Update On Fluid RE simulations in NIMROD

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The NIMROD fluid runaway model can now compute the linearized MHD+RE equations.

- NIMROD Simulations of linear $(m, n) = (2, 1)$ tearing modes and $(1, 1)$ resistive kink modes give similar results to previous work
- The linearized equations can use different models for the runaway velocity.

Different treatments of linear theory show qualitative differences

Helander finds that the linear growth rate of the tearing mode including runaways becomes the standard FKR expression for the case with only runaway current[1].

$$\frac{\gamma^{5/4}}{\eta^{3/4}k_{\parallel}'^{1/2}}\frac{2\pi\Gamma(3/4)}{\Gamma(1/4)} = \Delta' \tag{1}$$

Liu et al. find that the runaways introduce a real frequency to the eigenmode[2].

$$\frac{\gamma^{5/4}}{\eta^{3/4}k_c^{1/2}}\frac{2\pi\Gamma(3/4)}{\Gamma(1/4)} = \Delta' - i\pi\frac{mJ_{r0}'}{|k_c|r_s} \tag{2}$$

Avinash and Kaw find for a slab tearing mode (for small kinetic energy of runaways)[3]

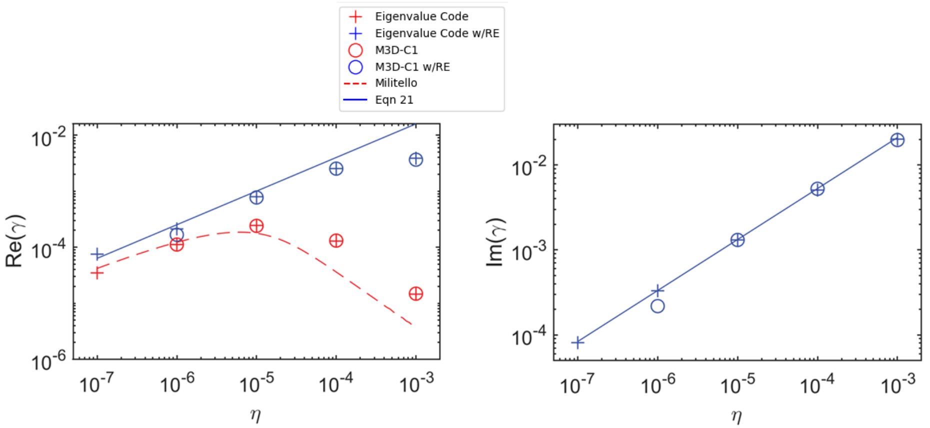
$$\gamma = \left(\frac{\Gamma(1/4)a\Delta'(1-\beta_t/2)}{\pi\Gamma(3/4)}\right)^{4/5} S^{-3/5}, \quad \beta_t \equiv \frac{2\mu_0\gamma n_r m_e c^2}{B_z^2} \tag{3}$$

[1] P. Helander et al., Physics of Plasmas **14**, 10.1063/1.2817016 (2007).

[2] C. Liu et al., Physics of Plasmas **27**, 10.1063/5.0018559 (2020).

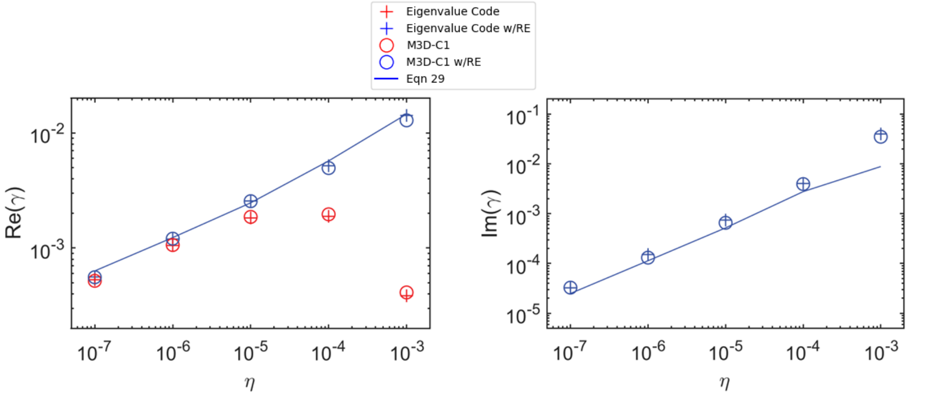
[3] Avinash and P. Kaw, Nuclear Fusion **28**, 10.1088/0029-5515/28/1/009 (1988).

Results from Liu, et al. compare the analytic growth rate scaling with linear M3D-C1 calculations for the (2,1) tearing mode.[4]



[4] C. Liu et al., Physics of Plasmas **27**, 10.1063/5.0018559 (2020).

Liu et al. also analyze the effect of runaways on the linear growth rate of the (1,1) resistive kink.



The reduced model evolves a beam-like runaway population density with volumetric sources.

Continuity equation for runaway electron population:

$$\frac{\partial n_r}{\partial t} + \boldsymbol{\nabla} \cdot (n_r \boldsymbol{u_r}) = S_D(\mathcal{E}_{||}) + S_A(E_{||}) + D_r \nabla^2 n_r, \tag{4}$$

where n_r is the number density of runaways, S_D, S_A are sources, D_r is a numerical diffusion coefficient and

$$\mathcal{E}_{||} \equiv \frac{E_{||}}{E_D}, \quad \boldsymbol{u_r} = -c_r \hat{\mathbf{b}} + \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2}, \quad c_r = \text{const.} \gg v_{th,e}, v_A$$

$$E_D = \frac{n_e e^3 \ln \Lambda}{4\pi \epsilon_0 T_e}.$$

The runaway electrons couple to the MHD evolution via a modified Ohm's law.

$$\mathbf{E} = -\boldsymbol{V} \times \boldsymbol{B} + \eta \left(\boldsymbol{J} + e n_r \boldsymbol{u}_r \right) \tag{5}$$

This Ohm's law is valid for

$$\frac{n_r}{n_e} \ll 1, \qquad m_e n_r \frac{\mathrm{d}\boldsymbol{u}_r}{\mathrm{d}t} \approx 0$$

- This model is similar the model employed in Bandaru[5] and Matsuyama[6], and the model in M3D-C1[7].

[5] V. Bandaru et al., Physical Review E **99**, 1–11 (2019)

[6] A. Matsuyama et al., Nuclear Fusion **57**, 10.1088/1741-4326/aa6867 (2017)

[7] C. Zhao et al., Nuclear Fusion **60**, 10.1088/1741-4326/ab96f4 (2020)

Quasi-neutrality may appear in different ways.

If we take $m_e \approx 0$ and $\sum_s m_s n_s \boldsymbol{u}_s \approx m_i n_i \boldsymbol{u}_i$, then the right-hand side of the resulting total momentum equation contains the following terms:

$$e(Zn_i - n_e - n_r)\boldsymbol{E} + e(Zn_i\boldsymbol{u}_i - n_e\boldsymbol{u}_e - n_r\boldsymbol{u}_r) \times \boldsymbol{B}$$

If $0 = Zn_i - n_e - n_r$, and $\boldsymbol{J} = e(Zn_i\boldsymbol{u}_i - n_e\boldsymbol{u}_e - n_r\boldsymbol{u}_r)$, then we get the typical momentum equation:

$$m_i n_i \frac{\mathrm{d}V}{\mathrm{d}t} = \boldsymbol{J} \times \boldsymbol{B} - \boldsymbol{\nabla} p$$

And taking $0 = \boldsymbol{E} + \boldsymbol{u}_e \times \boldsymbol{B} + \boldsymbol{R}_{ei} = \boldsymbol{E} + \boldsymbol{u}_r \times \boldsymbol{B}$ and substituting the definition of $\boldsymbol{J}$ yields the Ohm's law:

$$\left(1 + \frac{n_r}{n_e}\right)(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B}) = \frac{\boldsymbol{J}}{en_e} \times \boldsymbol{B} + \eta\,(\boldsymbol{J} - \boldsymbol{J}_r) \qquad (6)$$

Quasi-neutrality may appear in different ways.

One could also consider summing only the ion and bulk electron equations:

$$\boldsymbol{V} \equiv \frac{m_i n_i \boldsymbol{u}_i + m_e n_e \boldsymbol{u}_e}{m_i n_i + m_e n_e} \approx \boldsymbol{u}_i$$

$$m_i n_i \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = e n_r \boldsymbol{E} + (\boldsymbol{J} - \boldsymbol{J}_r) \times \boldsymbol{B} - \boldsymbol{\nabla} p$$

If $0 = \boldsymbol{E} + \boldsymbol{u}_r \times \boldsymbol{B}$ as before, these terms will cancel, but one could choose to work with this description and allow for terms of order m_e in the runaway momentum equation:

$$\boldsymbol{u}_r = \boldsymbol{u}_r^{(0)} + \boldsymbol{u}_r^{(1)} + \ldots \tag{7}$$

$$\boldsymbol{u}_r^{(0)} = -c_r \hat{\boldsymbol{b}} + \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2} \tag{8}$$

$$m_e \left(\boldsymbol{u}_r^{(0)} \cdot \boldsymbol{\nabla}\right) \boldsymbol{u}_r^{(1)} + m_e \left(\boldsymbol{u}_r^{(1)} \cdot \boldsymbol{\nabla}\right) \boldsymbol{u}_r^{(0)} = -e\boldsymbol{E} - e\boldsymbol{u}_r^{(1)} \times \boldsymbol{B} \tag{9}$$

The linearized runaway velocity can include the perturbed drift.

The full perturbed RE velocity is given by

$$\boldsymbol{u}_{r,\perp}(1+\alpha^2) = \boldsymbol{v}_\perp - c_r\frac{\boldsymbol{b}_\perp}{B} + \eta\alpha\boldsymbol{j}_\perp + \left(\eta\frac{\boldsymbol{j}}{B} + \alpha\boldsymbol{v} - \alpha c_r\frac{\boldsymbol{b}}{B}\right)\times\frac{\boldsymbol{B}}{B} \tag{10}$$

$$\alpha \equiv \frac{\eta e N_r}{B} \tag{11}$$

In the present form, we consider only the perturbed parallel

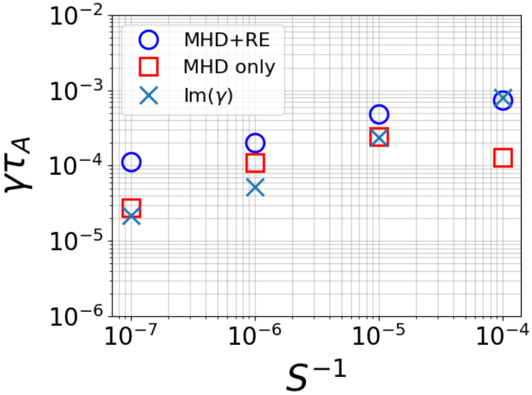
$$\boldsymbol{u}_r = -c_r\frac{\boldsymbol{b}_\perp}{B} \tag{12}$$

We also do not consider a linearized source term $\implies$ only the least-squares projection is used for the runaway continuity equation.

Results from cylindrical geometry tearing mode case reproduce Liu et. al analysis.

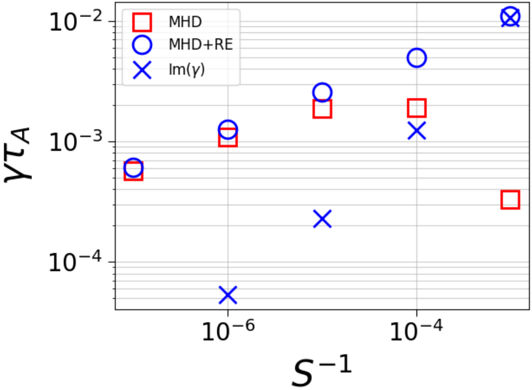
$\beta = 0$ screw pinch equilibrium in a cylinder

$$q = 1.15\left(1 + \frac{(r/a)^2}{0.6561}\right)$$



NIMROD also observes the modified scaling of the resistive kink in the presence of runaway current.

Kink equilibrium: $\beta = 0$, $R/a = 10$. $q = 0.9(1 + 1/2(r/a)^2)$



Summary and Future work

- The fluid runaway electron model in NIMROD reproduces results of implementations in other MHD codes in and analytics in linear calculations.
- There remain open questions about the importance of the drift velocity and runaway inertia in linear cases.
- Future work remains to complete the implementation of the perturbed drift velocity for linear calculations.