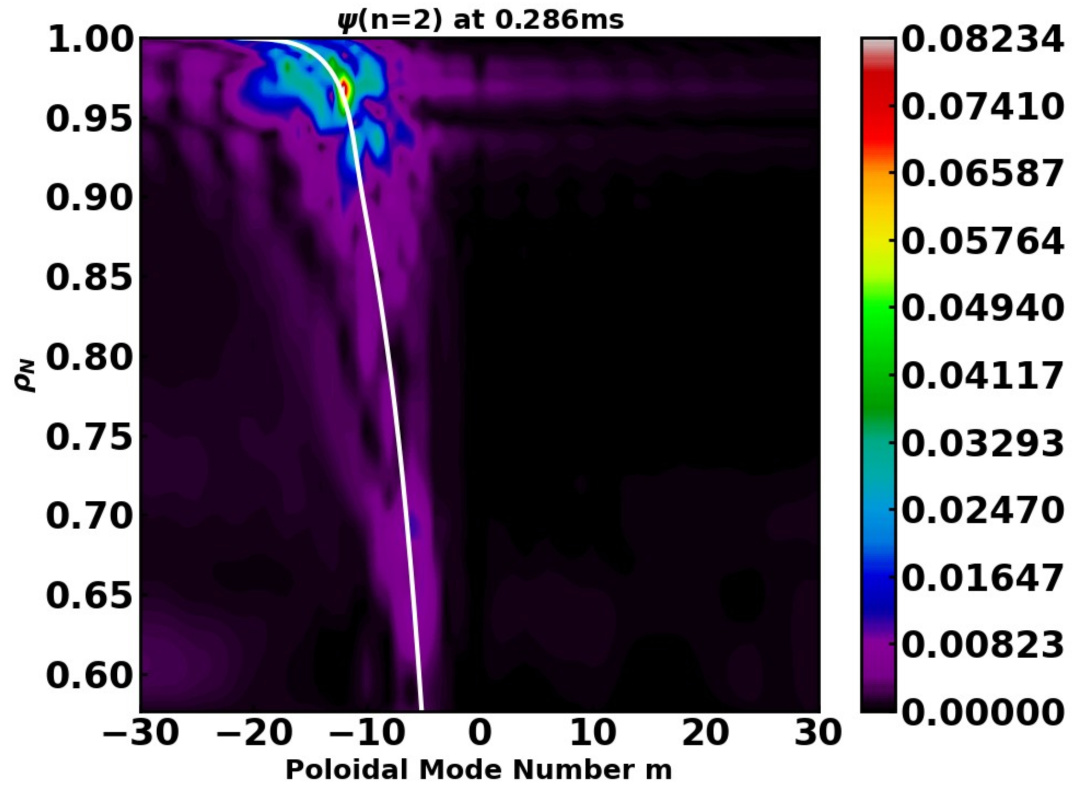




t=0.286ms

The n=2 is dominated by the m=12 first harmonic and a broad poloidal spectrum of subdominant modes





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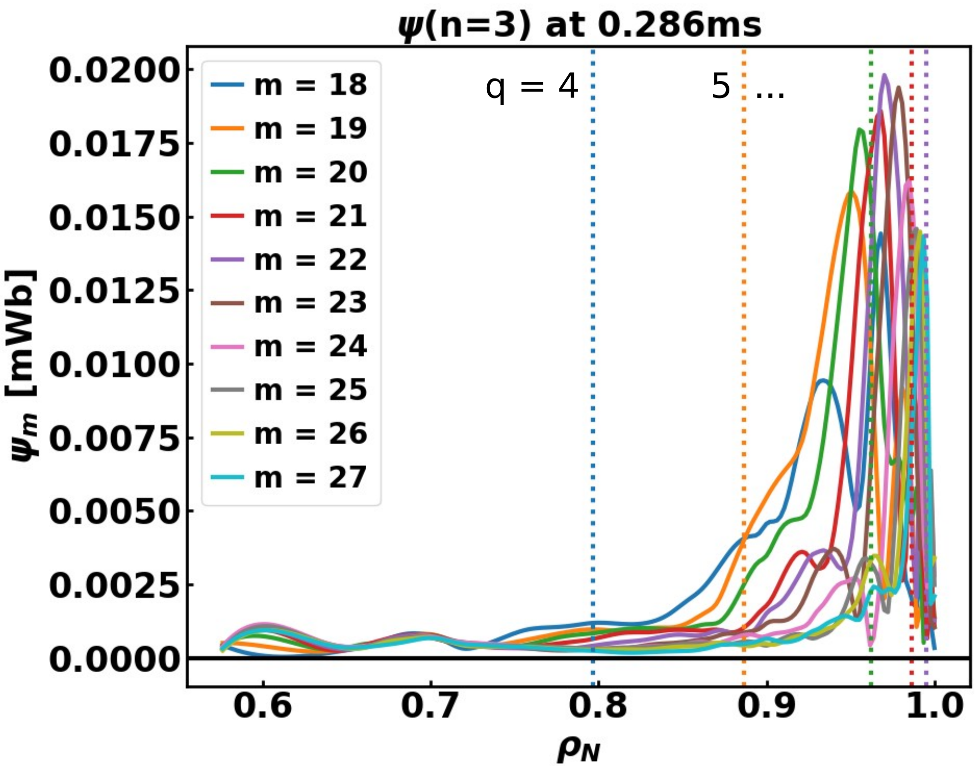
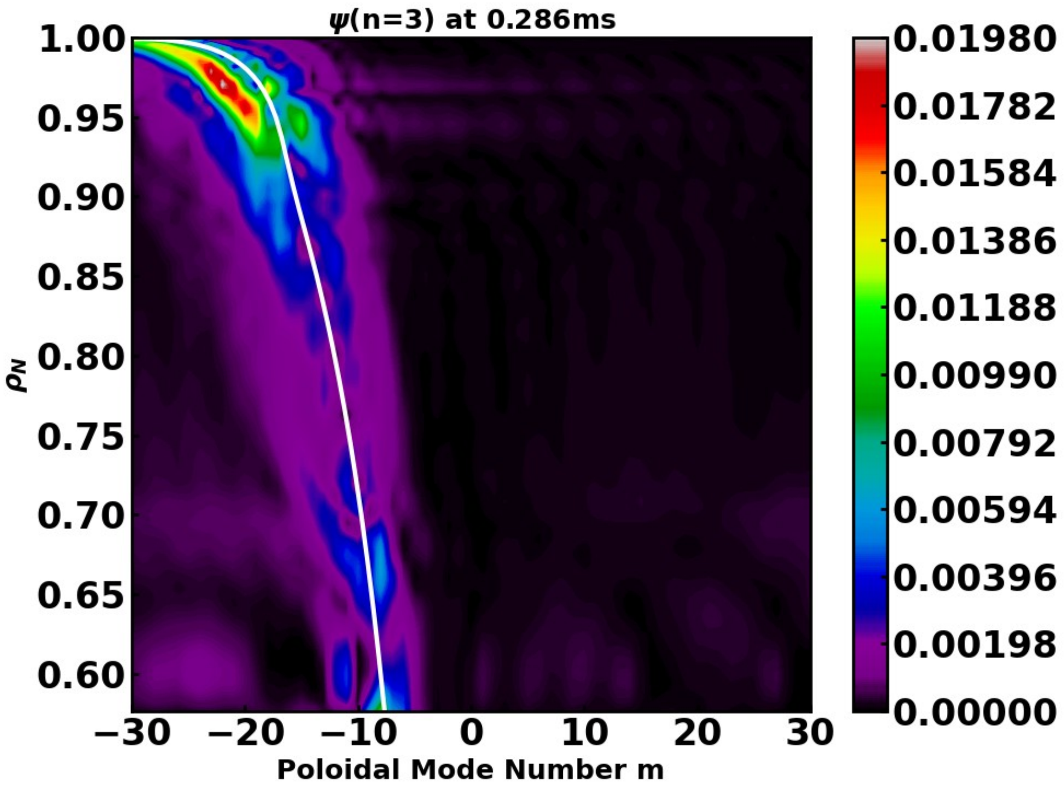
t=0.286ms

The n=3 modes are not resonant and small in amplitude





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t=0.286ms

Future work: energy saturation analysis and synthetic ECE and BES diagnostic comparisons

- **Energy analysis will resolve is saturation results from coupling to high-n modes or profile relaxation**
  – Need to reduce or treat hyperdiffusive terms with care
- **Relaxation frequency can be tuned to match amplitude of measured perturbations**
  – Compare frequency analysis, resulting mode structure
  – Does a single simulation agree with both BES and ECE measurements?
- **Quantify the role of the neoclassical closure**
- **Run full model on a standard QH-mode with EHO**

Conclusions

- **Unreduced, extended-MHD simulations have been carried out with the NIMROD code that incorporate**
  - Two-fluid, finite-gyroradius effects
  - Multispecies closures that include impact of carbon
  - Heuristic neoclassical closures
  - Relaxation sources
- **Non-field-aligned structure present in density perturbation, but produces minimal transport**
- **Multiple magnetic modes present in pedestal that are dominated by the 6/1 and 7/1**