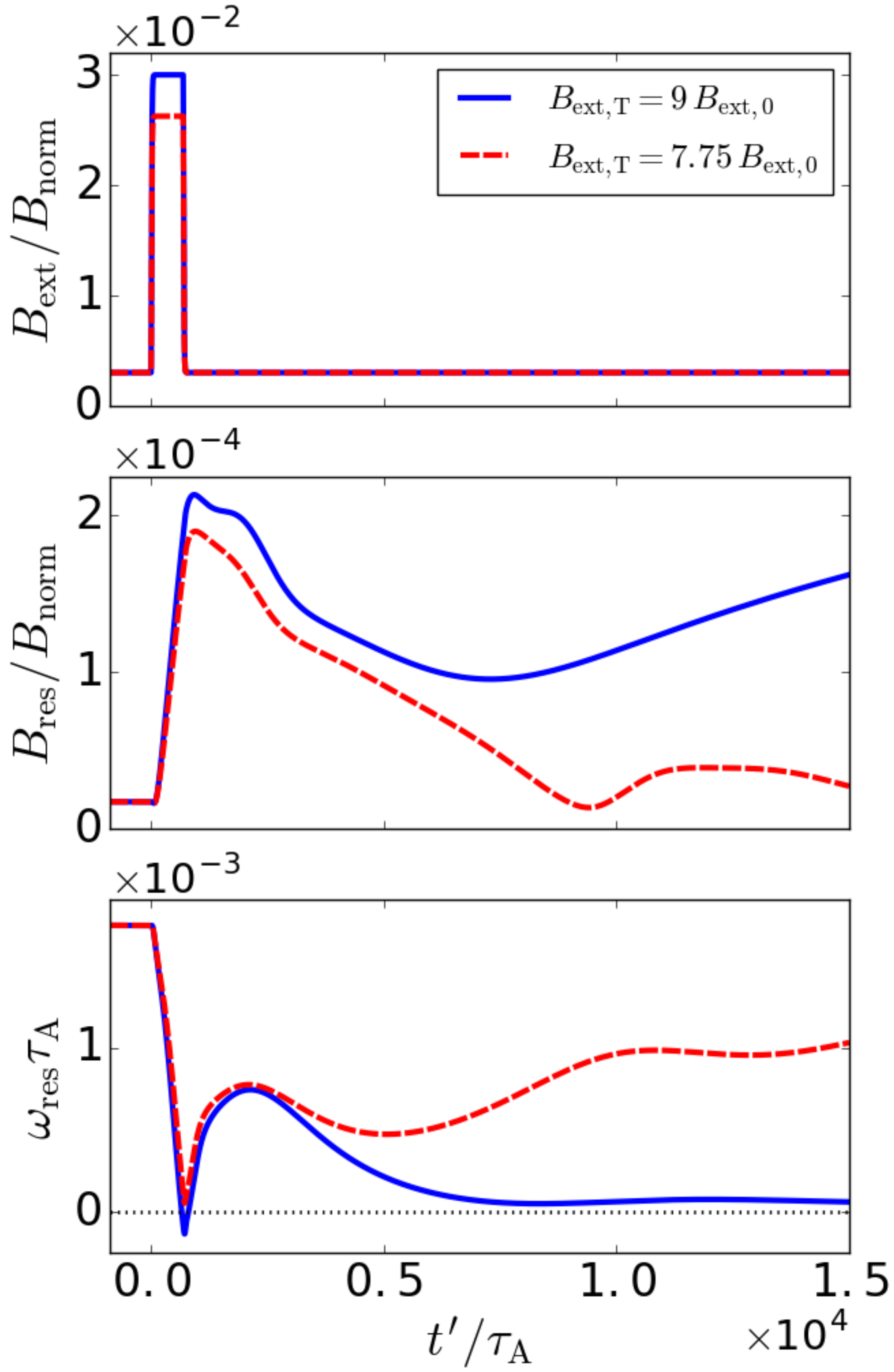
Nonlinear Mode Penetration Caused by Transient Magnetic Perturbations

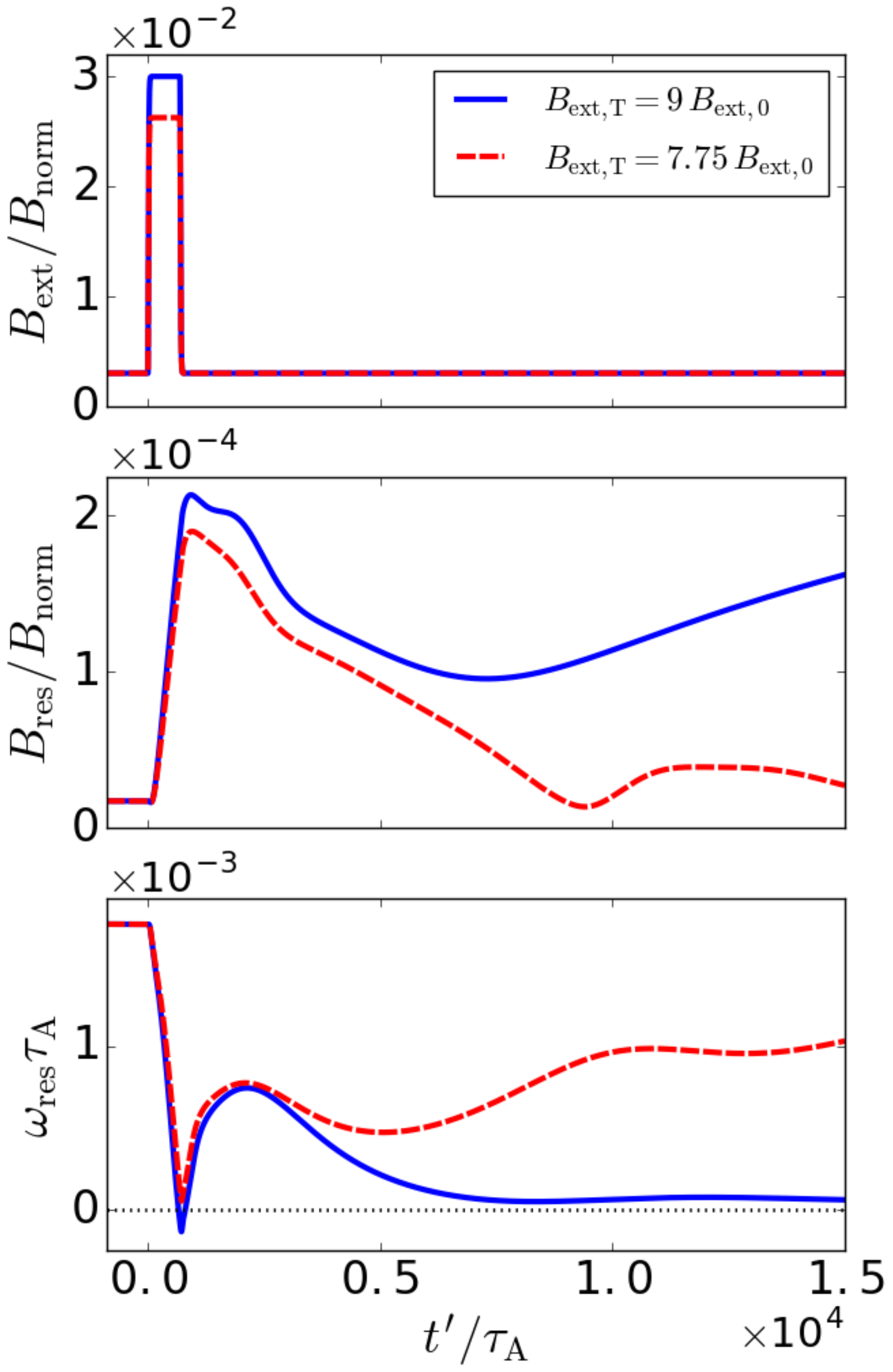
CTTS Meeting
April 22, 2018

M. T. Beidler, J. D. Callen,
C. C. Hegna, and C. R. Sovinec

Department of Engineering Physics,
University of Wisconsin - Madison

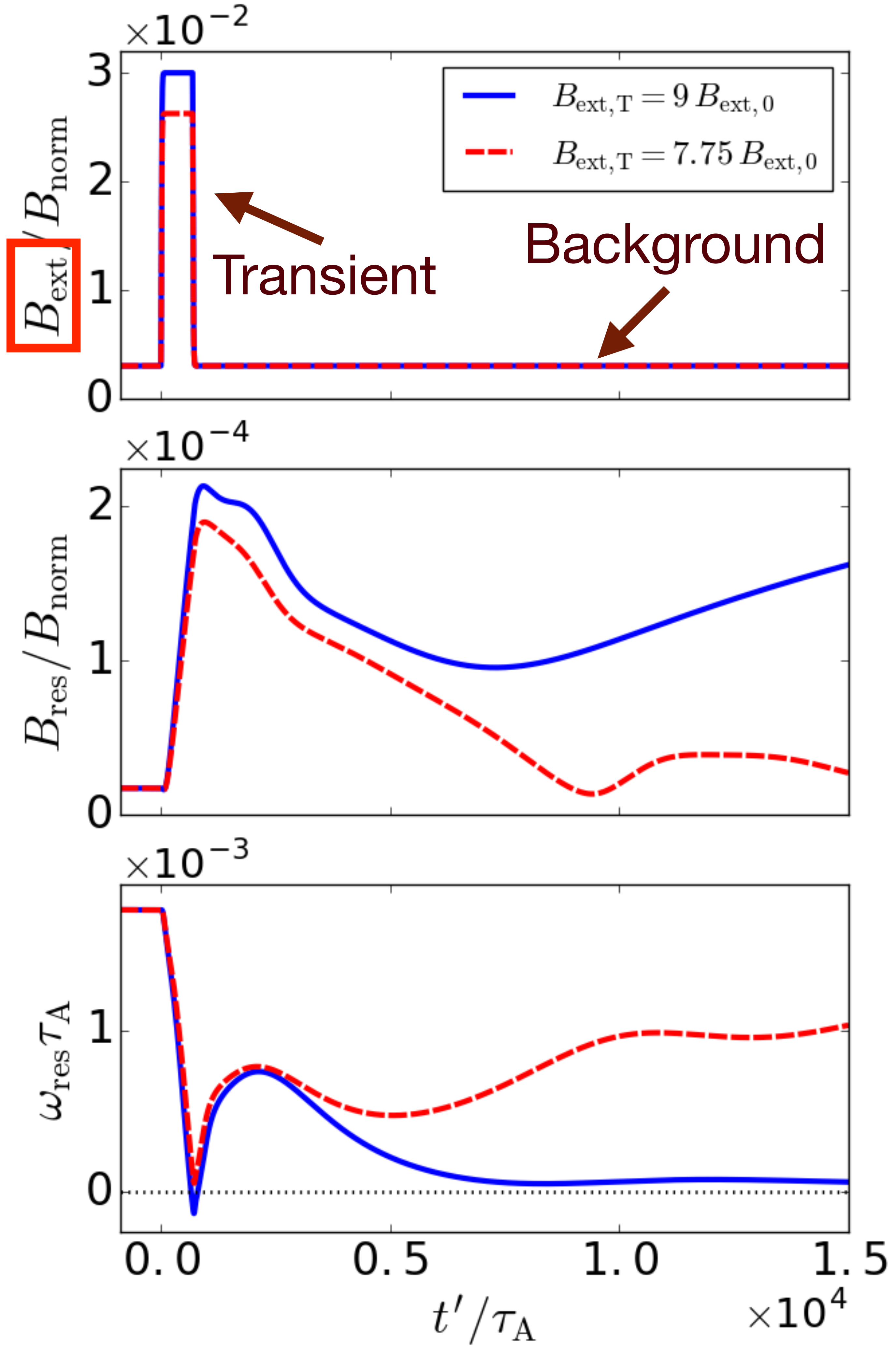


Frequently Used Quantities Need to Be Introduced



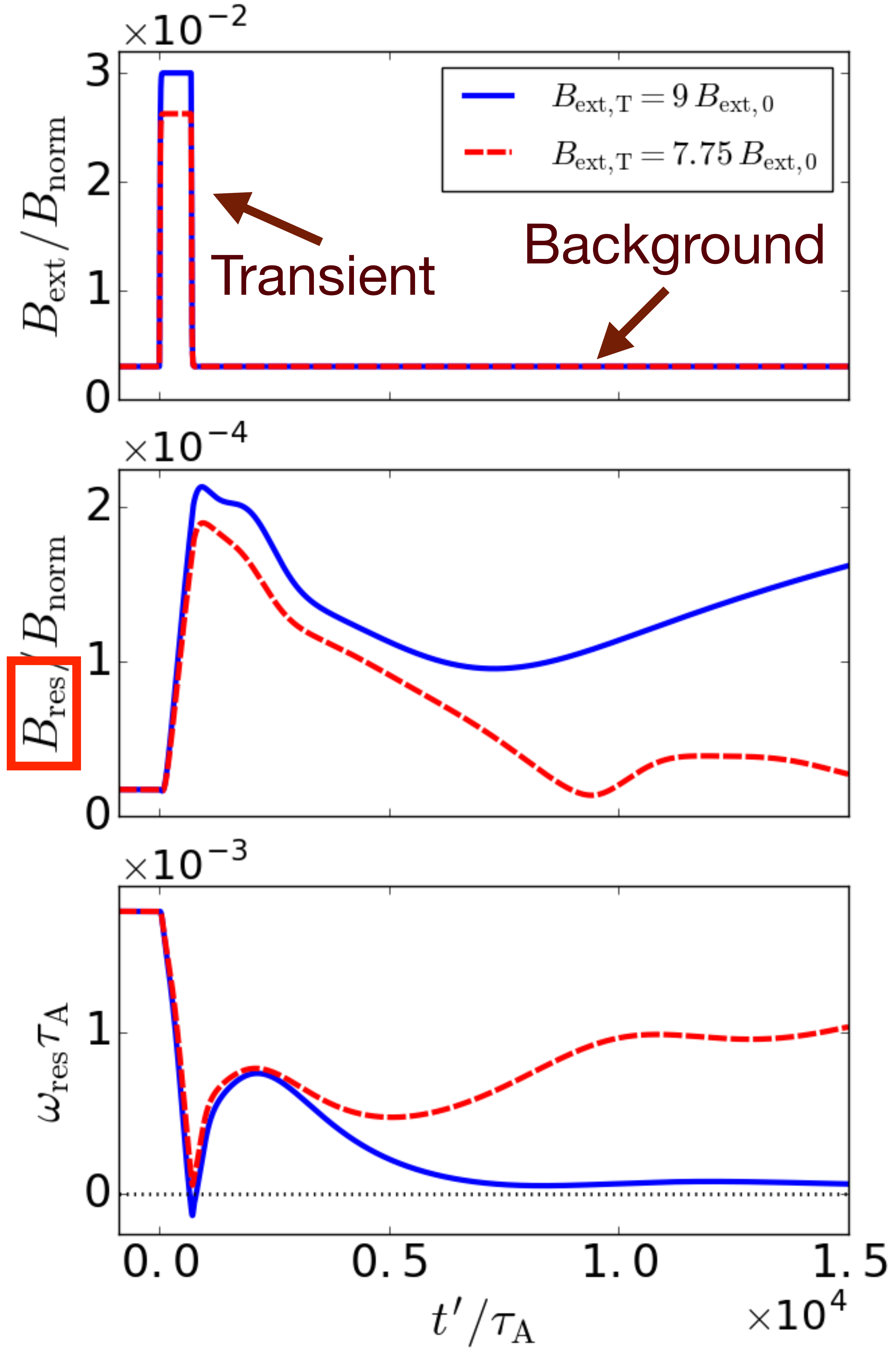
Frequently Used Quantities Need to Be Introduced

- **$B_{\rm ext}$: Magnitude of externally-applied field**
  - Background (0) and transient $(\rm T)$ contributions



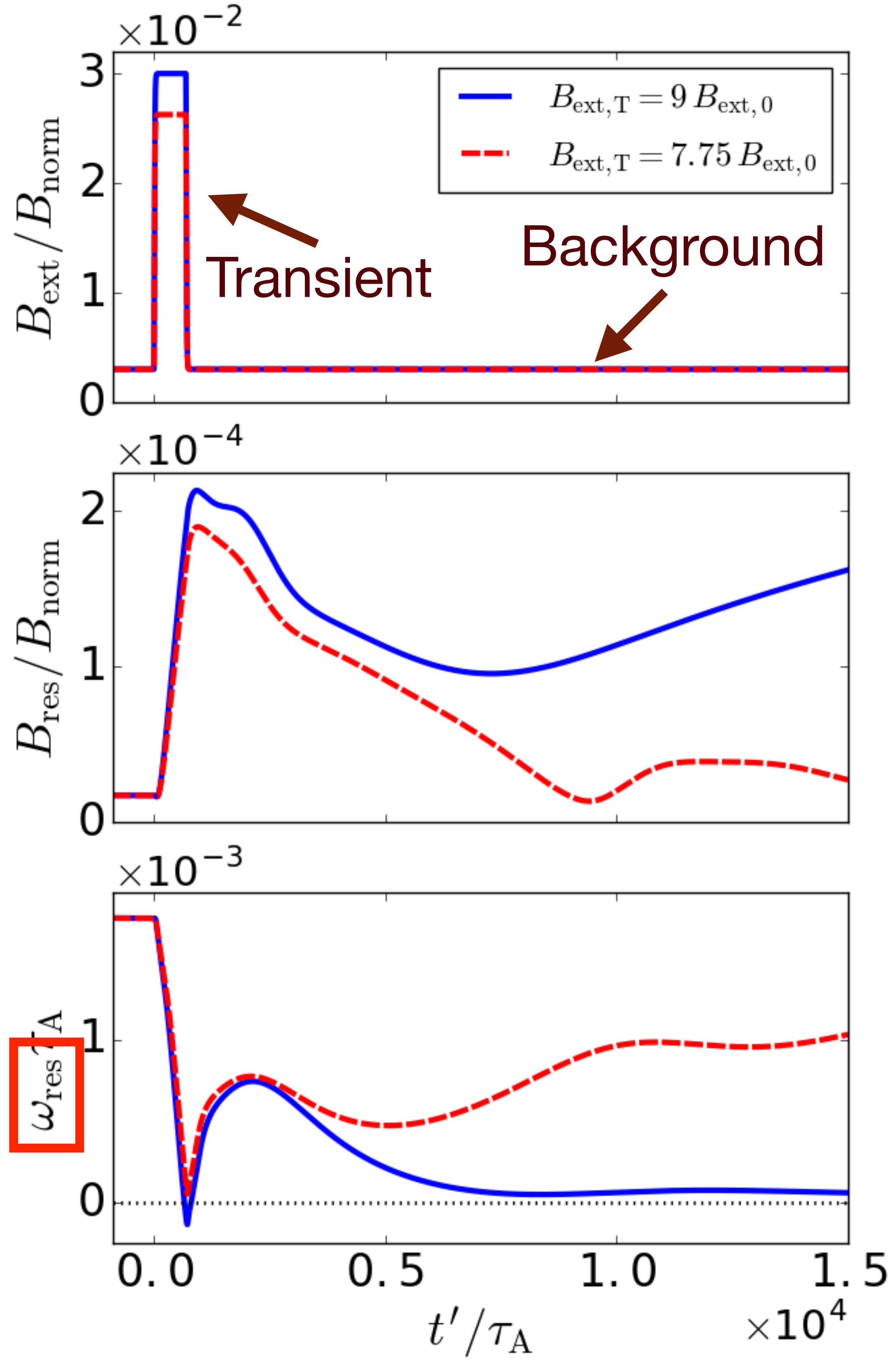
Frequently Used Quantities Need to Be Introduced

- **B_{ext} : Magnitude of externally-applied field**
  - Background (0) and transient (T) contributions
- **B_{res} : Magnitude of field response at resonant surface**



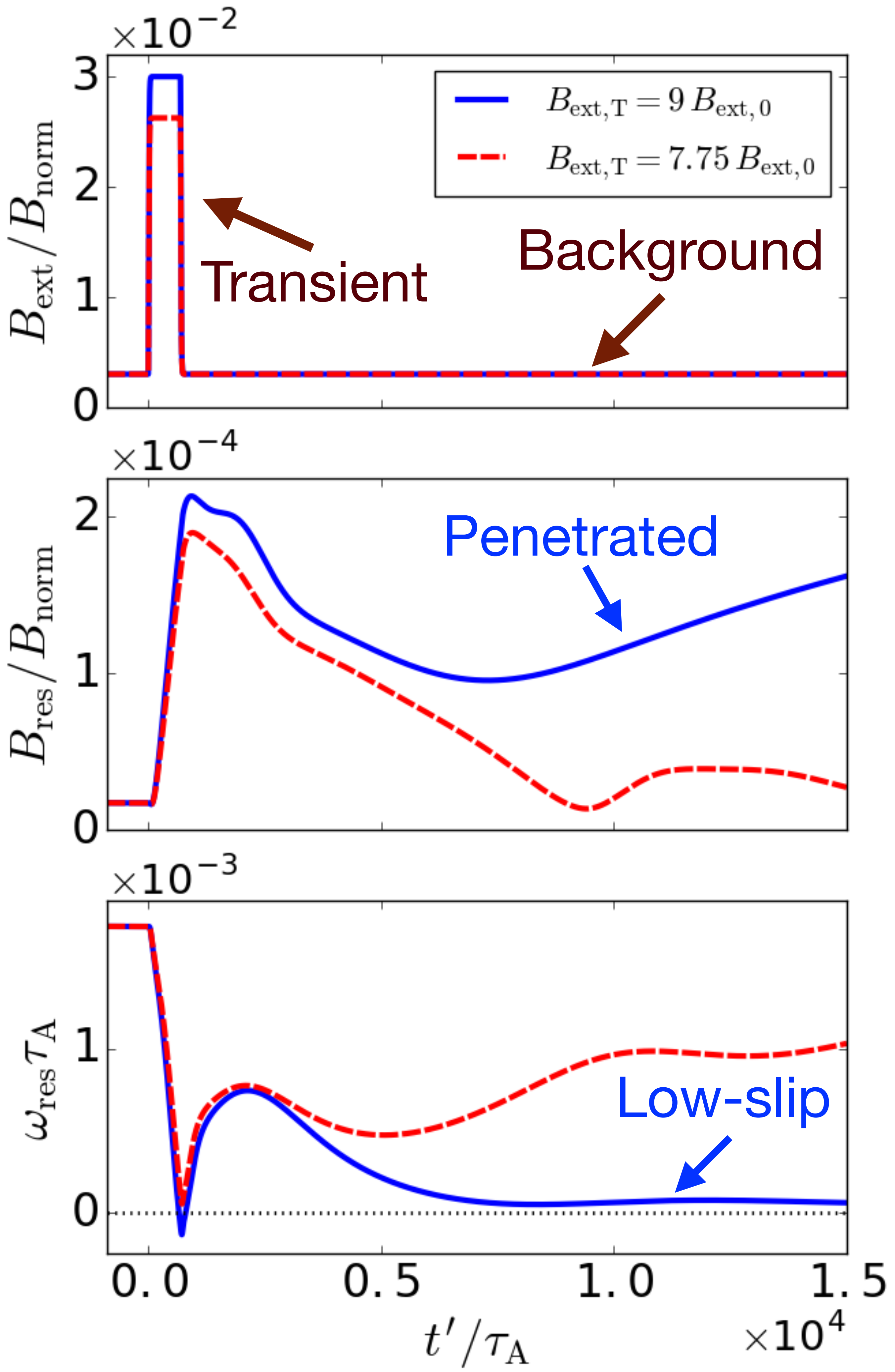
Frequently Used Quantities Need to Be Introduced

- B_{ext} **: Magnitude of externally-applied field**
  - Background (0) and transient (T) contributions
- B_{res} **: Magnitude of field response at resonant surface**
- ω_{res} **: Flow frequency at resonant surface**



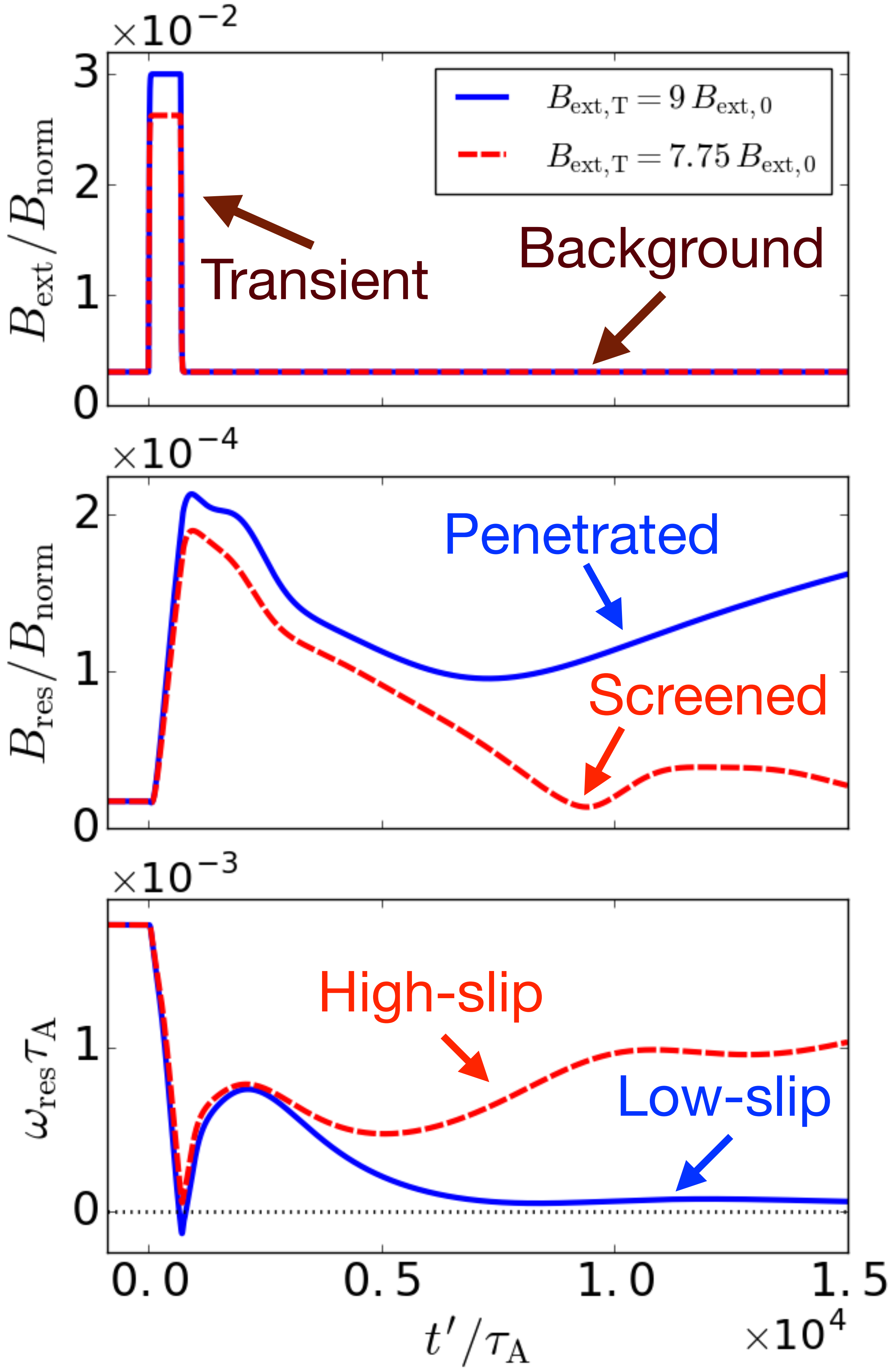
Frequently Used Quantities Need to Be Introduced

- B_{ext} **: Magnitude of externally-applied field**
  - Background (0) and transient (T) contributions
- B_{res} **: Magnitude of field response at resonant surface**
- ω_{res} **: Flow frequency at resonant surface**
  - Large transient precipitates transition to a low-slip state, with penetrated B_{res}



Frequently Used Quantities Need to Be Introduced

- B_{ext} **: Magnitude of externally-applied field**
  - Background (0) and transient (T) contributions
- B_{res} **: Magnitude of field response at resonant surface**
- ω_{res} **: Flow frequency at resonant surface**
  - Large transient precipitates transition to a low-slip state, with penetrated B_{res}
  - Small transient returns to a high-slip state, with screened B_{res}



Motivation: External 3D Fields Cause Forced Magnetic Reconnection

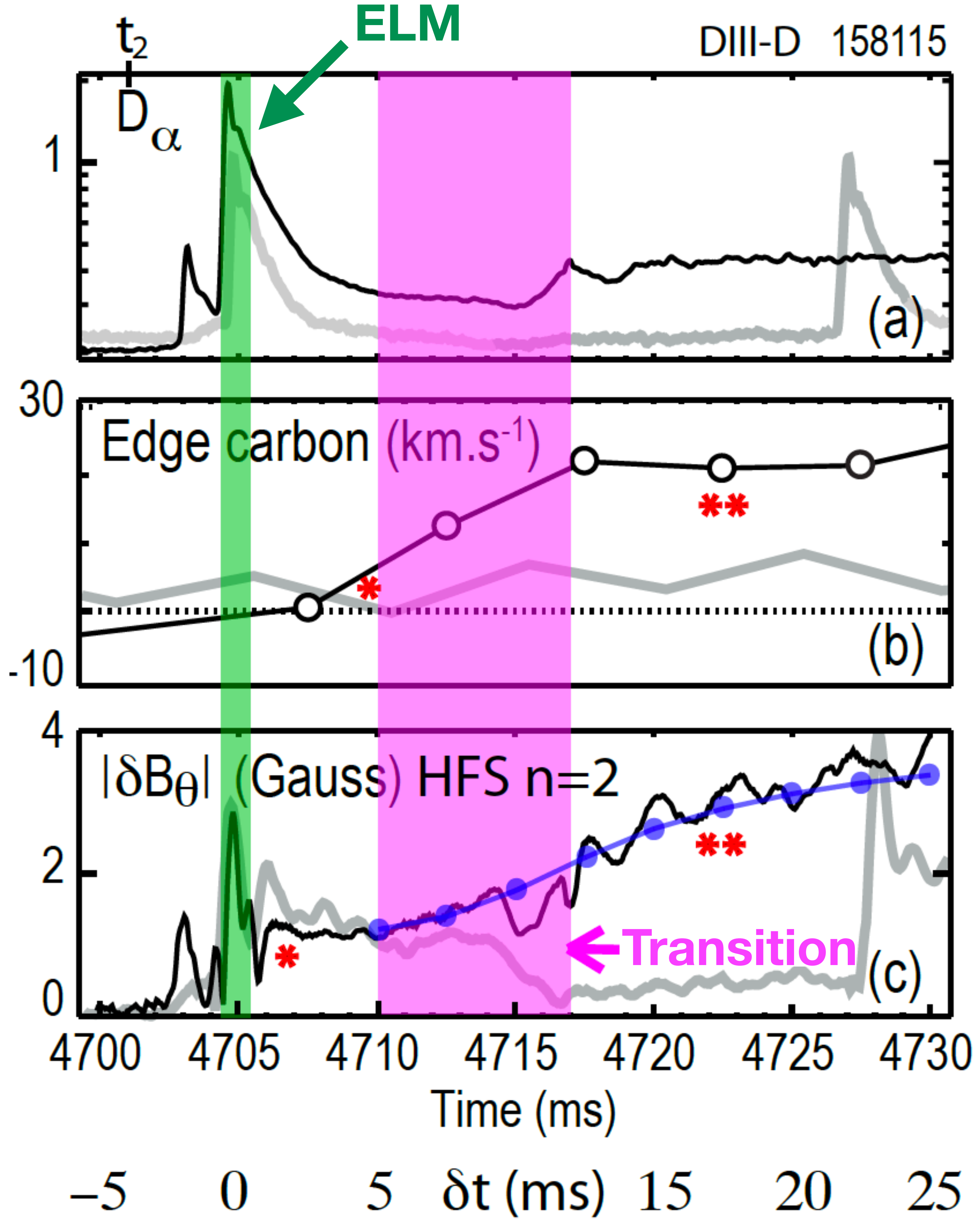
- **Externally applied 3D fields force magnetic reconnection (FMR)**
  - Islands can lock plasma to 3D field structure
  - Fundamental physics governed by external forcing, flow, resistivity, and viscosity

Motivation: Transient MHD Events Cause Forced Magnetic Reconnection

- **Externally applied 3D fields force magnetic reconnection (FMR)**
  - Islands can lock plasma to 3D field structure
  - Fundamental physics governed by external forcing, flow, resistivity, and viscosity
- **Transient MHD events are an additional source of 3D fields**

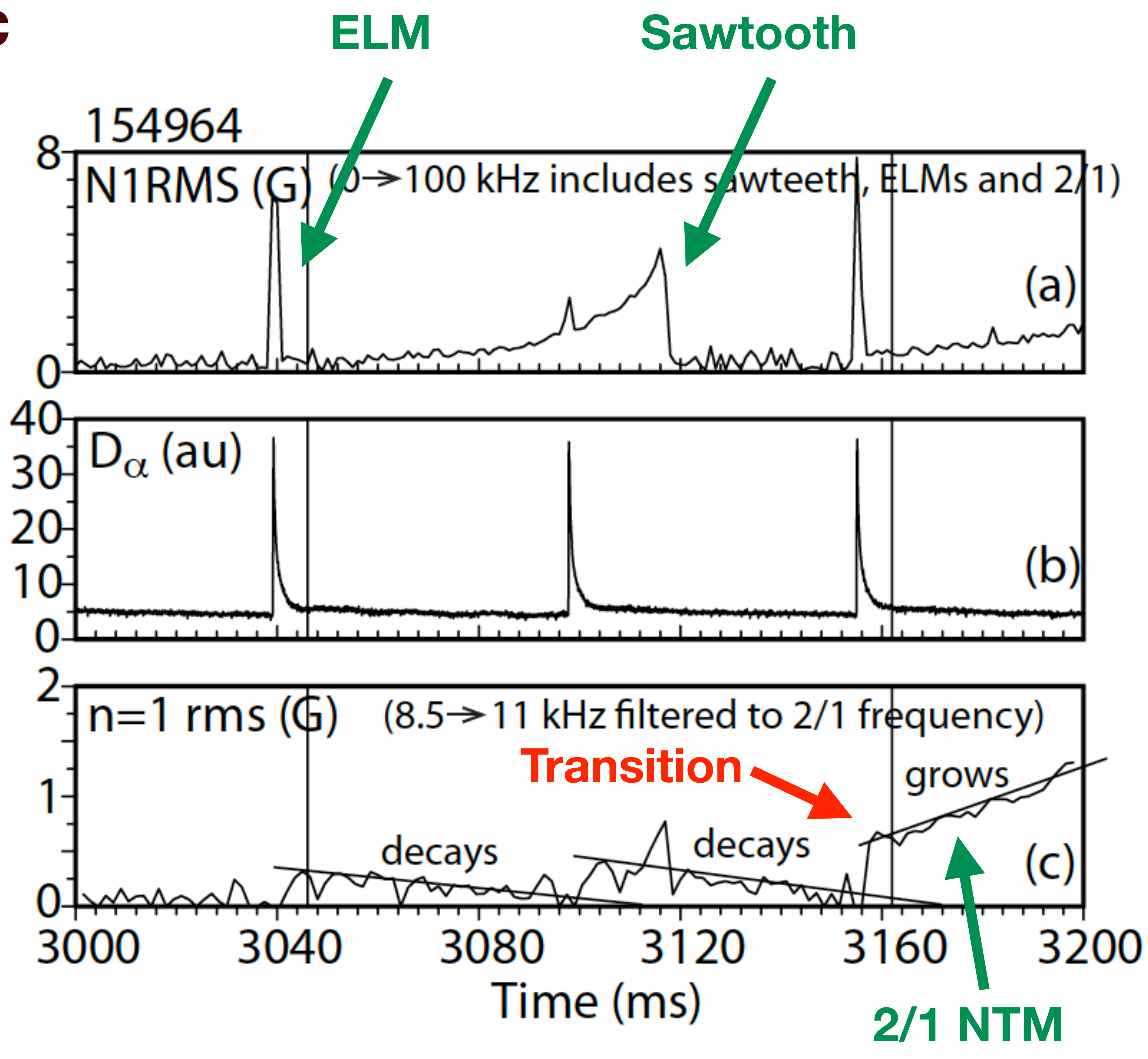
Motivation: ELM Can Precipitate Transition to ELM-Free State

- **Externally applied 3D fields force magnetic reconnection (FMR)**
  - Islands can lock plasma to 3D field structure
  - Fundamental physics governed by external forcing, flow, resistivity, and viscosity
- **Transient MHD events are an additional source of 3D fields; can induce transition**
  - ELM can trigger ELM-suppressed state for large resonant magnetic perturbation (RMP)
    - Paz Soldan et al., PRL (2015); Nazikian et al., PRL (2015); Callen et al., UW-CPTC Report 16-4



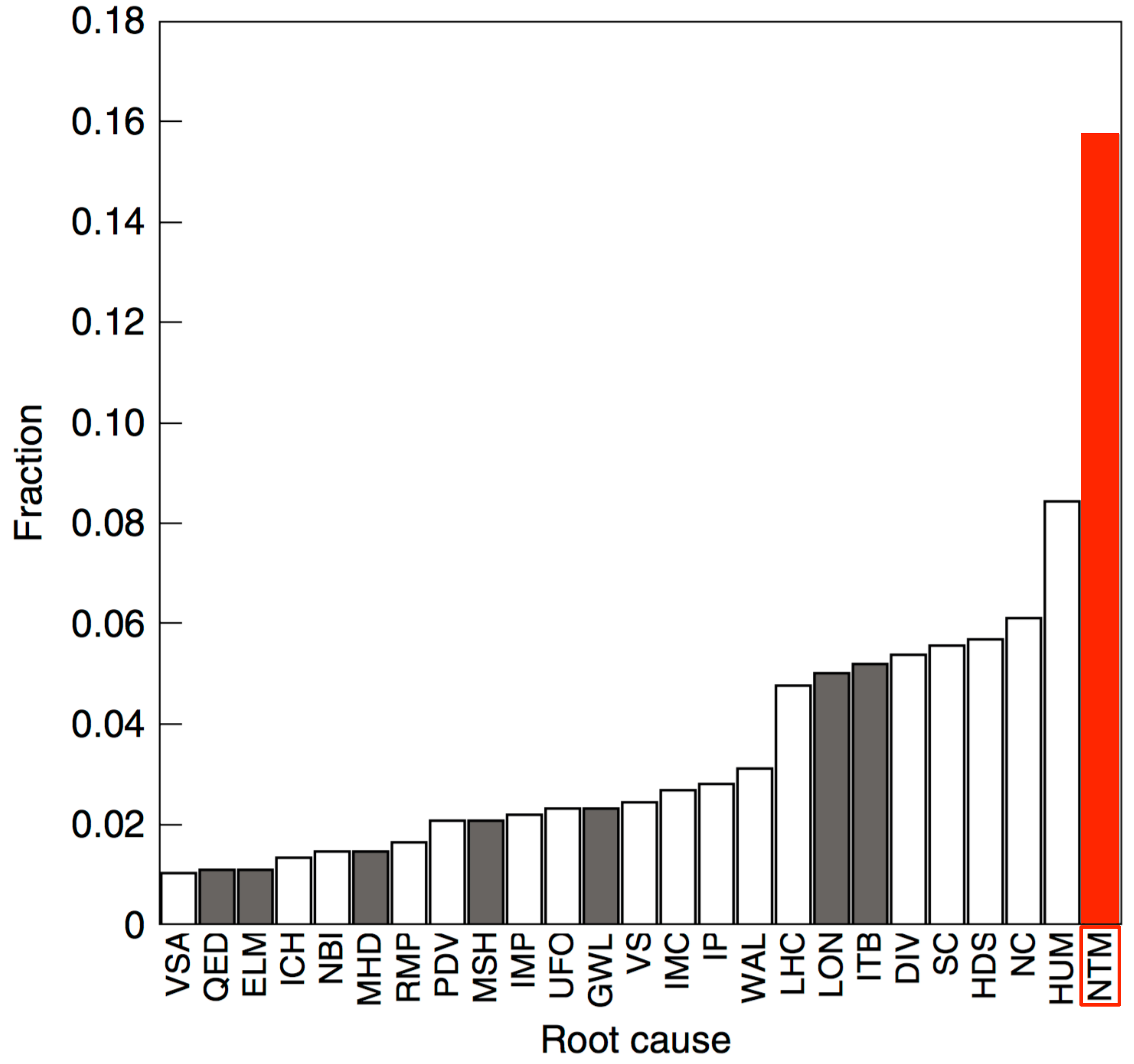
Motivation: ELMs and Sawteeth Can Precipitate NTM Growth

- **Externally applied 3D fields force magnetic reconnection (FMR)**
  - Islands can lock plasma to 3D field structure
  - Fundamental physics governed by external forcing, flow, resistivity, and viscosity
- **Transient MHD events are an additional source of 3D fields; can induce transition**
  - NTMs can be seeded by ELMs/sawteeth
    - La Haye, private communication (2016)



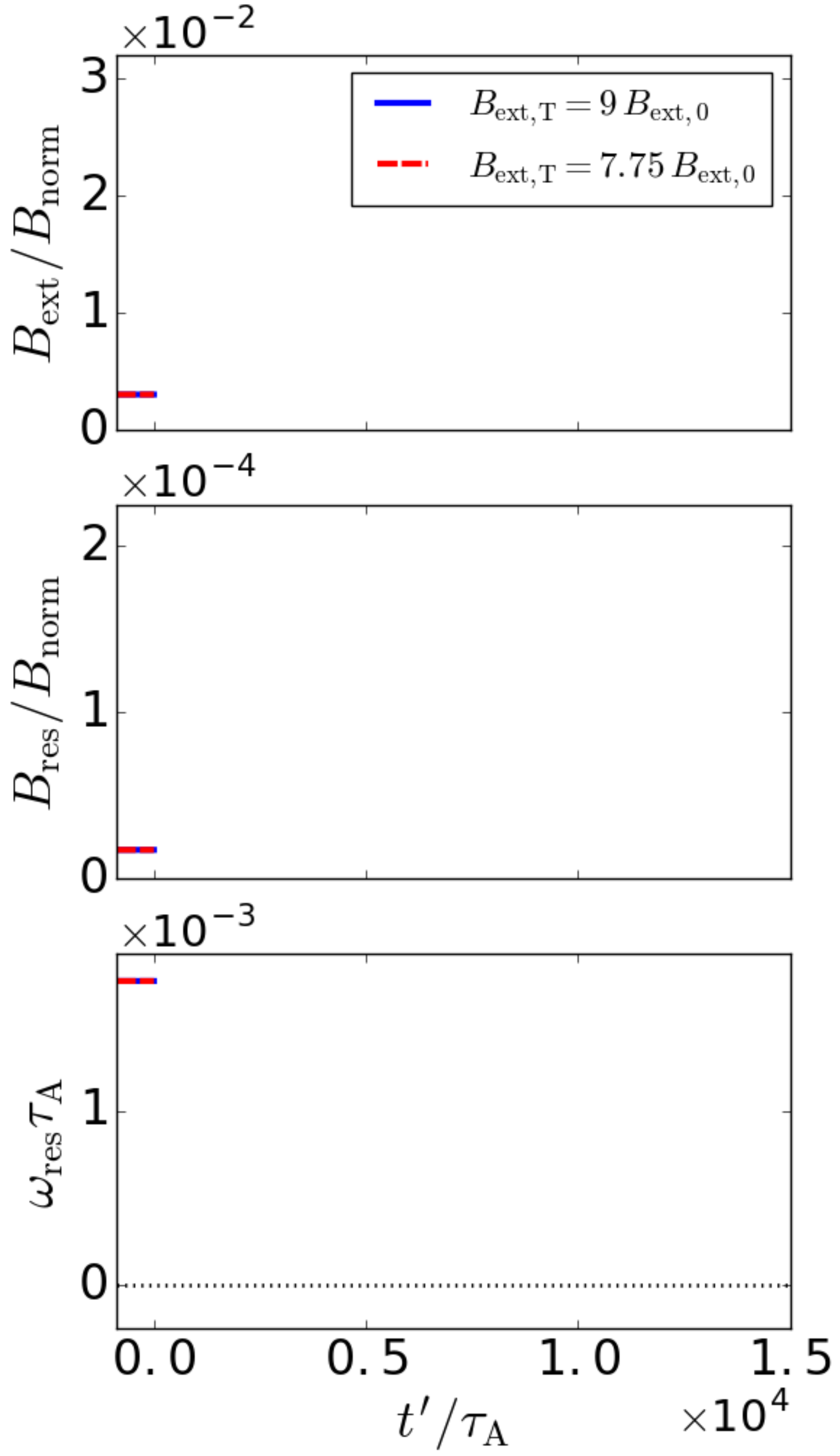
Motivation: NTMs Lead to Locked Mode Disruptions

- **The largest cause of disruptions in JET are NTMs that grow and lock**
  - de Vries et al., NF (2011)
  - 86% of NTMs triggered by sawteeth and 7% by ELMs



Mode Penetration Determined by Transient-Induced Force Evolution at Rational Surface

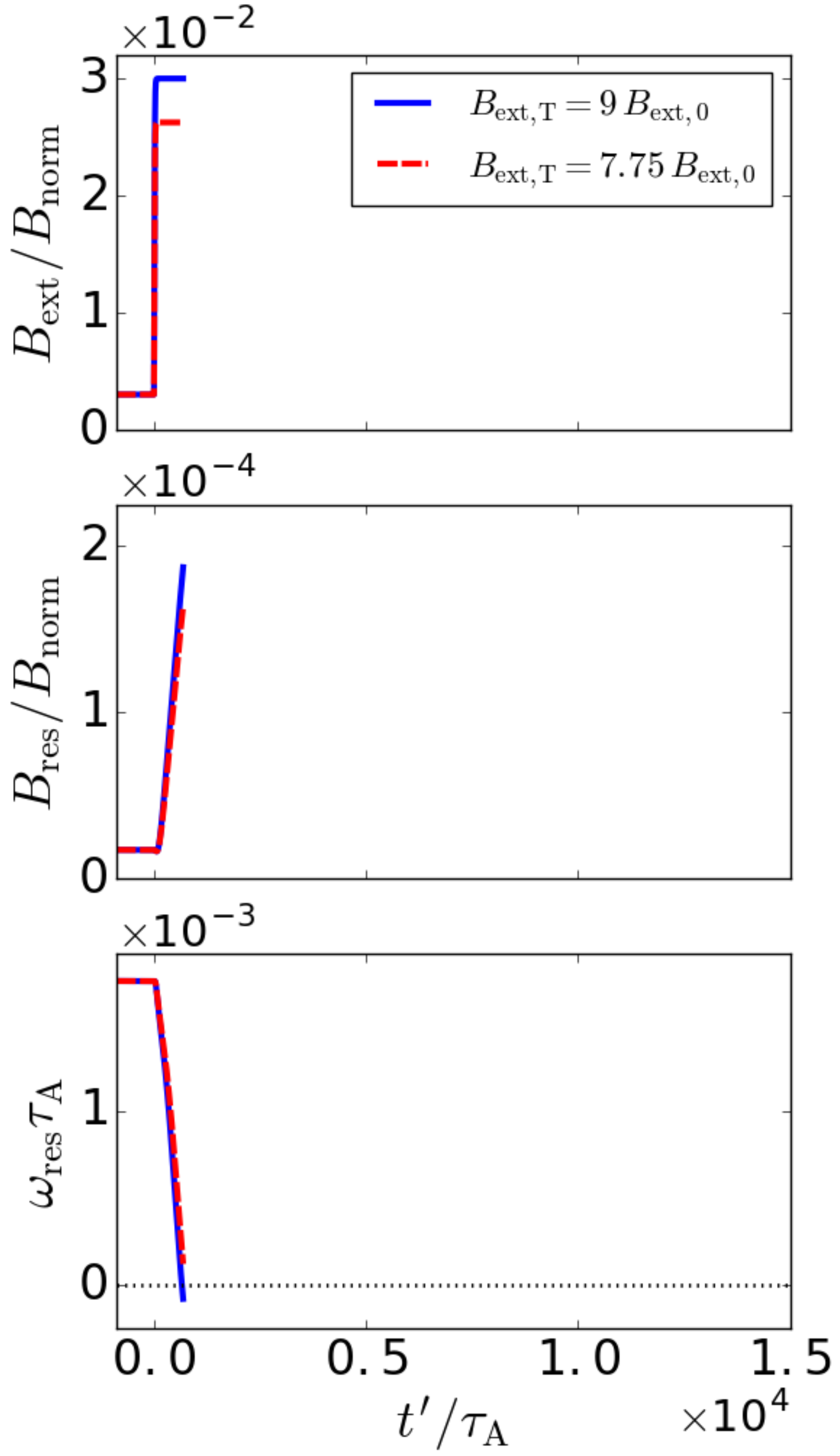
- **Begin in time-asymptotic, metastable state**
  - Background external 3D magnetic field $B_{\text{ext},0}$ is flow-screened
  - Electromagnetic (EM) and viscous forces balance



Mode Penetration Determined by Transient-Induced Force Evolution at Rational Surface

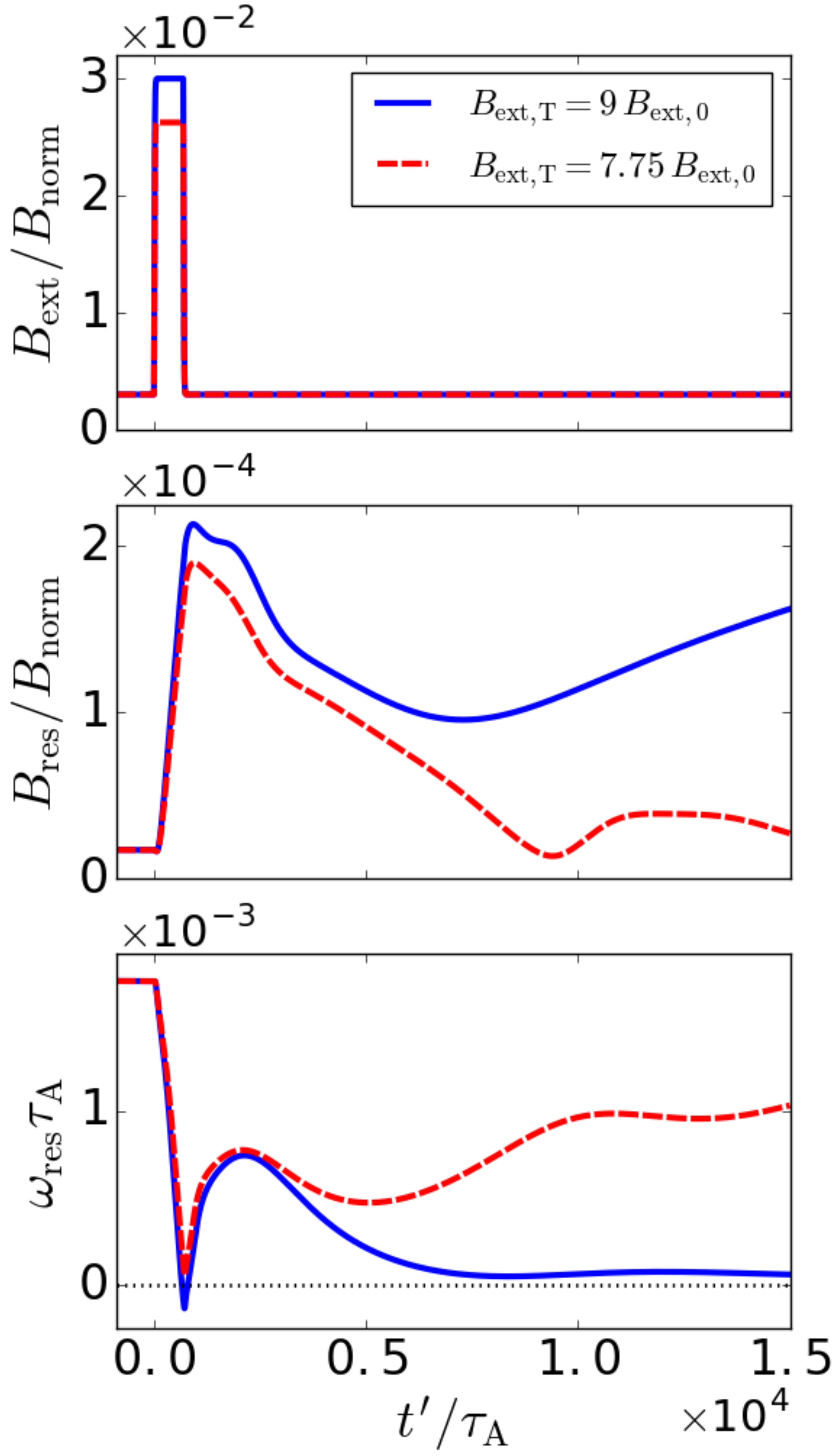
- **Begin in time-asymptotic, metastable state**
  - Background external 3D magnetic field $B_{\mathrm{ext},0}$ is flow-screened
  - Electromagnetic (EM) and viscous forces balance

- **Transient 3D field $B_{\mathrm{ext,T}}$ is added to $B_{\mathrm{ext},0}$**
  - EM force increases due to evolving current and magnetic field
  - Forcing decreases flow locally
    ➣ flow profile evolution induces viscous force



Mode Penetration Determined by Transient-Induced Force Evolution at Rational Surface

- **Begin in time-asymptotic, metastable state**
  - Background external 3D magnetic field $B_{\text{ext},0}$ is flow-screened
  - Electromagnetic (EM) and viscous forces balance
- **Transient 3D field $B_{\text{ext,T}}$ is added to $B_{\text{ext},0}$**
  - EM force increases due to evolving current and magnetic field
  - Forcing decreases flow locally
    ➛ flow profile evolution induces viscous force
- **Transient turns off and system continues to evolve**
  - Mutual evolution of forces determines final state



Outline

- Explore dynamics of transient perturbation in slab geometry

- Computational results elucidate mode penetration dynamics

- Develop analytic model of mode penetration dynamics

Outline

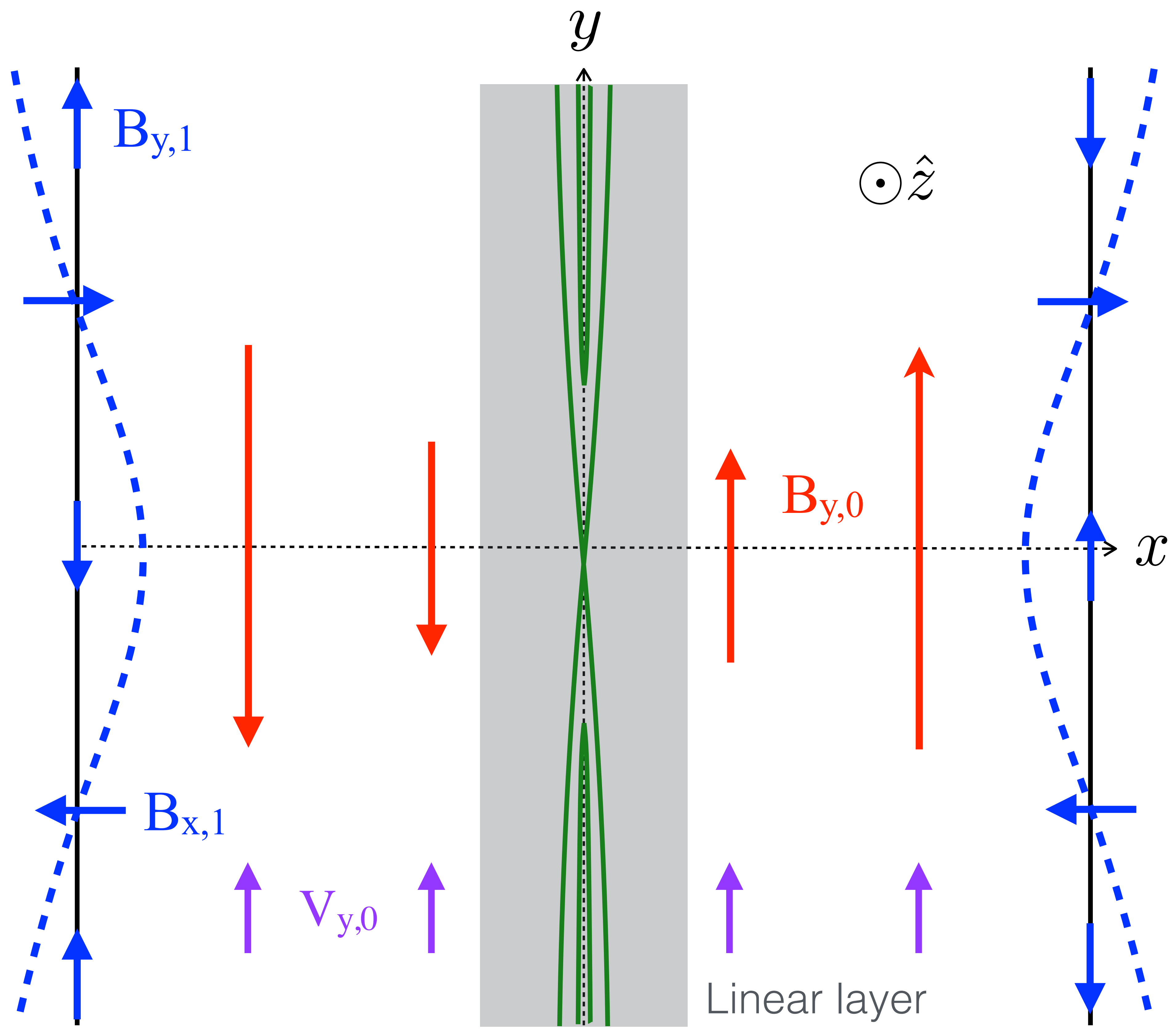
- **Explore dynamics of transient perturbation in slab geometry**

- **Computational results elucidate mode penetration dynamics**

- **Develop analytic model of mode penetration dynamics**

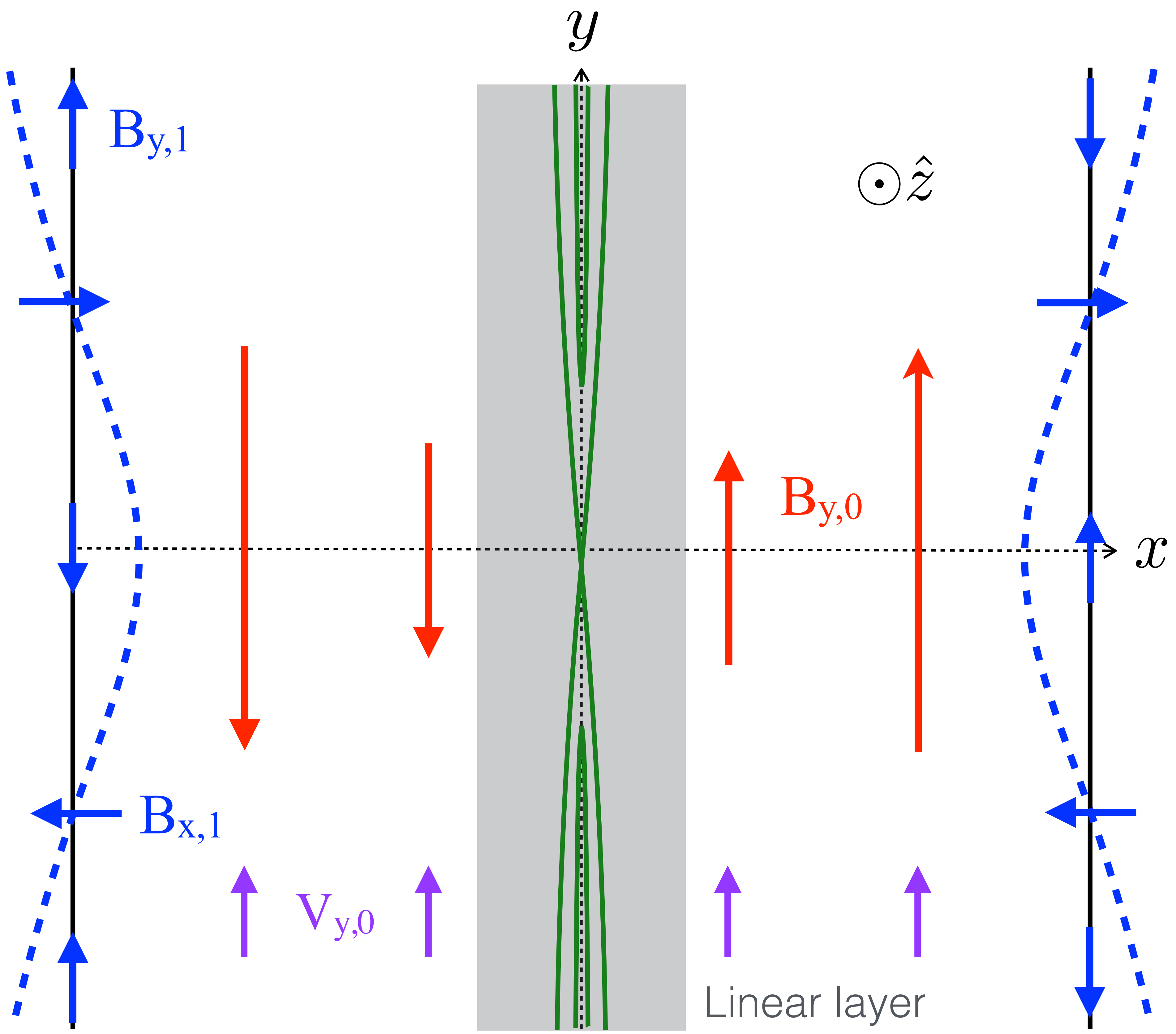
Slab Model Employed to Simplify Computations and Analytics

- **Slab geometry with uniform out-of-plane current density**
  - Stable equilibrium with $\Delta' a \cong -2k_y a < 0$



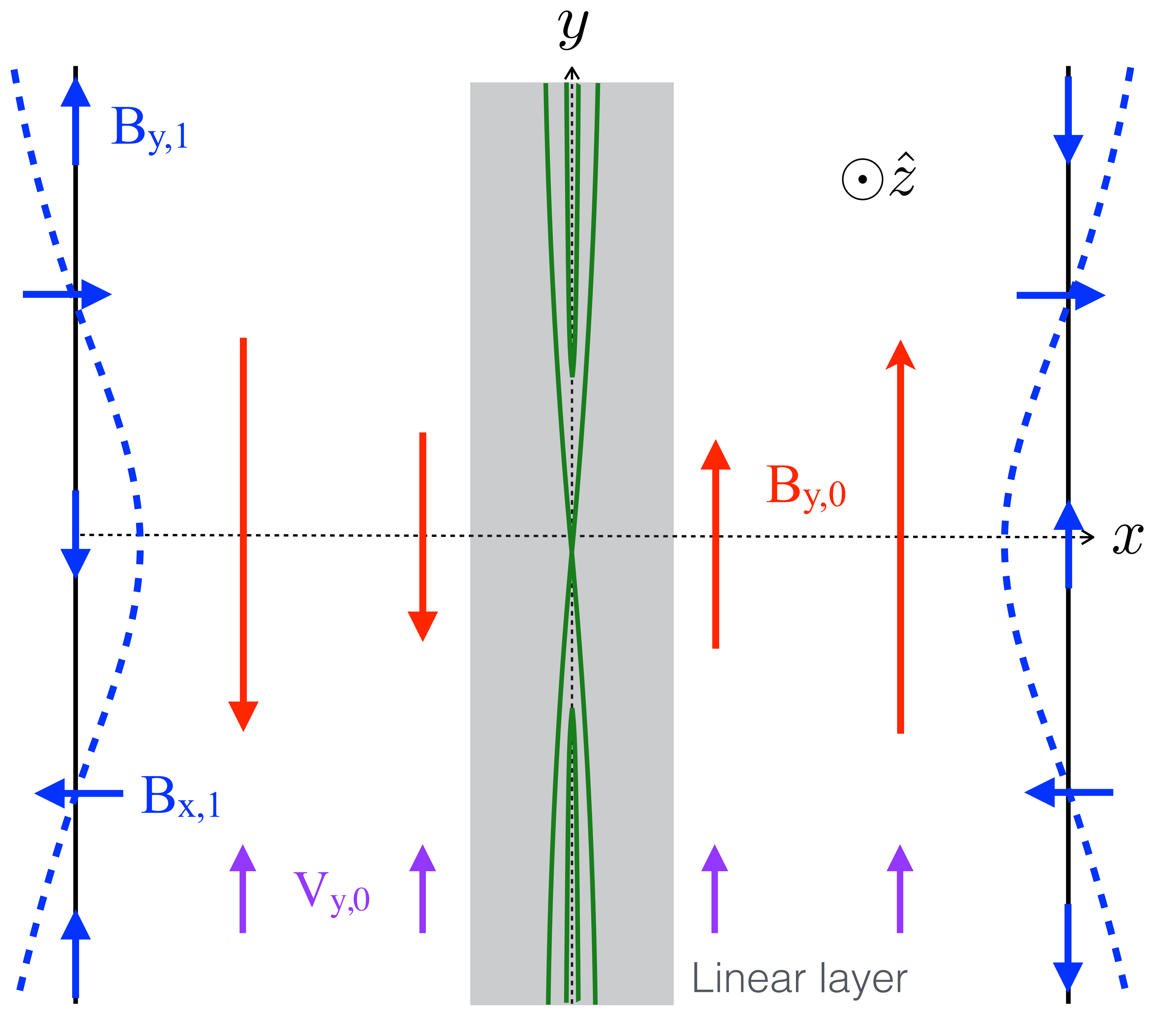
Slab Model Employed to Simplify Computations and Analytics

- **Slab geometry with uniform out-of-plane current density**
  - Stable equilibrium with $\Delta' a \cong -2k_y a < 0$
- **Apply normal magnetic field $B_{x,1}(|x|=a) = B_{ext}\sin(k_y y)$**
  - Drives reconnection at $x = 0$



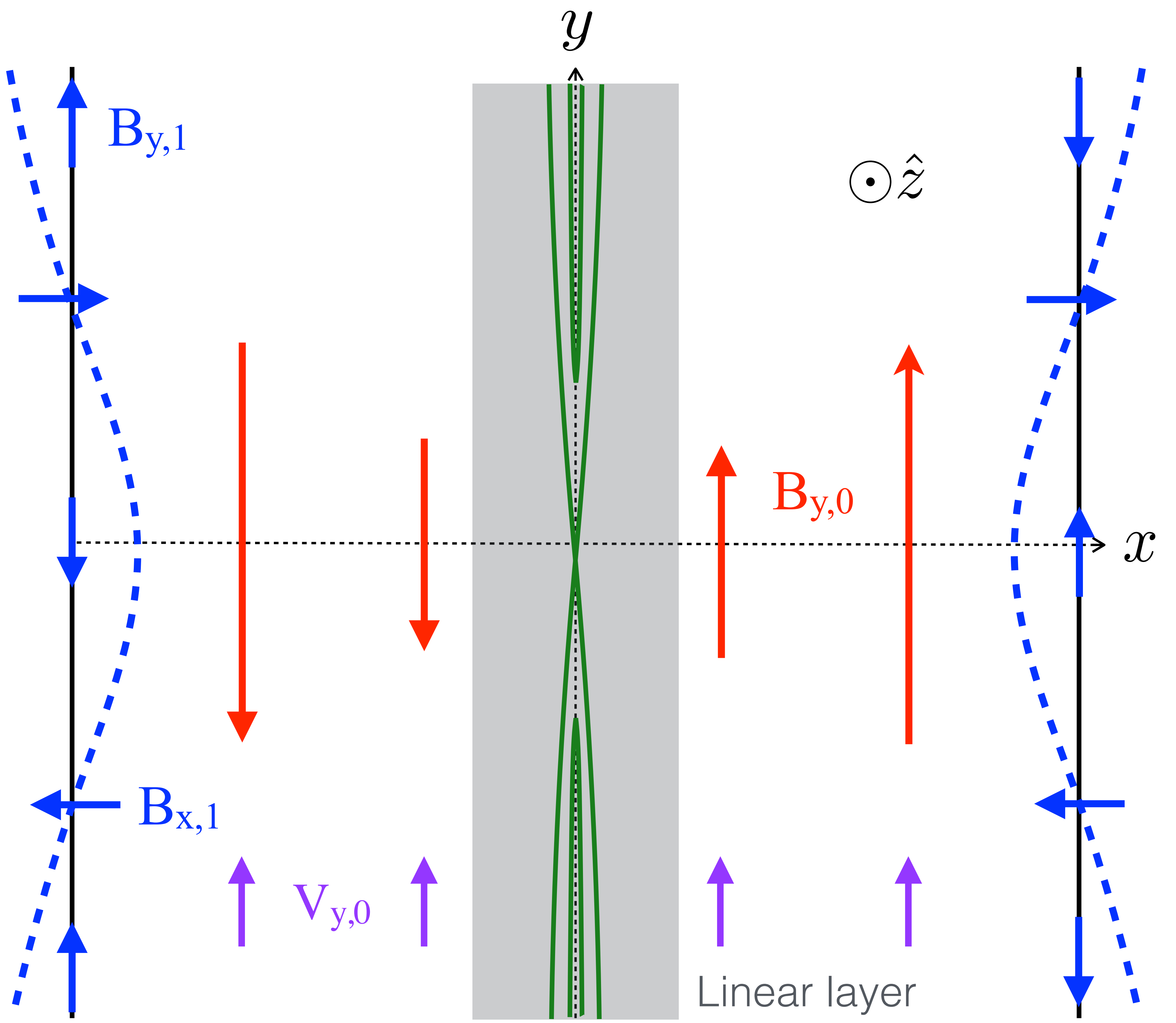
Slab Model Employed to Simplify Computations and Analytics

- **Slab geometry with uniform out-of-plane current density**
  - Stable equilibrium with $\Delta' a \cong -2k_y a < 0$
- **Apply normal magnetic field $B_{x,1}(|x|=a) = B_{ext}\sin(k_y y)$**
  - Drives reconnection at $x = 0$
- **Constant flow with $V_{y,0}$**
  - Provides flow-screening physics
  - Flow frequency at $x=0$: $\omega_{res} = k \cdot V = k_y V_y$



Slab Model Employed to Simplify Computations and Analytics

- **Slab geometry with uniform out-of-plane current density**
  - Stable equilibrium with $\Delta' a \cong -2k_y a < 0$
- **Apply normal magnetic field $B_{x,1}(|x|=a) = B_{\text{ext}} \sin(k_y y)$**
  - Drives reconnection at $x=0$
- **Constant flow with $V_{y,0}$**
  - Provides flow-screening physics
  - Flow frequency at $x=0$: $\omega_{\text{res}} = k \cdot V = k_y V_y$
- **Visco-Resistive dissipation parameters**
  - $S = 1.1\times10^{7}$, $P_m = 20$
  - Linear layer width: $\delta_{\text{VR}} = S^{-1/3} P_m^{1/6} a = 7.4\times10^{-3} a$



Time-Asymptotic Electromagnetic and Viscous Forces Balance At Rational Surface

- **n=0 EM force per unit length in *z* at *x*=0**

$$\hat{F}_{y,EM} = \int_{-\delta_{\mathrm{VR}}/2}^{\delta_{\mathrm{VR}}/2} dx \int_{-L_{y/2}}^{L_y/2} dy (\mathbf{J} \times \mathbf{B}) \cdot \hat{y} = -\frac{n\pi}{\mu_0 k_y^2} \mathrm{Im}\left\{B_{\mathrm{res}}^* [\![\partial_x B_{\mathrm{res}}]\!]_{x=0}\right\}$$

Time-Asymptotic Electromagnetic and Viscous Forces Balance At Rational Surface

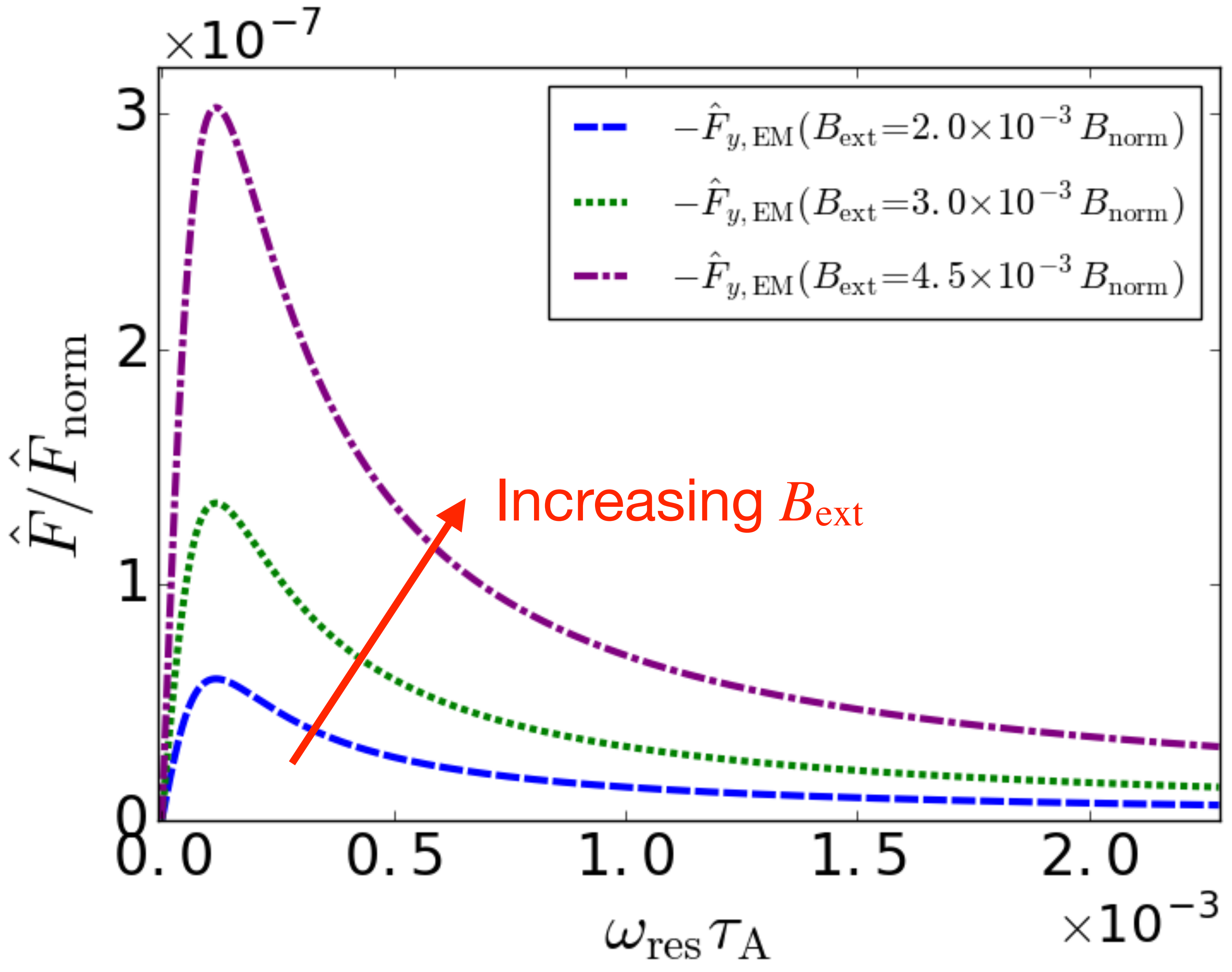
- **n=0 EM force per unit length in z at x=0**

$$\hat{F}_{y,EM} = \int_{-\delta_{\mathrm{VR}}/2}^{\delta_{\mathrm{VR}}/2} dx \int_{-L_y/2}^{L_y/2} dy (\mathbf{J}\times\mathbf{B})\cdot\hat{y} = -\frac{n\pi}{\mu_0 k_y^2}\mathrm{Im}\left\{B_{\mathrm{res}}^* [\![\partial_x B_{\mathrm{res}}]\!]_{x=0}\right\}$$

- Linear, time-asymptotic, visco-resistive response:

$$B_{\mathrm{res}} = \frac{a\Delta'_{\mathrm{ext}}}{-a\Delta' + i\omega_{\mathrm{res}}\tau_{\mathrm{VR}}} B_{\mathrm{ext}} \rightarrow \hat{F}_{y,EM} = -\frac{\omega_{\mathrm{res}}\tau_{\mathrm{VR}}}{(-a\Delta')^2 + (\omega_{\mathrm{res}}\tau_{\mathrm{VR}})^2}\frac{n\pi(a\Delta'_{\mathrm{ext}})^2}{ak_y^2}\frac{B_{\mathrm{ext}}^2}{\mu_0}$$

  - R. Fitzpatrick, NF **33**, 1049 (1993)
  - Quasilinear $F_{y,\mathrm{EM}}$ localized at x=0



Time-Asymptotic Electromagnetic and Viscous Forces Balance At Rational Surface

- **n=0 EM force per unit length in *z* at *x*=0**

$$\hat{F}_{y,EM} = \int_{-\delta_{\mathrm{VR}}/2}^{\delta_{\mathrm{VR}}/2} dx \int_{-L_{y/2}}^{L_y/2} dy(\mathbf{J}\times\mathbf{B})\cdot\hat{y} = -\frac{n\pi}{\mu_0 k_y^2}\mathrm{Im}\left\{B_{\mathrm{res}}^*[\![\partial_x B_{\mathrm{res}}]\!]_{x=0}\right\}$$

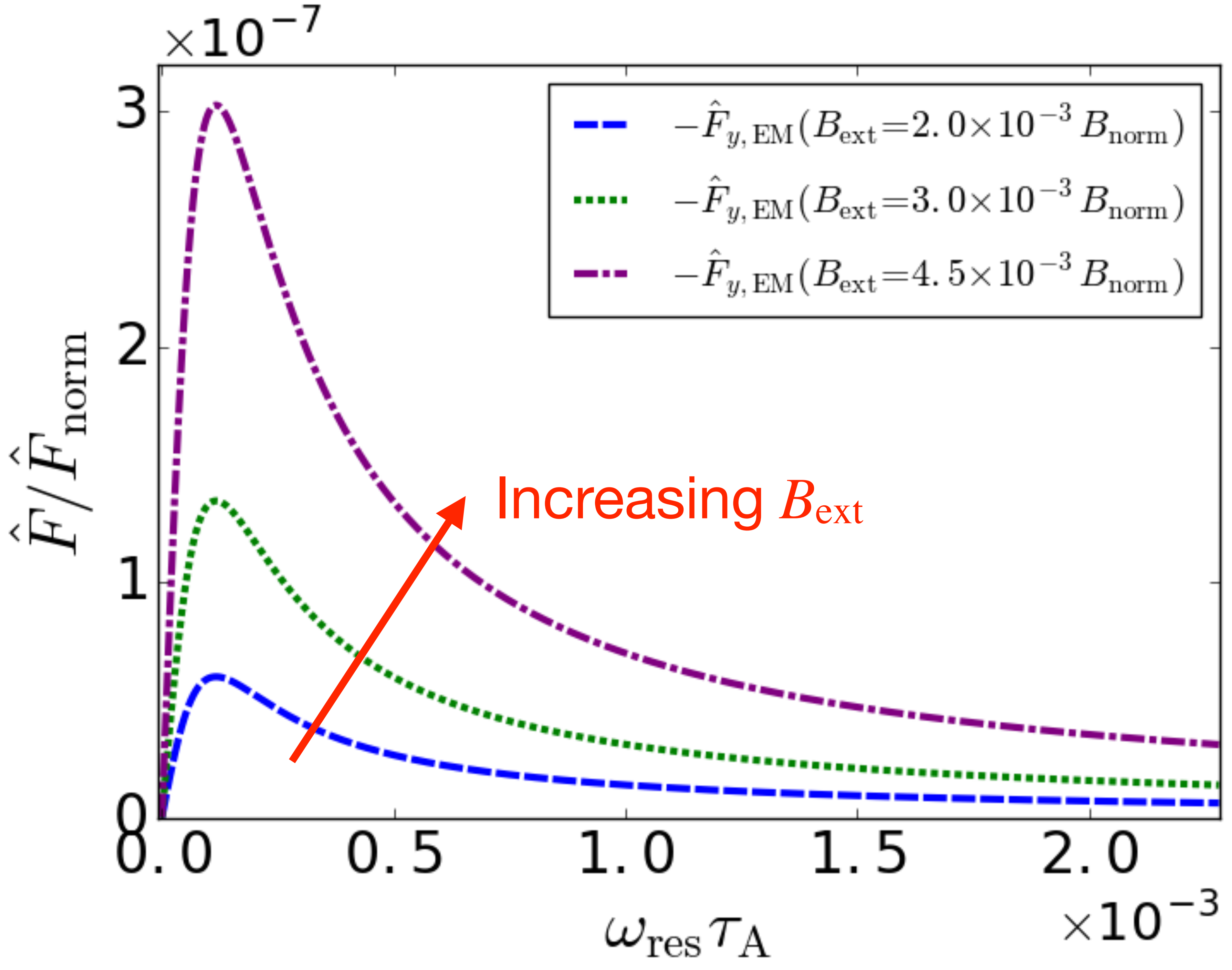
- Linear, time-asymptotic, visco-resistive response:

$$B_{\mathrm{res}} = \frac{a\Delta'_{\mathrm{ext}}}{-a\Delta' + i\omega_{\mathrm{res}}\tau_{\mathrm{VR}}} B_{\mathrm{ext}} \rightarrow \hat{F}_{y,EM} = -\frac{\omega_{\mathrm{res}}\tau_{\mathrm{VR}}}{(-a\Delta')^2 + (\omega_{\mathrm{res}}\tau_{\mathrm{VR}})^2}\frac{n\pi(a\Delta'_{\mathrm{ext}})^2}{ak_y^2}\frac{B_{\mathrm{ext}}^2}{\mu_0}$$

  - R. Fitzpatrick, NF **33**, 1049 (1993)
  - Quasilinear $F_{y,\mathrm{EM}}$ localized at x=0

- **Viscous force per unit length in *z* at *x*=0**

$$\hat{F}_{y,VS} = \int_{-\delta_\delta/2}^{\delta_\delta/2} dx \int_{-L_{y/2}}^{L_y/2} dy[\nabla\cdot\rho\nu\nabla\mathbf{V}]\cdot\hat{y} = L_y\rho\nu_0[\![\partial_x V(x,t)]\!]_{x=0}$$



Time-Asymptotic Electromagnetic and Viscous Forces Balance At Rational Surface

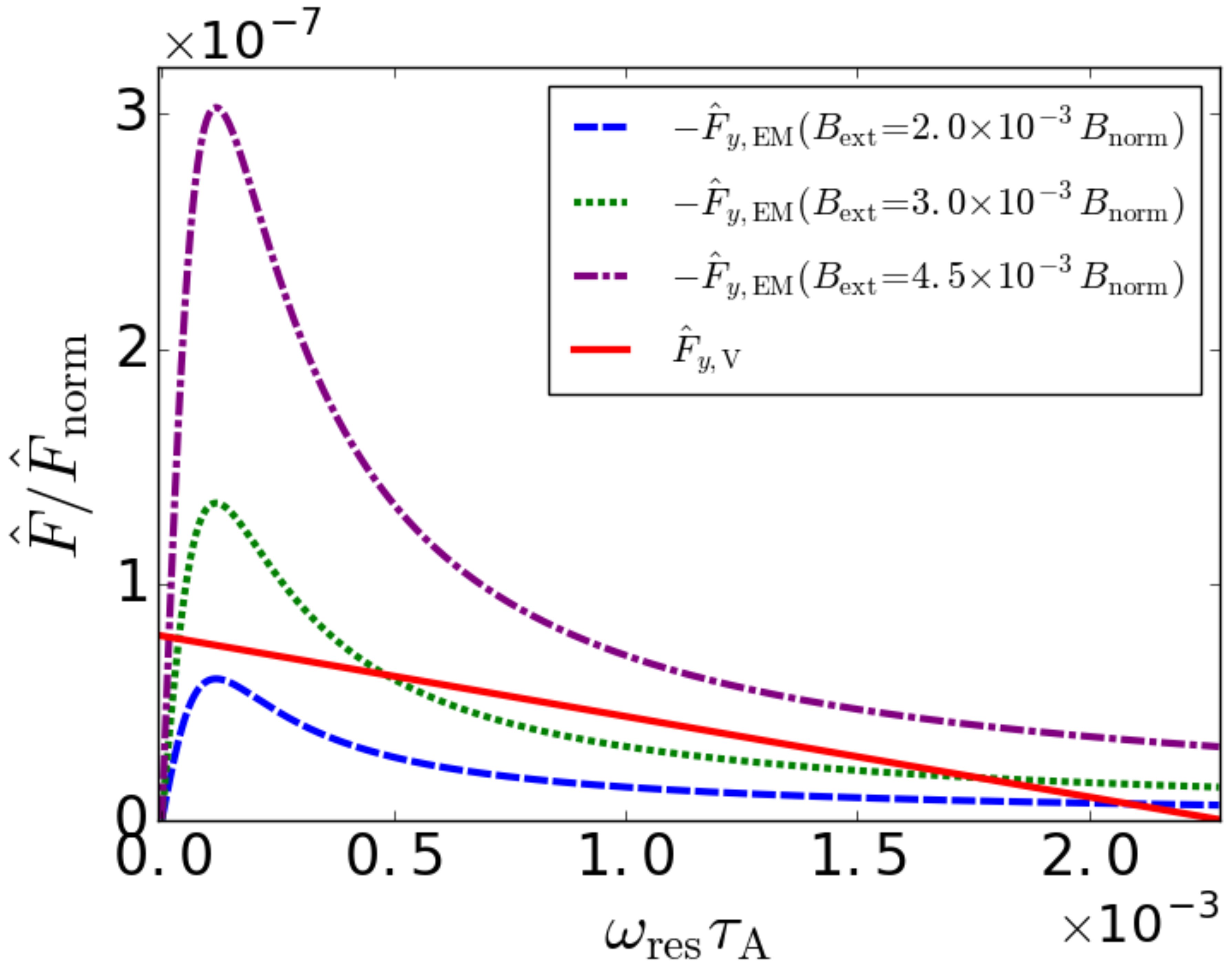
- **n=0 EM force per unit length in z at x=0**

$$\hat{F}_{y,EM} = \int_{-\delta_{VR}/2}^{\delta_{VR}/2} dx \int_{-L_y/2}^{L_y/2} dy(\mathbf{J}\times\mathbf{B})\cdot\hat{y} = -\frac{n\pi}{\mu_0 k_y^2}\mathrm{Im}\left\{B_{res}^{*}[\![\partial_x B_{res}]\!]_{x=0}\right\}$$

- Linear, time-asymptotic, visco-resistive response:

$$B_{res} = \frac{a\Delta'_{ext}}{-a\Delta' + i\omega_{res}\tau_{VR}} B_{ext} \rightarrow \hat{F}_{y,EM} = -\frac{\omega_{res}\tau_{VR}}{(-a\Delta')^2 + (\omega_{res}\tau_{VR})^2}\frac{n\pi(a\Delta'_{ext})^2}{ak_y^2}\frac{B_{ext}^2}{\mu_0}$$

  - R. Fitzpatrick, NF **33**, 1049 (1993)
  - Quasilinear $F_{y,EM}$ localized at x=0



- **Viscous force per unit length in z at x=0**

$$\hat{F}_{y,VS} = \int_{-\delta_\delta/2}^{\delta_\delta/2} dx \int_{-L_y/2}^{L_y/2} dy[\nabla\cdot\rho\nu\nabla\mathbf{V}]\cdot\hat{y} = L_y\rho\nu_0[\![\partial_x V(x,t)]\!]_{x=0}$$

- Because EM force is localized at x=0, $\partial^2_{xx}V_y$=0 in $0<x<|a_\nu|$ yields $\rightarrow \hat{F}_{y,V} = \frac{2L_y\rho\nu_0}{k_y a_\nu}(\omega_0 - \omega_{res})$

Time-Asymptotic Electromagnetic and Viscous Forces Balance At Rational Surface

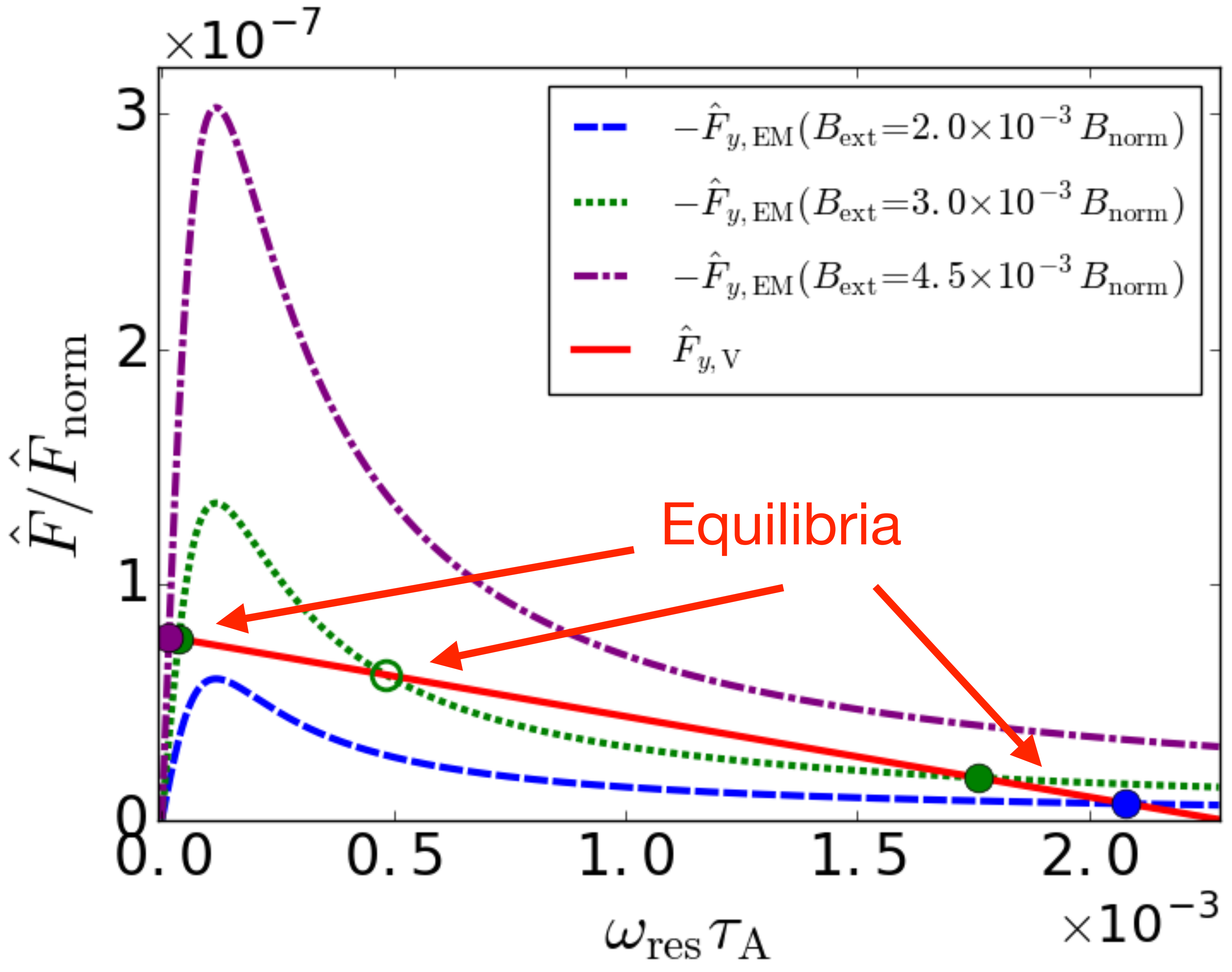
- **n=0 EM force per unit length in z at x=0**

$$\hat{F}_{y,EM} = \int_{-\delta_{VR}/2}^{\delta_{VR}/2} dx \int_{-L_y/2}^{L_y/2} dy (\mathbf{J}\times\mathbf{B})\cdot\hat{y} = -\frac{n\pi}{\mu_0 k_y^2} \mathrm{Im}\left\{B_{res}^* [\![\partial_x B_{res}]\!]_{x=0}\right\}$$

- Linear, time-asymptotic, visco-resistive response:

$$B_{res} = \frac{a\Delta'_{ext}}{-a\Delta' + i\omega_{res}\tau_{VR}} B_{ext} \rightarrow \hat{F}_{y,EM} = -\frac{\omega_{res}\tau_{VR}}{(-a\Delta')^2 + (\omega_{res}\tau_{VR})^2} \frac{n\pi(a\Delta'_{ext})^2}{ak_y^2}\frac{B_{ext}^2}{\mu_0}$$

  - R. Fitzpatrick, NF **33**, 1049 (1993)
  - Quasilinear $F_{y,EM}$ localized at x=0



- **Viscous force per unit length in z at x=0**

$$\hat{F}_{y,VS} = \int_{-\delta_\delta/2}^{\delta_\delta/2} dx \int_{-L_y/2}^{L_y/2} dy [\nabla\cdot\rho\nu\nabla\mathbf{V}]\cdot\hat{y} = L_y\rho\nu_0 [\![\partial_x V(x,t)]\!]_{x=0}$$

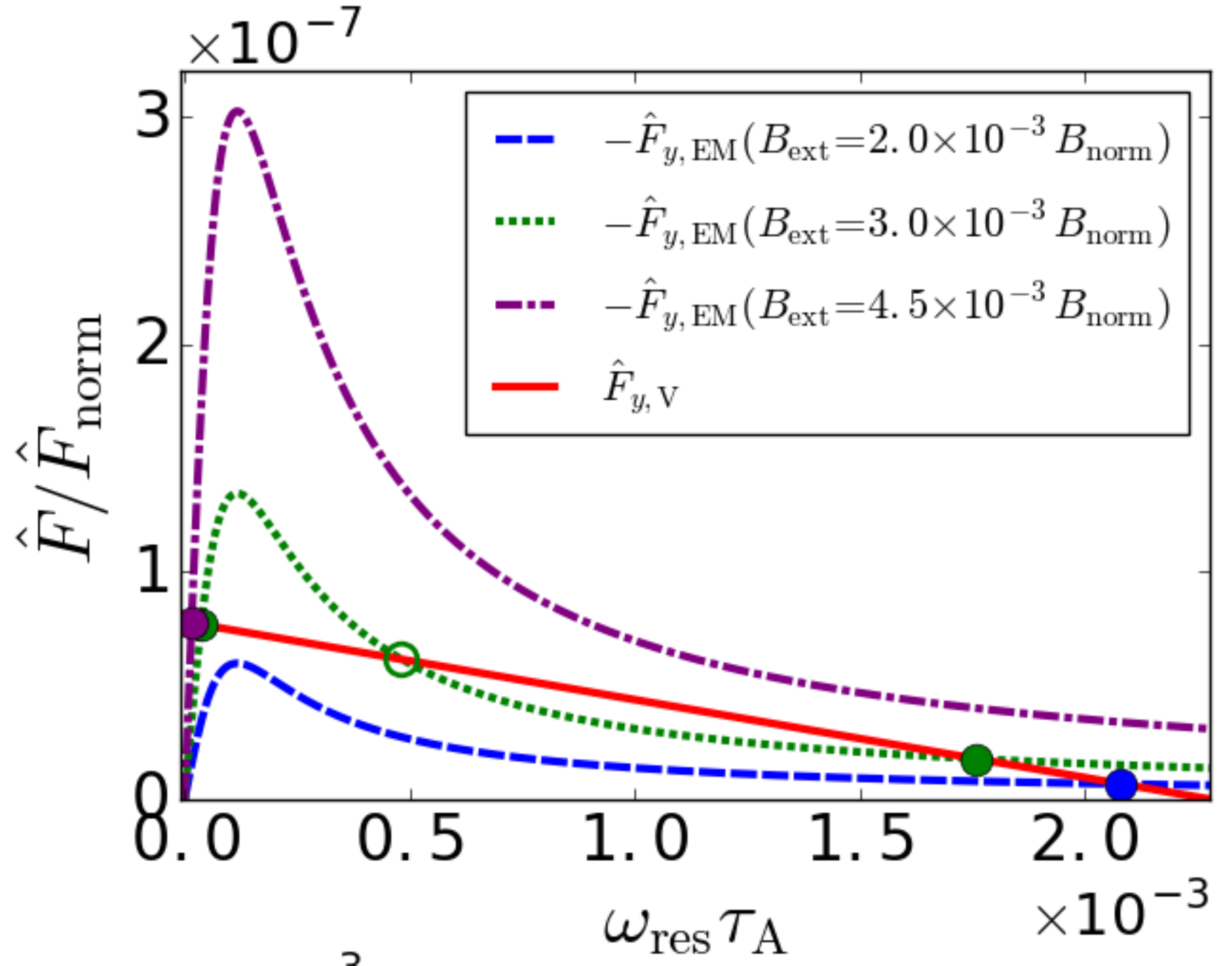
- Because EM force is localized at x=0, $\partial^2_{xx}V_y$=0 in $0<x<|a_\nu|$ yields $\rightarrow \hat{F}_{y,V} = \frac{2L_y\rho\nu_0}{k_y a_\nu}(\omega_0 - \omega_{res})$

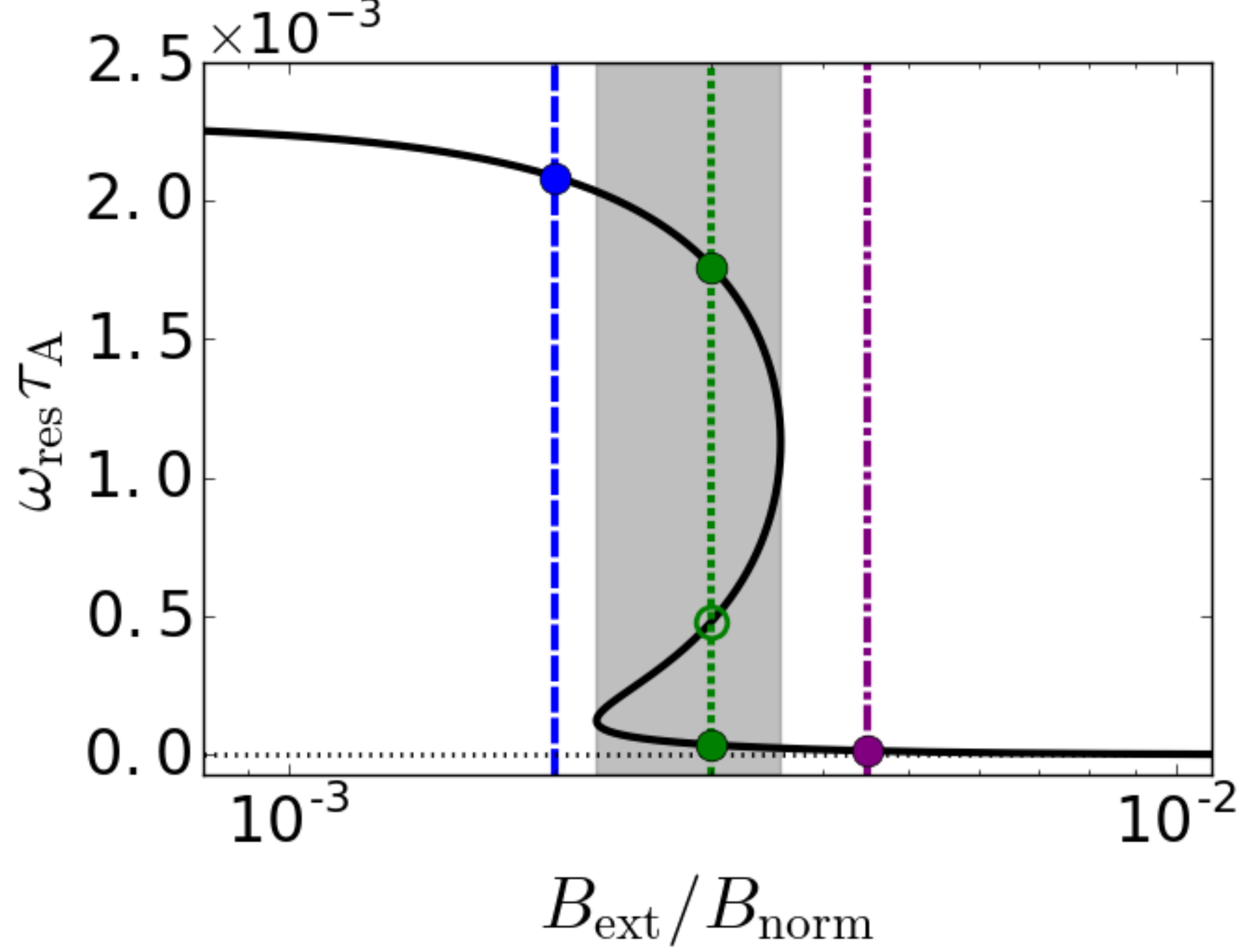
EM and Viscous Force Balance Gives Rise to Bifurcated, Metastable Equilibria

- **Force balance gives cubic relation for ω_{res}**

$$\frac{\omega_0}{\omega_{\text{res}}} - 1 + \omega_0\omega_{\text{res}}\tau_{\text{VR}}'^2 - \omega_{\text{res}}^2\tau_{\text{VR}}'^2 = \frac{a_\nu \tau_{\text{VR}}}{4a\rho\nu_0}\left(\frac{\Delta'_{\text{ext}}}{-\Delta'}\right)^2 \frac{B_{\text{ext}}^2}{\mu_0}$$

- Here, $\tau_{\text{VR}} = 2.104\tau_A S^{2/3} P_m{}^{1/6}$ and $\tau'_{\text{VR}} \equiv \tau_{\text{VR}}/(-a\Delta')$





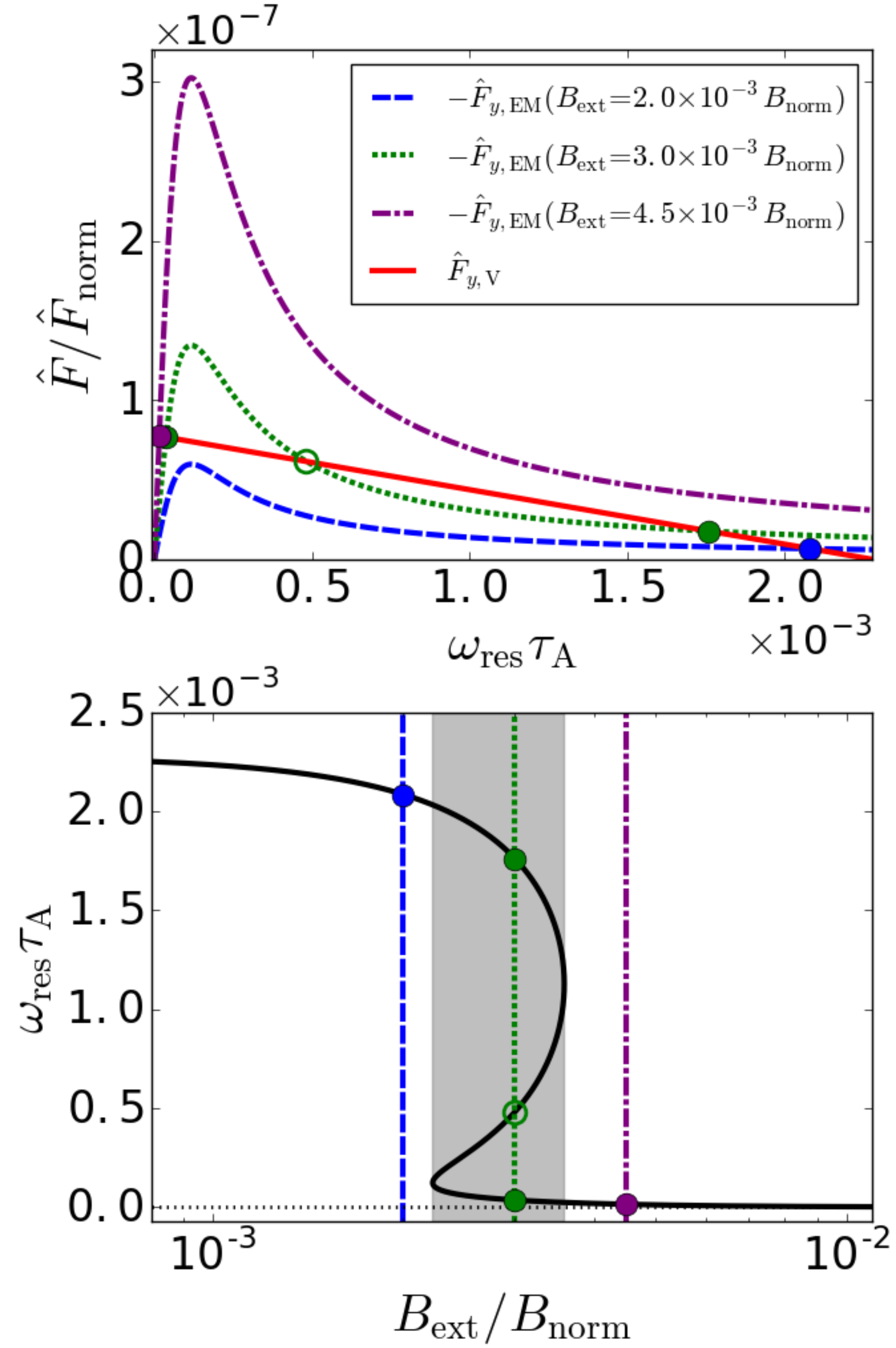
EM and Viscous Force Balance Gives Rise to Bifurcated, Metastable Equilibria

- **Force balance gives cubic relation for ω_{res}**

$$\frac{\omega_0}{\omega_{res}} - 1 + \omega_0 \omega_{res} \tau_{VR}'^2 - \omega_{res}^2 \tau_{VR}'^2 = \frac{a_\nu \tau_{VR}}{4 a \rho \nu_0} \left(\frac{\Delta'_{ext}}{-\Delta'} \right)^2 \frac{B_{ext}^2}{\mu_0}$$

  - Here, $\tau_{VR} = 2.104 \tau_A S^{2/3} P_m{}^{1/6}$ and $\tau'_{VR} \equiv \tau_{VR}/(-a\Delta')$

- **System bifurcates for $\omega_0 > 3\sqrt{3}/\tau'_{VR}$**
  - R. Fitzpatrick, NF **33**, 1049 (1993)



WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

OAK RIDGE INSTITUTE FOR SCIENCE AND EDUCATION
Managed by ORAU for DOE

EM and Viscous Force Balance Gives Rise to Bifurcated, Metastable Equilibria

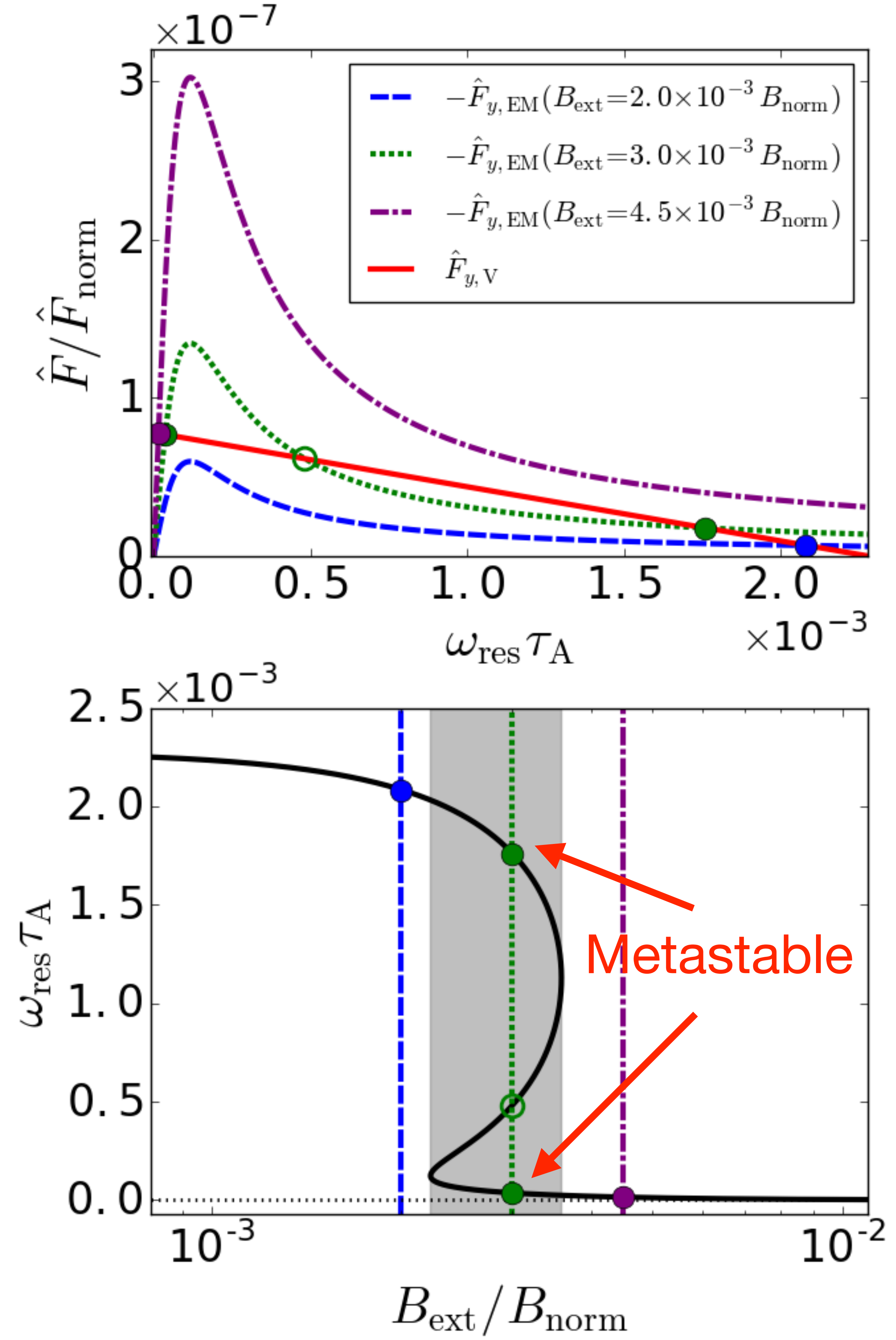
- **Force balance gives cubic relation for ω_{res}**

$$\frac{\omega_0}{\omega_{res}} - 1 + \omega_0\omega_{res}\tau_{VR}'^2 - \omega_{res}^2\tau_{VR}'^2 = \frac{a_\nu \tau_{VR}}{4a\rho\nu_0}\left(\frac{\Delta_{ext}'}{-\Delta'}\right)^2 \frac{B_{ext}^2}{\mu_0}$$

  - Here, $\tau_{VR} = 2.104\tau_A S^{2/3} P_m^{1/6}$ and $\tau_{VR}' \equiv \tau_{VR}/(-a\Delta')$

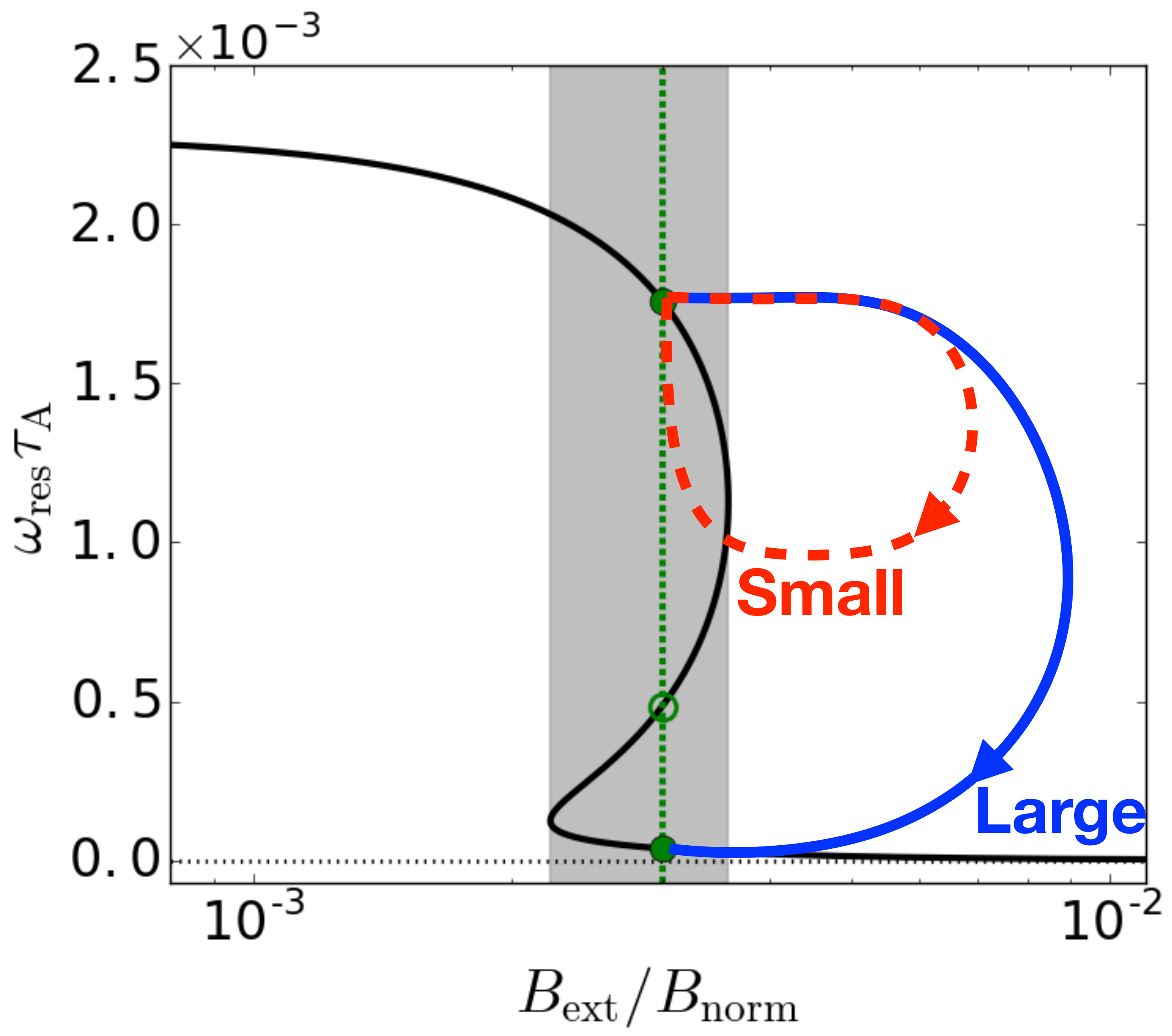
- **System bifurcates for $\omega_0 > 3\sqrt{3}/\tau_{VR}'$**
  - R. Fitzpatrick, NF **33**, 1049 (1993)

- **Two metastable equilibria: flow-screened and mode-penetrated**
  - Shaded region is metastable
  - Existence of metastable equilibria enables transient-induced mode penetration



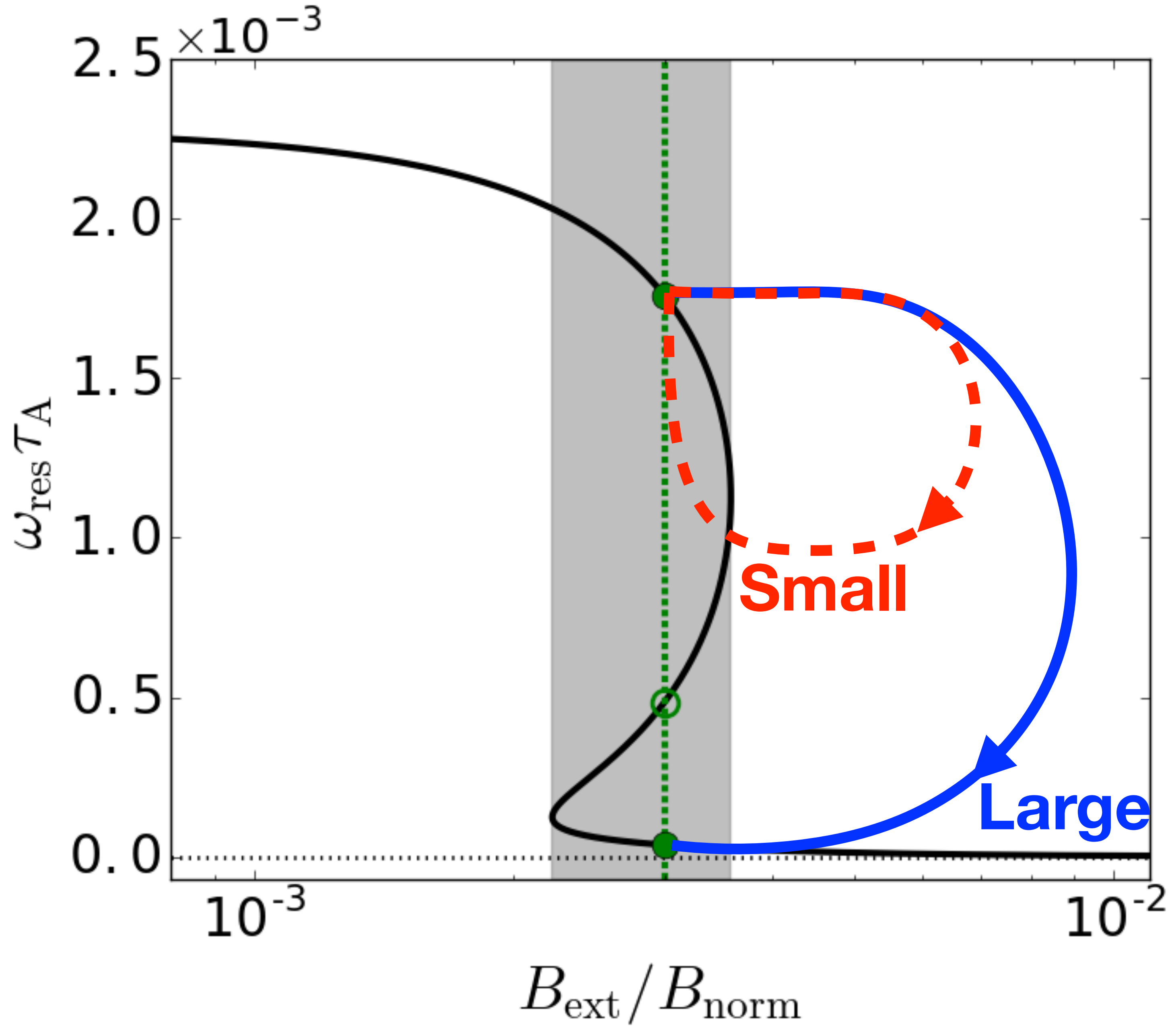
Transient Can Precipitate Transition Between Metastable Equilibria

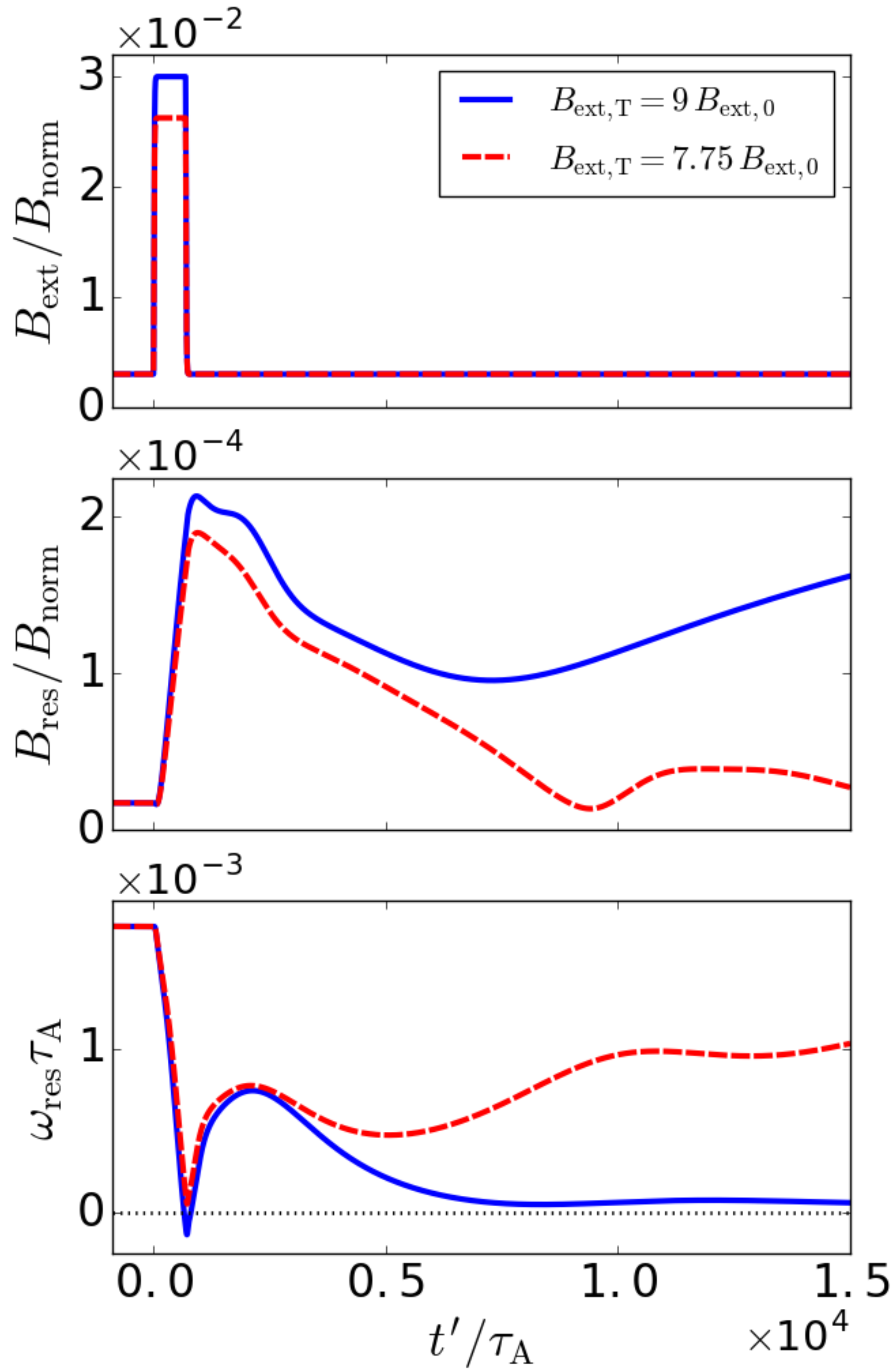
- **Hypothesis: If transient causes enough flow evolution, mode penetration occurs**



Transient Can Precipitate Transition Between Metastable Equilibria

- **Hypothesis: If transient causes enough flow evolution, mode penetration occurs**





Outline

- **Explore dynamics of transient perturbation in slab geometry**
  - Time-asymptotic EM and viscous force balance
  - Transient induced mode penetration needs metastable equilibrium
- **Computational results elucidate mode penetration dynamics**
- **Develop analytic model of mode penetration dynamics**

Outline

- Explore dynamics of transient perturbation in slab geometry
- **Computational results elucidate mode penetration dynamics**
- Develop analytic model of mode penetration dynamics

NIMROD Code Employed to Solve Visco-Resistive MHD Equations

- **NIMROD capable of solving extended-MHD equations**
  - Presently, assume cold plasma and ignore two-fluid effects

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{\Pi}_i \,,$$

$$\mathbf{\Pi}_i \equiv -\rho\nu \left[\nabla \mathbf{V} + \nabla \mathbf{V}^T - \frac{2}{3} \nabla \cdot \mathbf{V} \right],$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \mu_0 \mathbf{J} = \nabla \times \mathbf{B},$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J}$$

NIMROD Code Employed to Solve Visco-Resistive MHD Equations

- **NIMROD capable of solving extended-MHD equations**
  - Presently, assume cold plasma and ignore two-fluid effects
- **Time discretization uses finite difference**
  - Implicit leapfrog time evolution
  - Evolve perturbation fields about a fixed equilibrium

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\boldsymbol{\nabla}\mathbf{V}\right) = \mathbf{J}\times\mathbf{B} - \boldsymbol{\nabla}\cdot\boldsymbol{\Pi}_i\,,$$

$$\boldsymbol{\Pi}_i \equiv -\rho\nu\left[\boldsymbol{\nabla}\mathbf{V} + \boldsymbol{\nabla}\mathbf{V}^T - \frac{2}{3}\boldsymbol{\nabla}\cdot\mathbf{V}\right],$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\boldsymbol{\nabla}\times\mathbf{E},\quad \mu_0\mathbf{J} = \boldsymbol{\nabla}\times\mathbf{B},$$

$$\mathbf{E} = -\mathbf{V}\times\mathbf{B} + \eta\mathbf{J}$$

NIMROD Code Employed to Solve Visco-Resistive MHD Equations

- **NIMROD capable of solving extended-MHD equations**
  - Presently, assume cold plasma and ignore two-fluid effects
- **Time discretization uses finite difference**
  - Implicit leapfrog time evolution
  - Evolve perturbation fields about a fixed equilibrium
- **Spatial discretization uses 2D, C^0, spectral elements**
  - Employ mesh packing at rational surface and edge

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}\right) = \mathbf{J}\times\mathbf{B} - \nabla\cdot\mathbf{\Pi}_i\,,$$

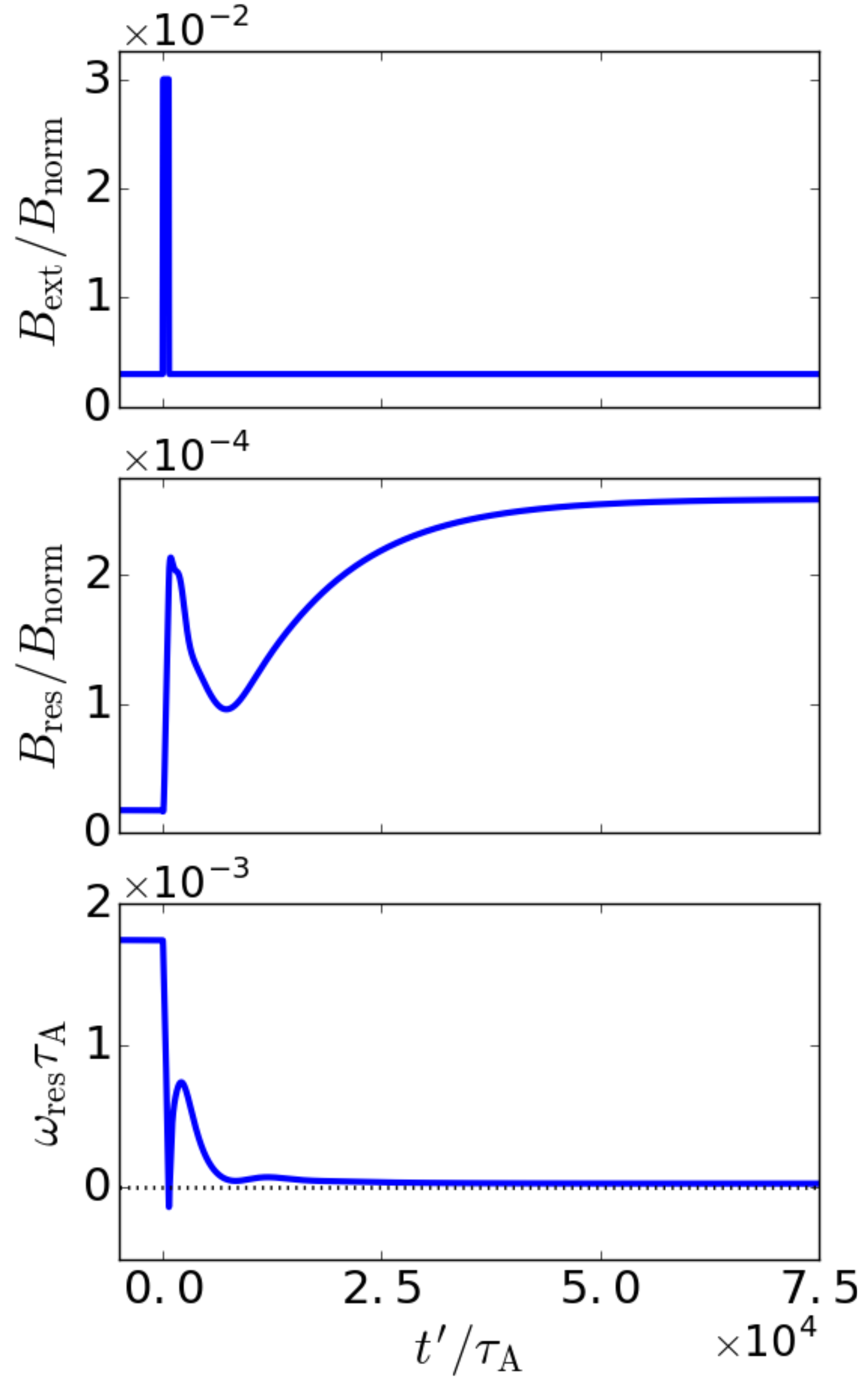
$$\mathbf{\Pi}_i \equiv -\rho\nu\left[\nabla\mathbf{V} + \nabla\mathbf{V}^T - \frac{2}{3}\nabla\cdot\mathbf{V}\right],$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E},\quad \mu_0\mathbf{J} = \nabla\times\mathbf{B},$$

$$\mathbf{E} = -\mathbf{V}\times\mathbf{B} + \eta\mathbf{J}$$

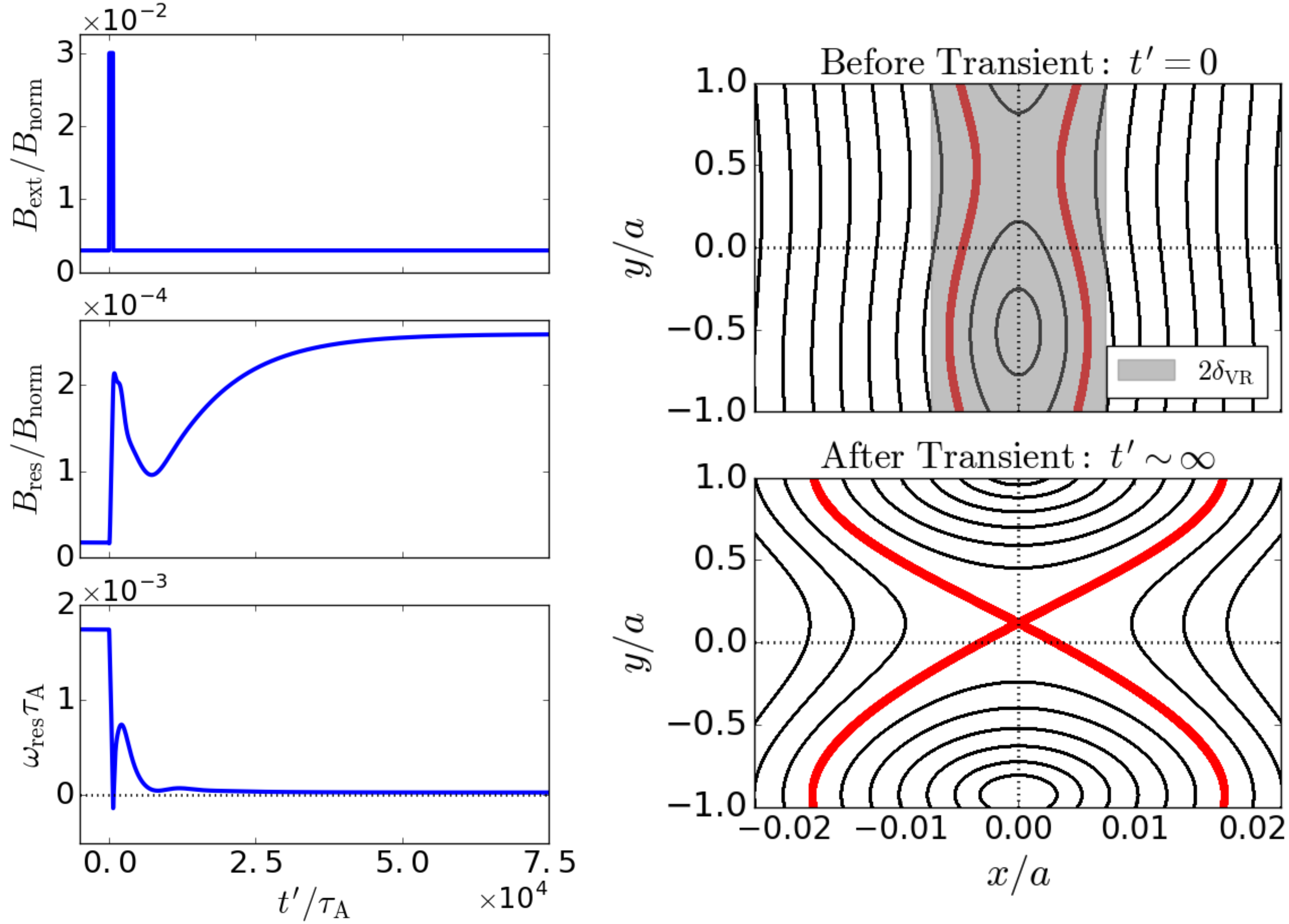
Large Transient Induces Mode Penetration

- **System properties**
  - $S = 1.1\times10^7$
  - $P_m = 20$
  - $V_0 = 500$ m/s
  - $B_{ext,0} = 3\times10^{-4}$ T
- **Transient properties**
  - $B_{ext,T} = 9\ B_{ext,0}$
  - $\Delta t_T = 690\ \tau_A$ duration
  - Approximately square



Large Transient Induces Mode Penetration

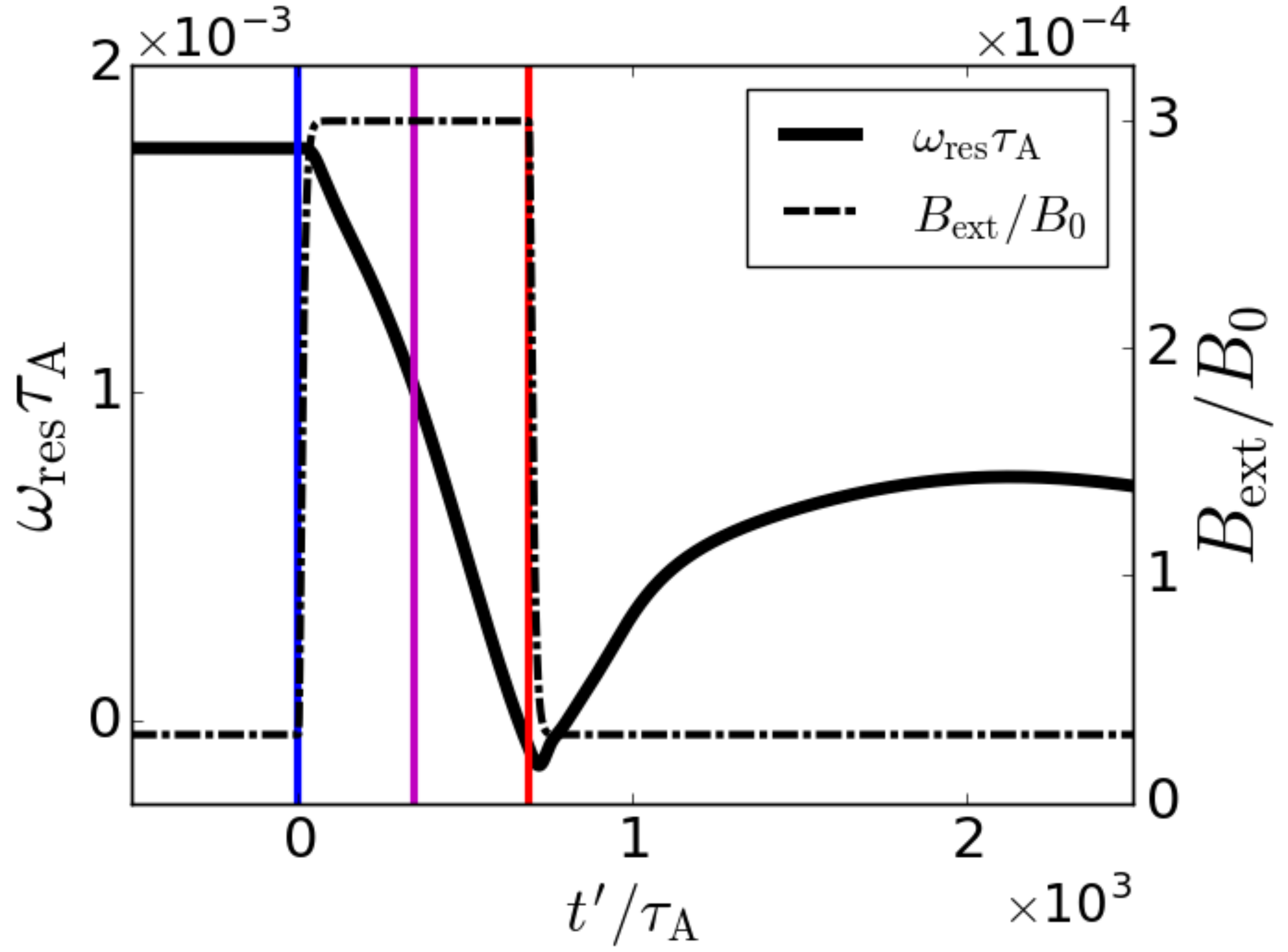
- **System properties**
  - $S = 1.1\times10^{7}$
  - $P_m = 20$
  - $V_0 = 500$ m/s
  - $B_{ext,0} = 3\times10^{-4}$ T
- **Transient properties**
  - $B_{ext,T} = 9\ B_{ext,0}$
  - $\Delta t_T = 690\ \tau_A$ duration
  - Approximately square
- **Mode penetration forms nonlinear magnetic island**

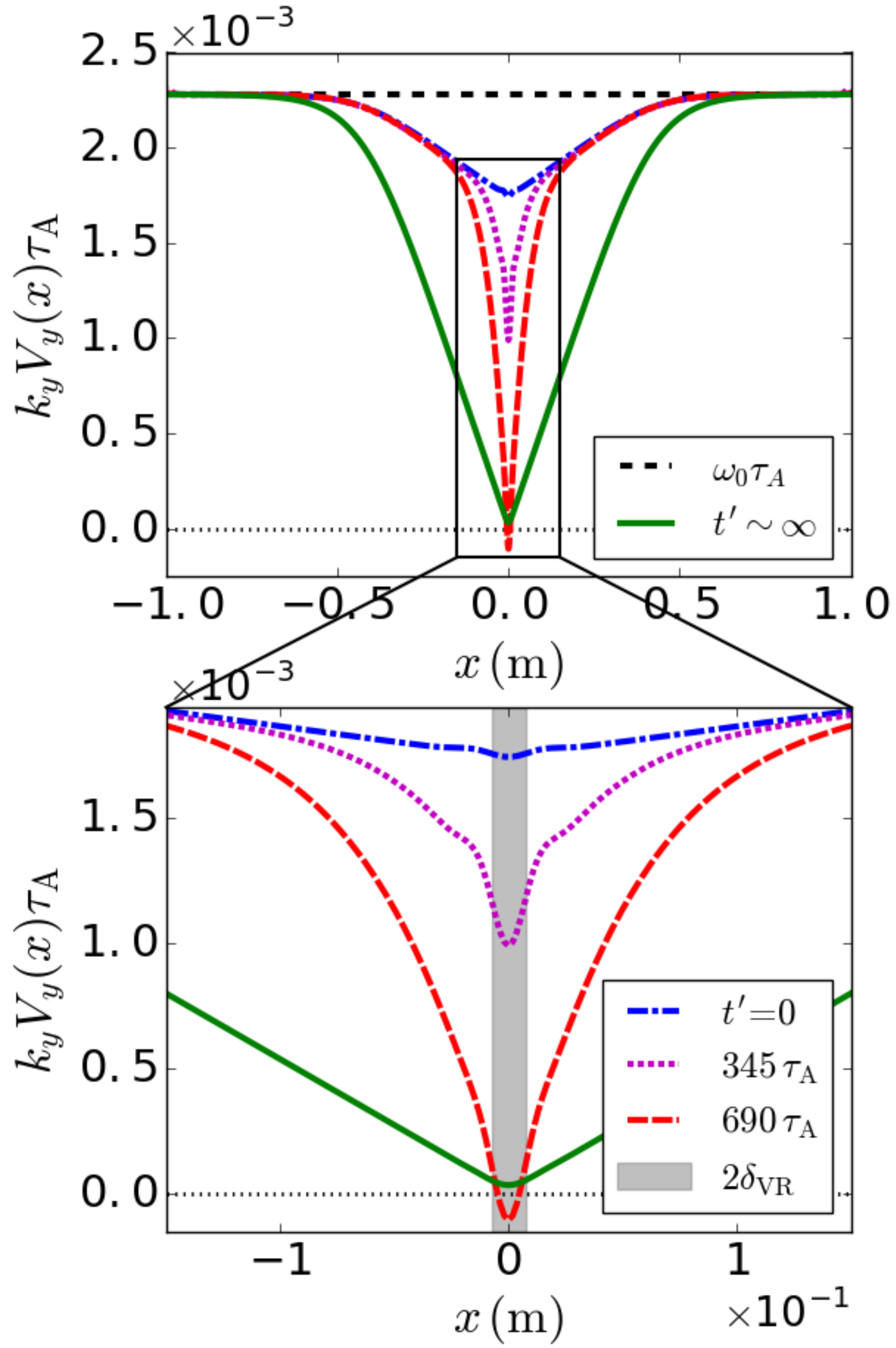


Flow Profile Evolution Determines Magnitude of Viscous Force

- **Transient enhancement of viscous force due to local flow evolution**

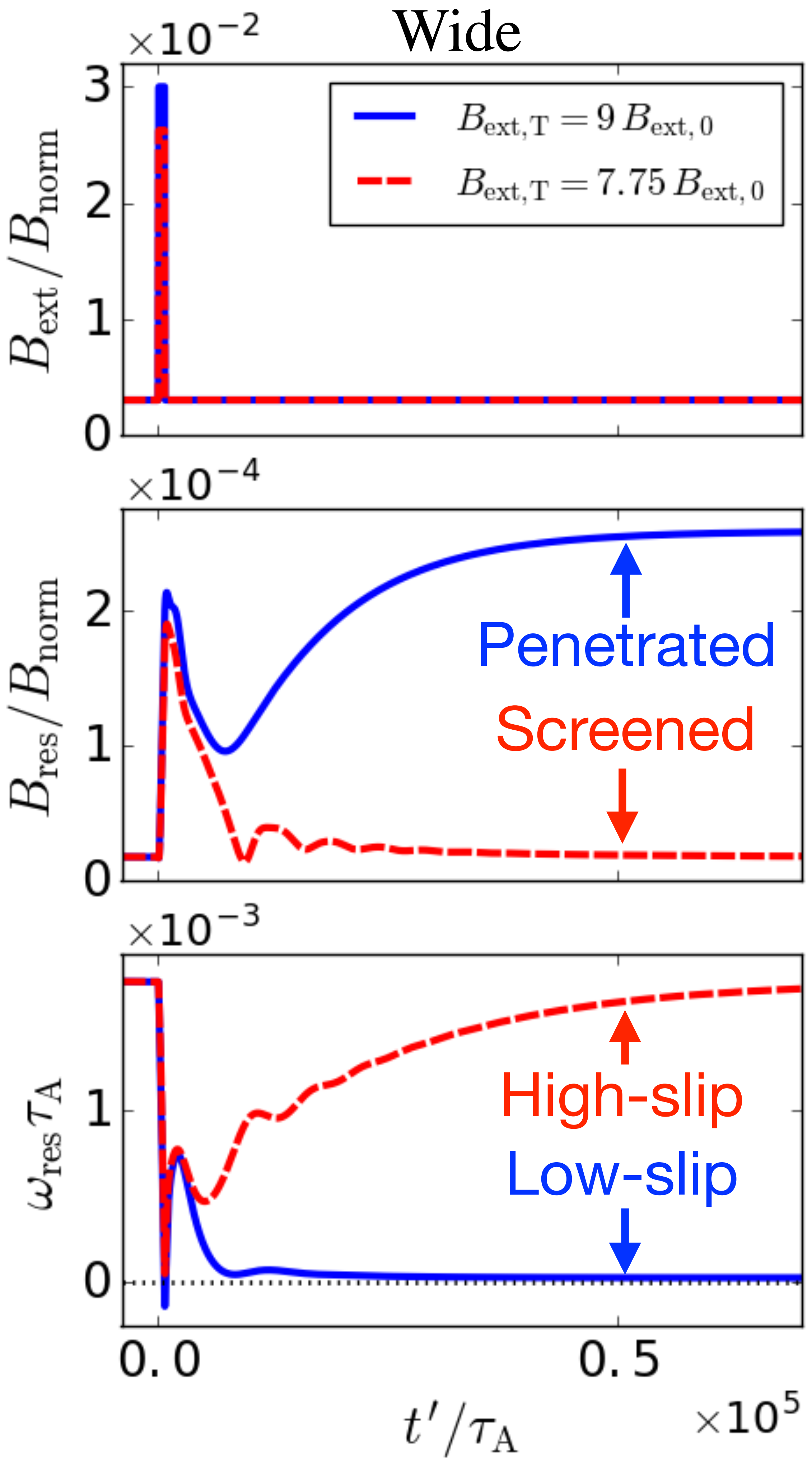
$$\hat{F}_{y,VS} = L_y \rho \nu_0 [\![\partial_x V(x,t')]\!]_{x=0}$$





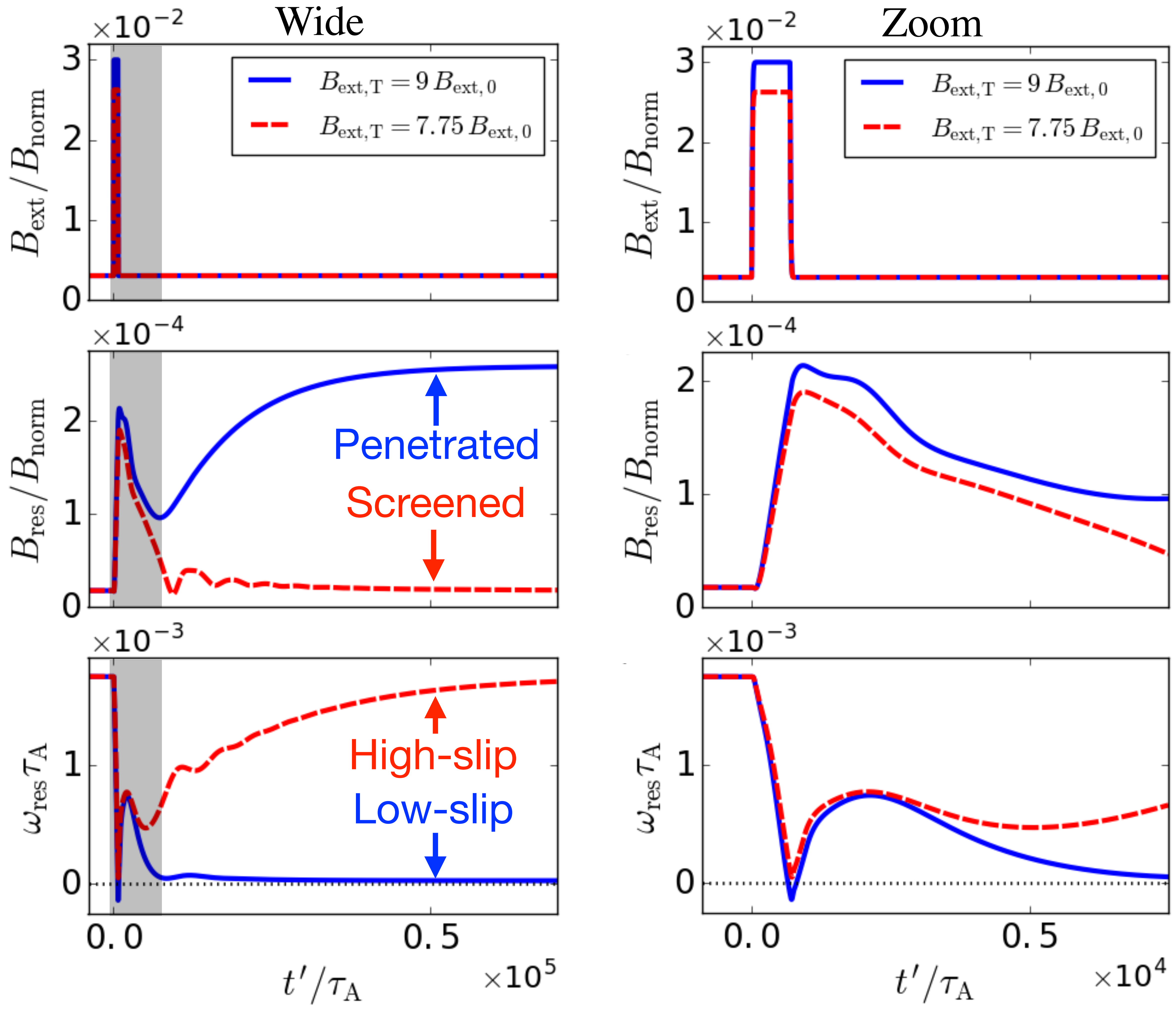
Magnitude of Transient Is Critical For Flow Response and Mode Penetration

- **System properties**
  - $S = 1.1\times10^7$
  - $P_m = 20$
  - $V_0 = 500$ m/s
  - $B_{ext,0} = 3\times10^{-4}$ T
- **Transient properties**
  - $\Delta t_T = 690\, \tau_A$ duration
  - Approximately square
- **$B_{ext,T} = 9\, B_{ext,0}$ mode penetrates**
  - $B_{ext,T} = 7.75\, B_{ext,0}$ returns to high slip state



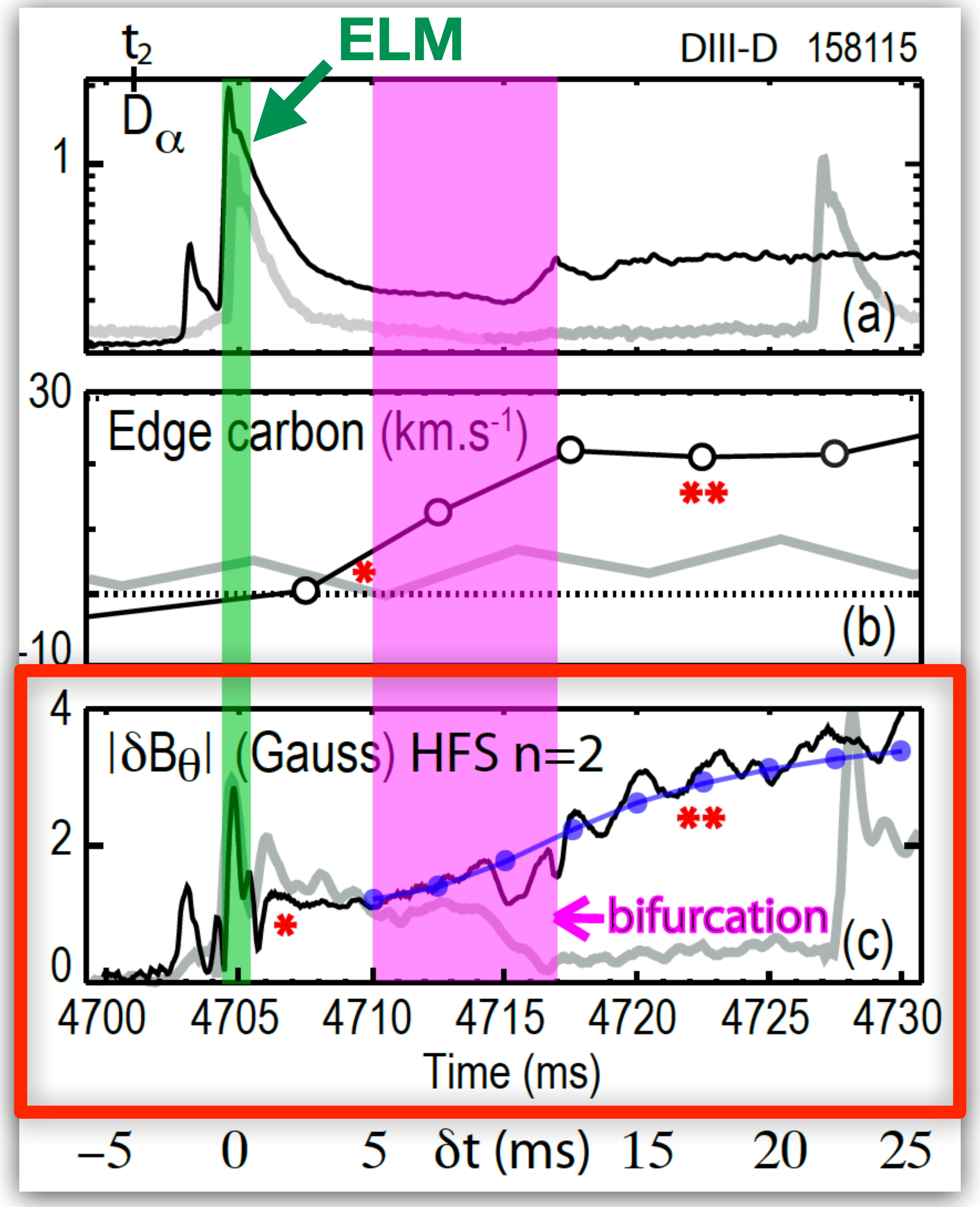
Magnitude of Transient Is Critical For Flow Response and Mode Penetration

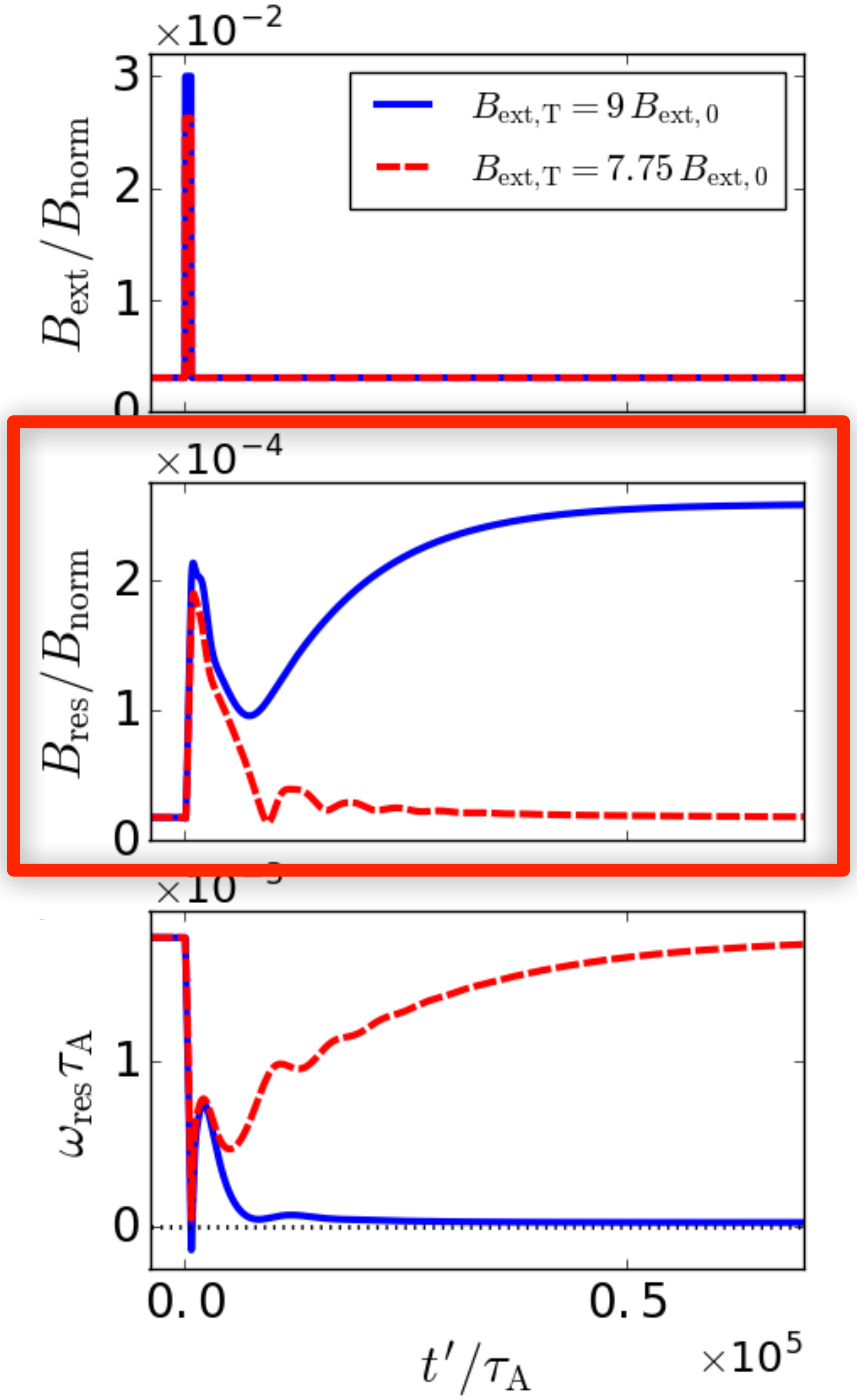
- **System properties**
  - $S = 1.1\times10^7$
  - $P_m = 20$
  - $V_0 = 500$ m/s
  - $B_{ext,0} = 3\times10^{-4}$ T
- **Transient properties**
  - $\Delta t_T = 690\,\tau_A$ duration
  - Approximately square
- **$B_{ext,T} = 9\,B_{ext,0}$ mode penetrates**
  - $B_{ext,T} = 7.75\,B_{ext,0}$ returns to high slip state



WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON
OAK RIDGE INSTITUTE FOR SCIENCE AND EDUCATION
Managed by ORAU for DOE

Computed Field Response Is Similar to Experimental Observations



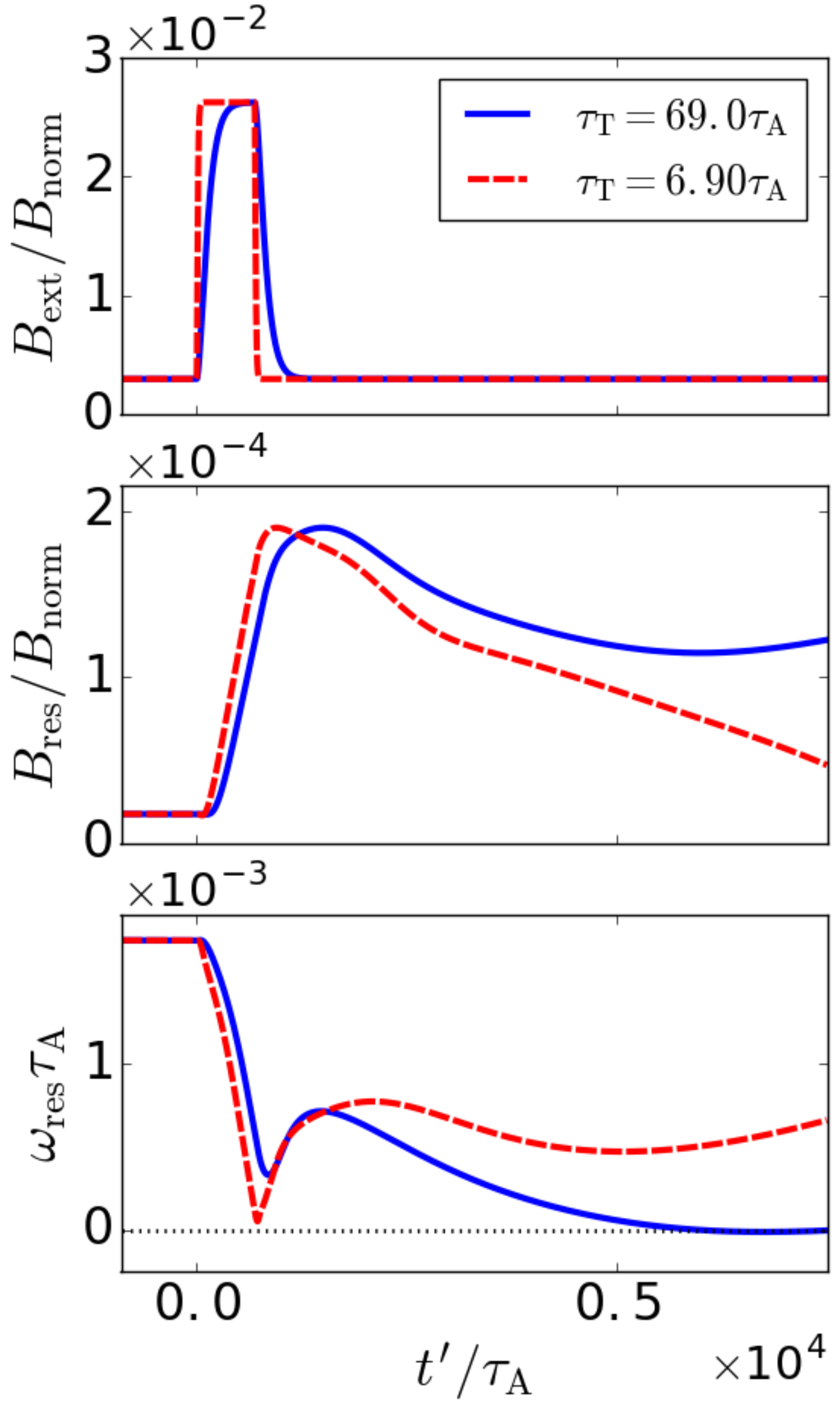


Mode Penetration Threshold Is Sensitive to Transient Shape

- **Rise and fall time of transient parameterized as**

$$T(t) = 1 - e^{-t/\tau_T} - \frac{t}{\tau_T} e^{-t/\tau_T}$$

- **Transient properties**
  - $B_{ext,T} = 7.75\, B_{ext,0}$
  - $\Delta t_T = 690\, \tau_A$ duration
- **$\tau_T = 69\, \tau_A$ mode penetrates**
  - $\tau_T = 6.9\, \tau_A$ returns to high slip state

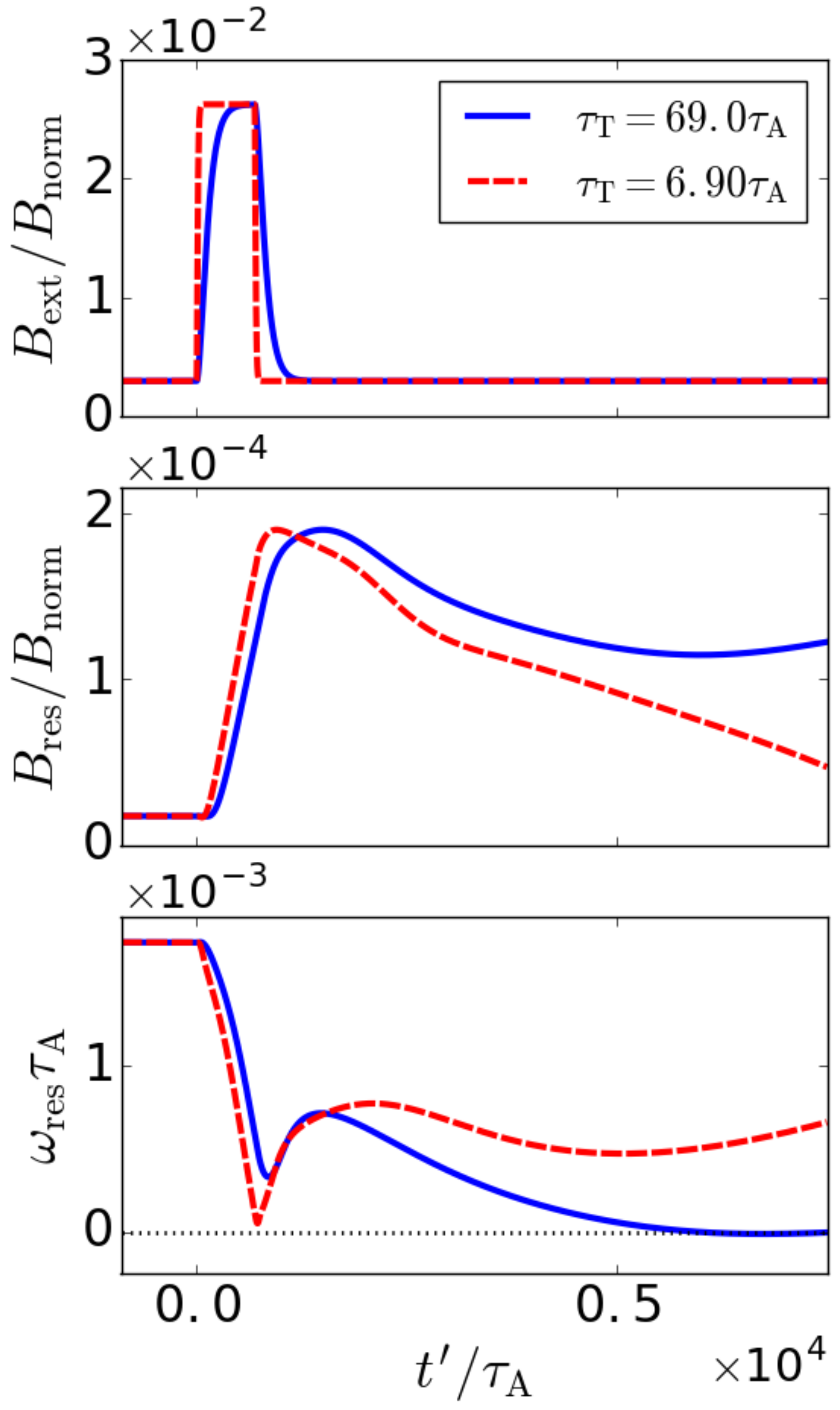


Mode Penetration Threshold Is Sensitive to Transient Shape

- **Rise and fall time of transient parameterized as**

$$T(t) = 1 - e^{-t/\tau_T} - \frac{t}{\tau_T} e^{-t/\tau_T}$$

- **Transient properties**
  - $B_{ext,T} = 7.75\ B_{ext,0}$
  - $\Delta t_T = 690\ \tau_A$ duration
- $\boldsymbol{\tau_T = 69\ \tau_A}$ **mode penetrates**
  - $\tau_T = 6.9\ \tau_A$ returns to high slip state
- **Same time-integrated transient RMP can yield different final state!**



Outline

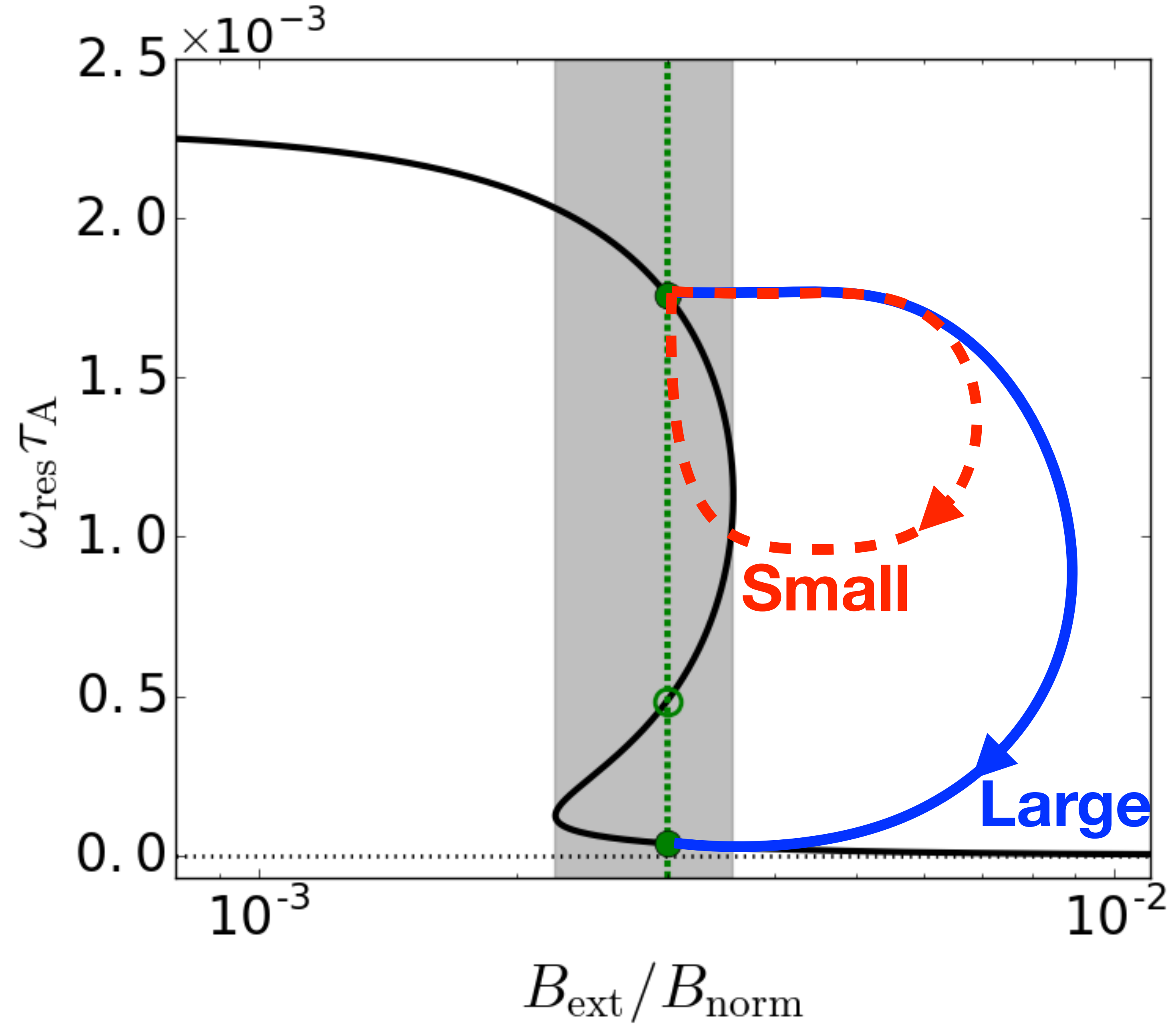
- Explore dynamics of transient perturbation in slab geometry
- **Computational results elucidate mode penetration dynamics**
  - Effects of transient perturbation on metastable equilibrium
  - Parametric tests illustrate sensitivity of mode penetration
- Develop analytic model of mode penetration dynamics

Outline

- Explore dynamics of transient perturbation in slab geometry
- Computational results elucidate mode penetration dynamics
- **Develop analytic model of mode penetration dynamics**

Effect of Magnetic Transient Depends on EM and Viscous Force Evolution

- **Hypothesis: If transient causes enough flow evolution, mode penetration occurs**

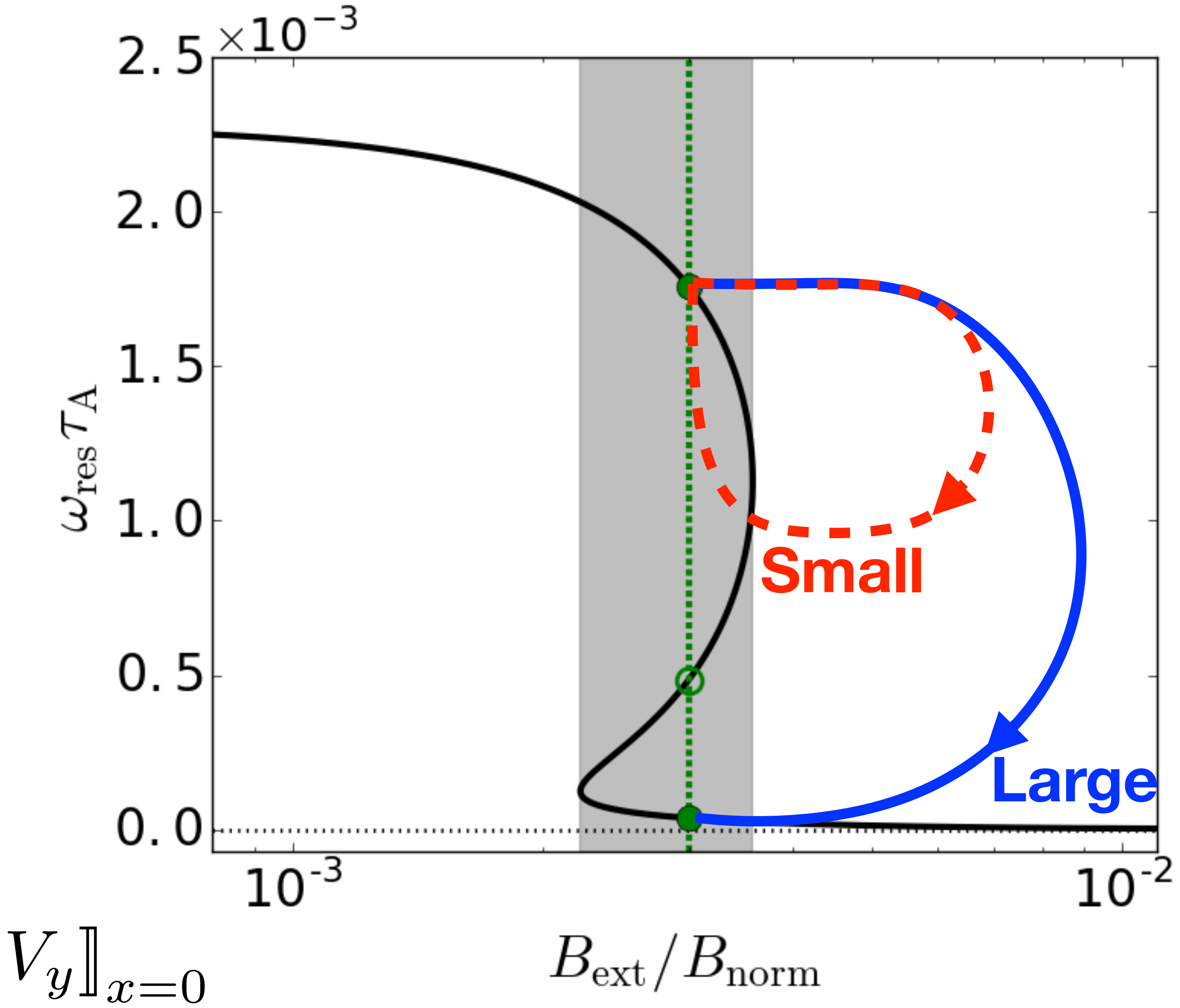


Effect of Magnetic Transient Depends on EM and Viscous Force Evolution

- **Hypothesis: If transient causes enough flow evolution, mode penetration occurs**

- **Flow evolution equation with EM and viscous forces:**

$$\frac{\delta_{\rm VR} L_y \rho}{k_y}\frac{d\omega_{\rm res}}{dt} = \hat{F}_{\rm EM} + \hat{F}_{\rm V}$$
$$= -\frac{n\pi}{\mu_0 k_y^2}{\rm Im}\left\{B^*_{\rm res}\left[\!\left[\partial_x B_{\rm res}\right]\!\right]_{x=0}\right\} + L_y\rho\nu_0\left[\!\left[\partial_x V_y\right]\!\right]_{x=0}$$



- **Transient magnetic perturbation causes forces to evolve**
  - Directly increases EM force local to the rational surface
  - Local change in flow profile increases viscous force

Model For Accurate EM Force Depends on History of Flow Evolution

- **Evolution of penetrated field governed by asymptotic matching of induction equation**

$$\frac{dB_{\mathrm{res}}(t')}{dt} + \left[i\omega_{\mathrm{res}}(t') + \frac{a\Delta'}{\tau_{\mathrm{VR}}} \right] B_{\mathrm{res}}(t') = \frac{a\Delta'_{\mathrm{ext}}}{\tau_{\mathrm{VR}}} \left[B_{\mathrm{ext,0}} + B_{\mathrm{ext,T}} T(t') \right]$$

  - Background $B_{\mathrm{ext,0}}$ is constant in time; transient $B_{\mathrm{ext,T}}$ is applied with time-dependence $T(t')$

Model For Accurate EM Force Depends on History of Flow Evolution

- **Evolution of penetrated field governed by asymptotic matching of induction equation**

$$\frac{dB_{\rm res}(t')}{dt} + \left[i\omega_{\rm res}(t') + \frac{a\Delta'}{\tau_{\rm VR}}\right] B_{\rm res}(t') = \frac{a\Delta'_{\rm ext}}{\tau_{\rm VR}} \left[B_{\rm ext,0} + B_{\rm ext,T} T(t')\right]$$

  - Background $B_{\rm ext,0}$ is constant in time; transient $B_{\rm ext,T}$ is applied with time-dependence $T(t')$

- **Solve ODE for $B_{\rm res}$ by using integration factor**

$$B_{\rm res}(t') = \frac{\Delta'_{\rm ext}}{-\Delta'} \exp\left[-\frac{t'}{\tau'_{\rm VR}} - i\varphi_{\rm res}(t')\right] \left\{ \frac{B_{\rm ext,0}}{1 + i\omega_{\rm res}(0)\tau'_{\rm VR}} + \frac{B_{\rm ext,0}}{\tau'_{\rm VR}} \int_0^{t'} ds \exp\left[\frac{s}{\tau'_{\rm VR}} + i\varphi_{\rm res}(s)\right] + \frac{B_{\rm ext,T}}{\tau'_{\rm VR}} \int_0^{t'} ds \exp\left[\frac{s}{\tau'_{\rm VR}} + i\varphi_{\rm res}(s)\right] T(s) \right\}$$

  - History of flow frequency evolution quantified by $\varphi_{\rm res}(t') \equiv \int_0^{t'} ds\, \omega_{\rm res}(s)$

WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

OAK RIDGE INSTITUTE FOR SCIENCE AND EDUCATION
Managed by ORAU for DOE

Model For Accurate EM Force Depends on History of Flow Evolution

- **Evolution of penetrated field governed by asymptotic matching of induction equation**

$$\frac{dB_{\text{res}}(t')}{dt} + \left[i\omega_{\text{res}}(t') + \frac{a\Delta'}{\tau_{\text{VR}}}\right] B_{\text{res}}(t') = \frac{a\Delta'_{\text{ext}}}{\tau_{\text{VR}}} \left[B_{\text{ext},0} + B_{\text{ext,T}} T(t')\right]$$

  - Background $B_{\text{ext},0}$ is constant in time; transient $B_{\text{ext,T}}$ is applied with time-dependence $T(t')$

- **Solve ODE for $\boldsymbol{B_{\text{res}}}$ by using integration factor**

$$B_{\text{res}}(t') = \frac{\Delta'_{\text{ext}}}{-\Delta'} \exp\left[-\frac{t'}{\tau'_{\text{VR}}} - i\varphi_{\text{res}}(t')\right] \left\{ \frac{B_{\text{ext},0}}{1 + i\omega_{\text{res}}(0)\tau'_{\text{VR}}} + \frac{B_{\text{ext},0}}{\tau'_{\text{VR}}} \int_0^{t'} ds \exp\left[\frac{s}{\tau'_{\text{VR}}} + i\varphi_{\text{res}}(s)\right] + \frac{B_{\text{ext,T}}}{\tau'_{\text{VR}}} \int_0^{t'} ds \exp\left[\frac{s}{\tau'_{\text{VR}}} + i\varphi_{\text{res}}(s)\right] T(s) \right\}$$

  - History of flow frequency evolution quantified by $\varphi_{\text{res}}(t') \equiv \int_0^{t'} ds\, \omega_{\text{res}}(s)$
  - Separate contributions due to initially penetrated field and transient RMP penetration

Model For Accurate EM Force Depends on History of Flow Evolution

- **Evolution of penetrated field governed by asymptotic matching of induction equation**

$$\frac{dB_{\rm res}(t')}{dt} + \left[i\omega_{\rm res}(t') + \frac{a\Delta'}{\tau_{\rm VR}}\right] B_{\rm res}(t') = \frac{a\Delta'_{\rm ext}}{\tau_{\rm VR}} \left[B_{\rm ext,0} + B_{\rm ext,T} T(t')\right]$$

  - Background $B_{\rm ext,0}$ is constant in time; transient $B_{\rm ext,T}$ is applied with time-dependence $T(t')$

- **Solve ODE for $B_{\rm res}$ by using integration factor**

$$B_{\rm res}(t') = \frac{\Delta'_{\rm ext}}{-\Delta'} \exp\left[-\frac{t'}{\tau'_{\rm VR}} - i\varphi_{\rm res}(t')\right] \left\{ \frac{B_{\rm ext,0}}{1 + i\omega_{\rm res}(0)\tau'_{\rm VR}} + \frac{B_{\rm ext,0}}{\tau'_{\rm VR}} \int_0^{t'} ds \exp\left[\frac{s}{\tau'_{\rm VR}} + i\varphi_{\rm res}(s)\right] + \frac{B_{\rm ext,T}}{\tau'_{\rm VR}} \int_0^{t'} ds \exp\left[\frac{s}{\tau'_{\rm VR}} + i\varphi_{\rm res}(s)\right] T(s) \right\}$$

  - History of flow frequency evolution quantified by $\varphi_{\rm res}(t') \equiv \int_0^{t'} ds\, \omega_{\rm res}(s)$
  - Separate contributions due to initially penetrated field and transient RMP penetration

- **Quasilinear EM force** $\hat{F}_{y,\rm EM}(t') = -\frac{n\pi}{\mu_0 k_y^2} \Delta'_{\rm ext} \left[B_{\rm ext,0} + B_{\rm ext,T} T(t')\right] \mathrm{Im}\left\{B^*_{\rm res}(t')\right\}$
  - Separate contributions of $B_{\rm res}$ interact with transiently-induced current

Model For Viscous Force Evolution Depends on Evolution of Flow Profile

- **Evaluate** $\hat{F}_{y,V} = L_y \rho \nu_0 \left[\!\left[\partial_x V(x,t') \right]\!\right]_{x=0}$ **with evolving** $\boldsymbol{V(x,t')}$

Model For Viscous Force Evolution Depends on Evolution of Flow Profile

- **Evaluate** $\hat{F}_{y,V} = L_y \rho \nu_0 \left[\!\left[\partial_x V(x,t') \right]\!\right]_{x=0}$ **with evolving** $\boldsymbol{V(x,t')}$
- **Because EM force is localized at** $\boldsymbol{x=0}$**, solve for flow profile in** $\boldsymbol{0 < |x| < a_\nu}$
  - $\partial_t V(x,t') = \nu_0 \partial^2_{xx} V(x,t')$ with time-dependent BCs solved using infinite series expansion

Model For Viscous Force Evolution Depends on Evolution of Flow Profile

- **Evaluate** $\hat{F}_{y,V} = L_y \rho \nu_0 \left[\!\left[\partial_x V(x,t') \right]\!\right]_{x=0}$ **with evolving** $V(x,t')$
- **Because EM force is localized at** $x=0$**, solve for flow profile in** $0 < |x| < a_\nu$
  - $\partial_t V(x,t') = \nu_0 \, \partial^2_{xx} V(x,t')$ with time-dependent BCs solved using infinite series expansion

$$V(x,t') = V_{\rm res}(t') + [V_0 - V_{\rm res}(t')]\left(\frac{x}{a_\nu}\right) - \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{a_\nu}\right) e^{-(n\pi)^2 \frac{t'}{\tau_\nu}}$$

$$\times \left\{ \frac{2}{a_\nu}\int_0^{a_\nu} dx \sin\left(\frac{n\pi x}{a_\nu}\right)\left[V(x,0) - \left\{V_{\rm res}(0) + [V_0 - V_{\rm res}(0)]\left(\frac{x}{a_\nu}\right)\right\}\right] - \frac{2}{n\pi}\int_0^{t'} ds \frac{dV_{\rm res}(s)}{ds} e^{(n\pi)^2 \frac{s}{\tau_\nu}} \right\}$$

  - Separate contributions due to time-asymptotic and transient $V_{\rm res}(t)$

Model For Viscous Force Evolution Depends on Evolution of Flow Profile

- **Evaluate $\hat{F}_{y,V} = L_y \rho \nu_0 \left[\!\left[\partial_x V(x,t') \right]\!\right]_{x=0}$ with evolving $V(x,t')$**
- **Because EM force is localized at x=0, solve for flow profile in $0 < |x| < a_\nu$**
  - $\partial_t V(x,t') = \nu_0 \partial^2_{xx} V(x,t')$ with time-dependent BCs solved using infinite series expansion

$$V(x,t') = V_{\mathrm{res}}(t') + [V_0 - V_{\mathrm{res}}(t')]\left(\frac{x}{a_\nu}\right) - \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{a_\nu}\right) e^{-(n\pi)^2 \frac{t'}{\tau_\nu}}$$

$$\times \left\{ \frac{2}{a_\nu}\int_0^{a_\nu} dx \sin\left(\frac{n\pi x}{a_\nu}\right)\left[V(x,0) - \left\{V_{\mathrm{res}}(0) + [V_0 - V_{\mathrm{res}}(0)]\left(\frac{x}{a_\nu}\right)\right\}\right] - \frac{2}{n\pi}\int_0^{t'} ds \frac{dV_{\mathrm{res}}(s)}{ds} e^{(n\pi)^2 \frac{s}{\tau_\nu}} \right\}$$

  - Separate contributions due to time-asymptotic and transient $V_{\mathrm{res}}(t)$

- **Using derived flow profile yields viscous force:**

$$\hat{F}_{y,V}(t') = \frac{2 L_y \rho \nu_0}{k_y a_\nu}\left\{ [\omega_0 - \omega_{\mathrm{res}}(t')] + 2\,[\omega_{\mathrm{res}}(0) - \omega_{\mathrm{res}}(t')] \sum_{n=1}^{\infty} \exp\left[-(n\pi)^2 \frac{t'}{\tau_\nu}\right] \right\}$$

Model Of Self-Consistent Force Balance Exhibits Mode Penetration

- **Balance EM and viscous forces against inertia, yields system of coupled PDEs:**

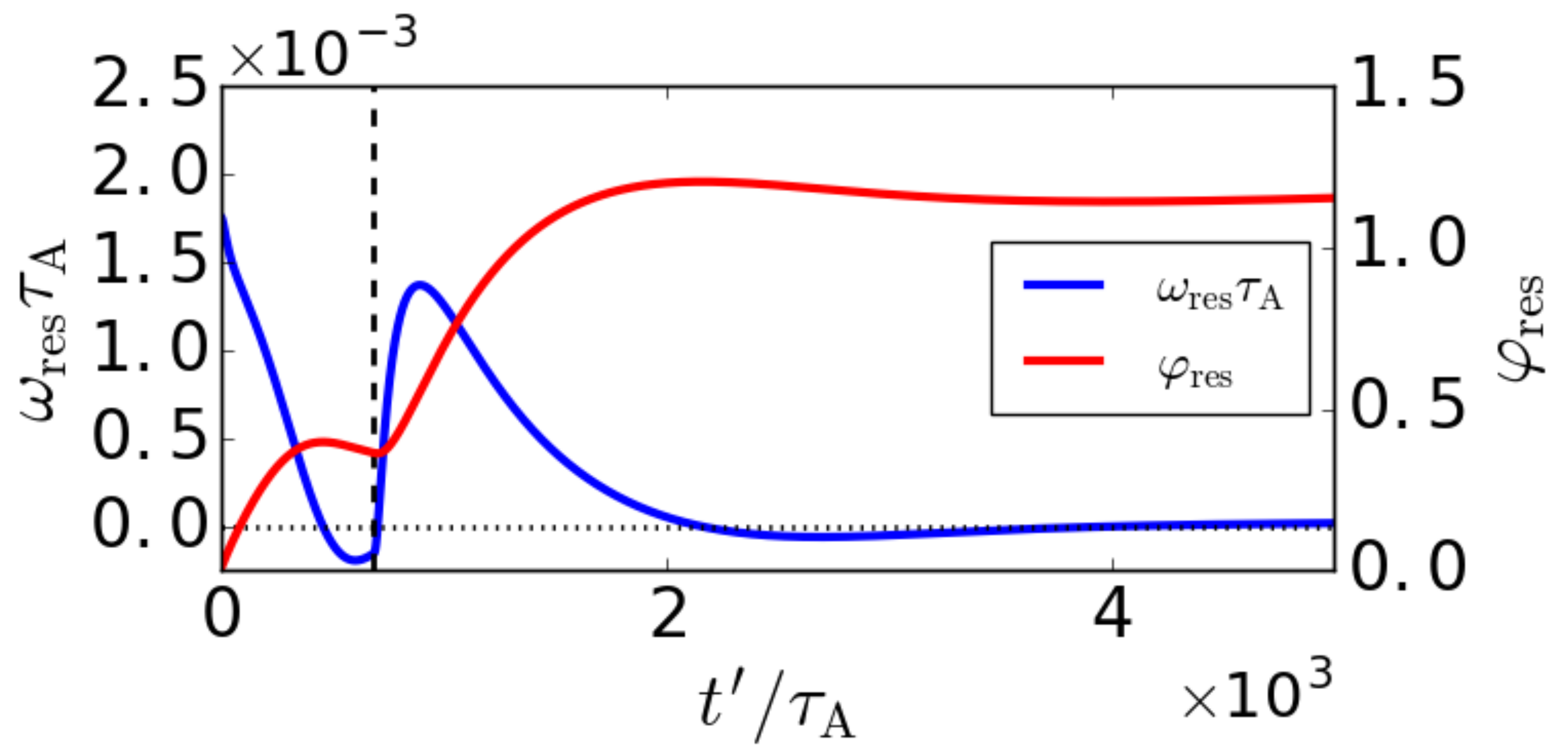
$$\frac{\delta_{\mathrm{VR}} L_y \rho}{k_y} \frac{d\omega_{\mathrm{res}}(t')}{dt'} = \hat{F}_{\mathrm{EM}}(t') + \hat{F}_{\mathrm{V}}(t')\,, \quad \frac{d\varphi_{\mathrm{res}}(t')}{dt'} = \omega_{\mathrm{res}}(t')$$

Model Of Self-Consistent Force Balance Exhibits Mode Penetration

- **Balance EM and viscous forces against inertia, yields system of coupled PDEs:**

$$\frac{\delta_{\mathrm{VR}} L_y \rho}{k_y} \frac{d\omega_{\mathrm{res}}(t')}{dt'} = \hat{F}_{\mathrm{EM}}(t') + \hat{F}_{\mathrm{V}}(t') \,, \quad \frac{d\varphi_{\mathrm{res}}(t')}{dt'} = \omega_{\mathrm{res}}(t')$$

- **Numerically solve system**
  - Solution shown for transient with $B_{\mathrm{ext,T}} = 9\, B_{\mathrm{ext,0}}$, $\Delta t_{\mathrm{T}} = 690\, \tau_{\mathrm{A}}$, $\tau_{\mathrm{T}} = 6.9\, \tau_{\mathrm{A}}$

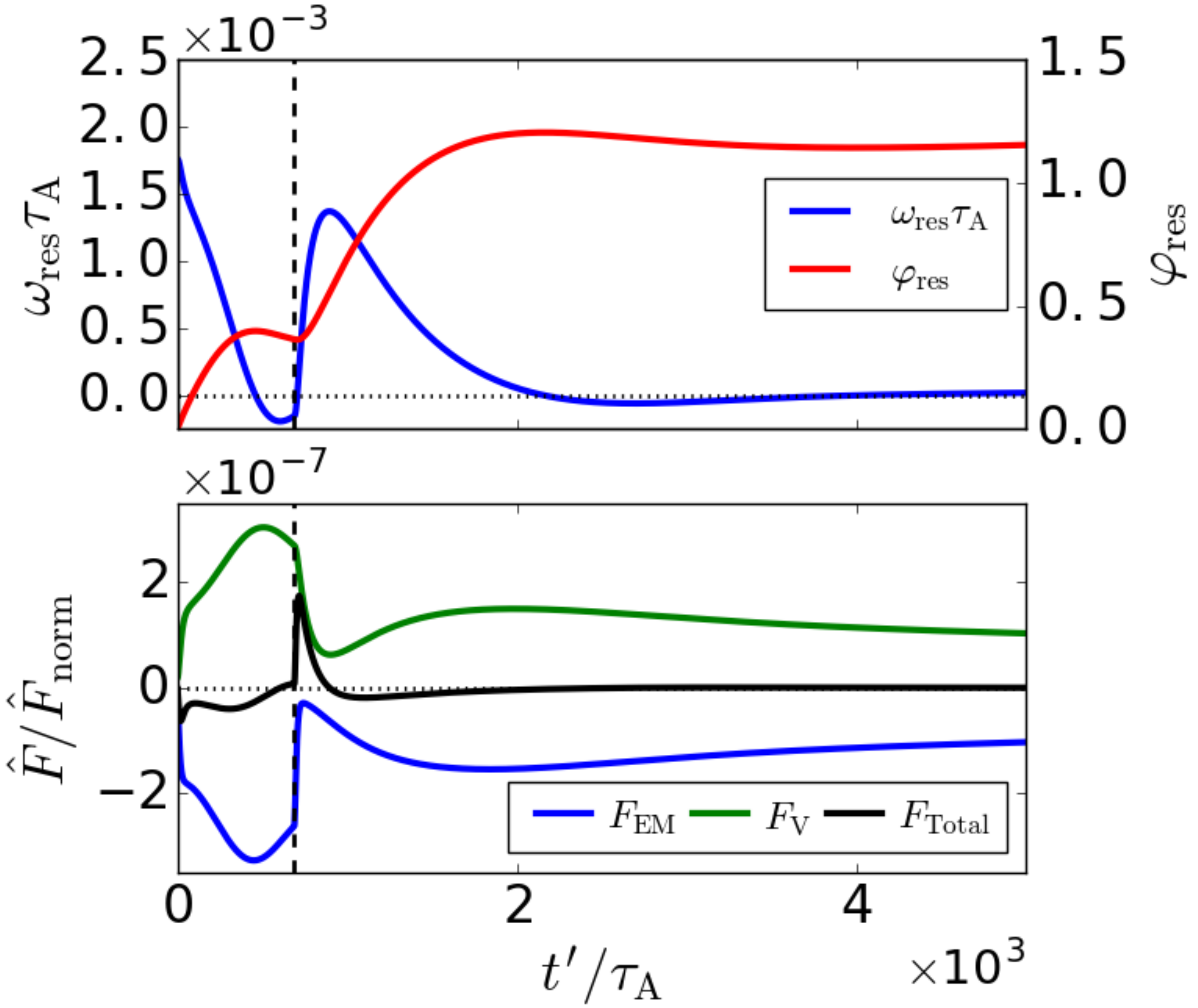


Model Of Self-Consistent Force Balance Exhibits Mode Penetration

- **Balance EM and viscous forces against inertia, yields system of coupled PDEs:**

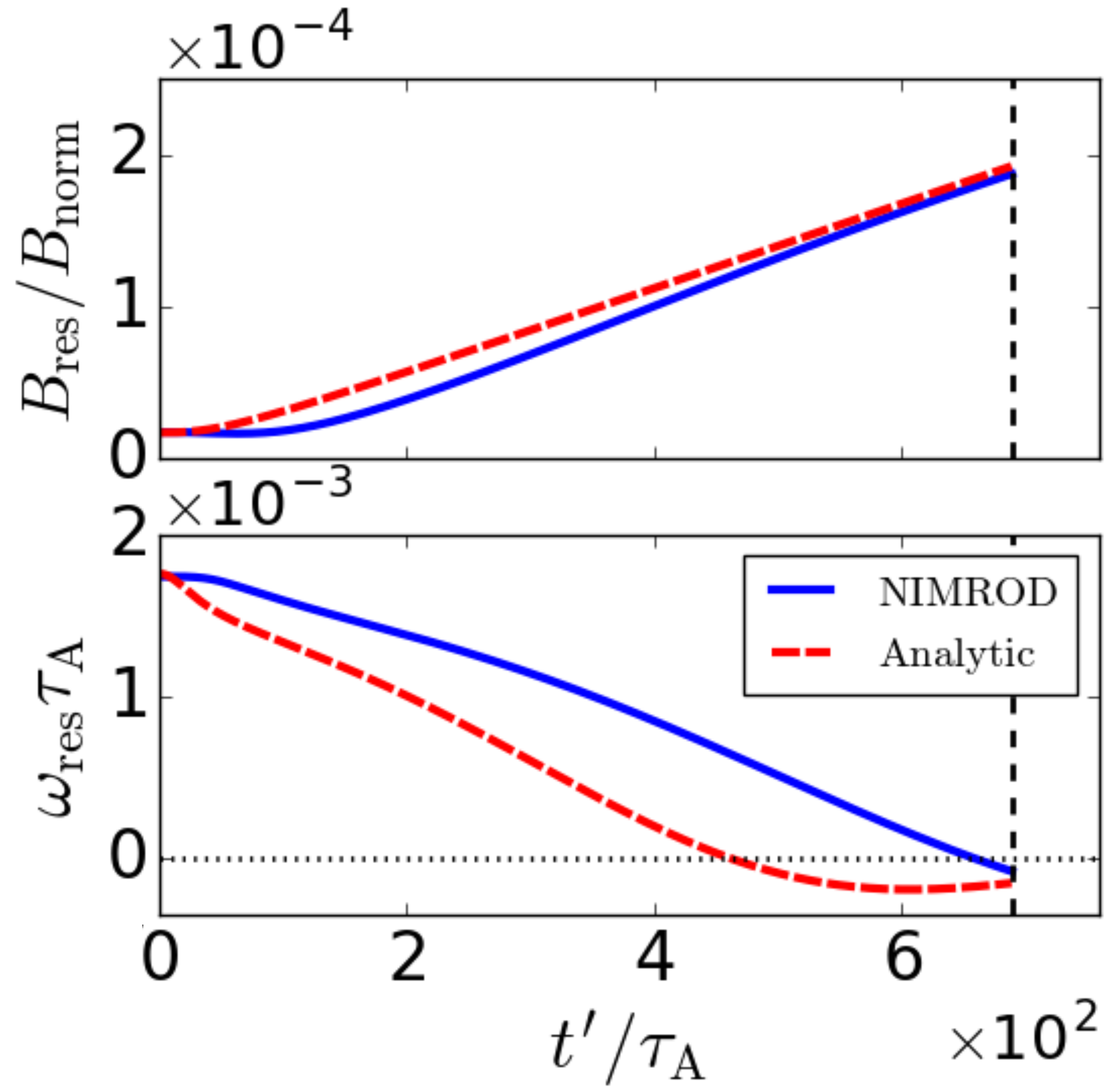
$$\frac{\delta_{\mathrm{VR}} L_y \rho}{k_y} \frac{d\omega_{\mathrm{res}}(t')}{dt'} = \hat{F}_{\mathrm{EM}}(t') + \hat{F}_{\mathrm{V}}(t')\,, \quad \frac{d\varphi_{\mathrm{res}}(t')}{dt'} = \omega_{\mathrm{res}}(t')$$

- **Numerically solve system**
  - Solution shown for transient with $B_{\mathrm{ext,T}} = 9\, B_{\mathrm{ext,0}}$, $\Delta t_{\mathrm{T}} = 690\ \tau_{\mathrm{A}}$, $\tau_{\mathrm{T}} = 6.9\ \tau_{\mathrm{A}}$
- **EM and viscous forces balance in time-asymptotic, mode penetrated state**
  - Recoil directly following transient due to slow response of viscous force



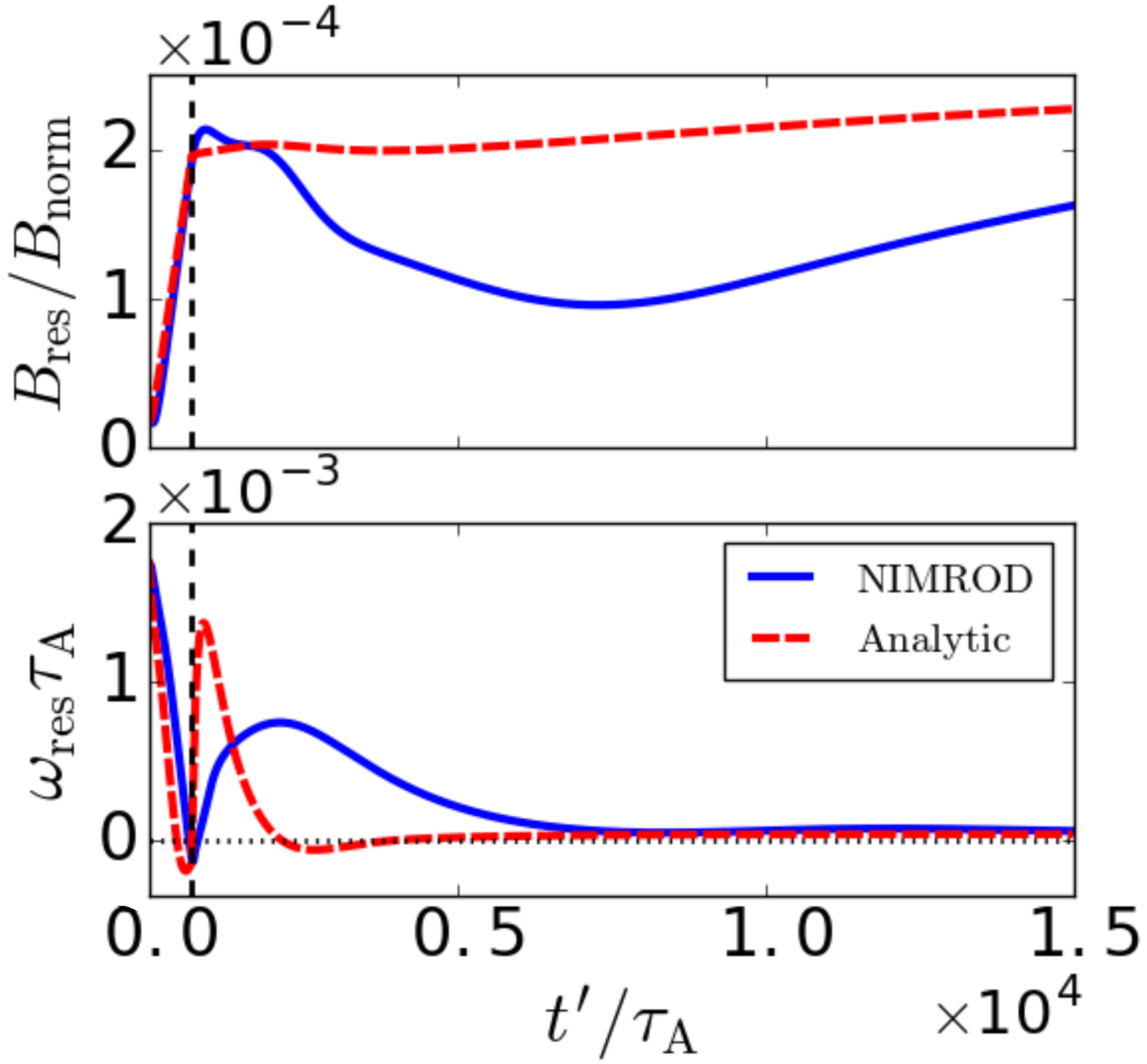
Qualitative Agreement Between Analytical Model and Computational Results

- **Agreement with NIMROD during transient**
  - Analytics yield mode penetration threshold at $B_{\mathrm{ext,T}} = 4.75\ B_{\mathrm{ext,0}}$ (not shown)
    - Within factor ~ 2 of NIMROD results
  - In line with analytics from J.D.Callen poster P3.017



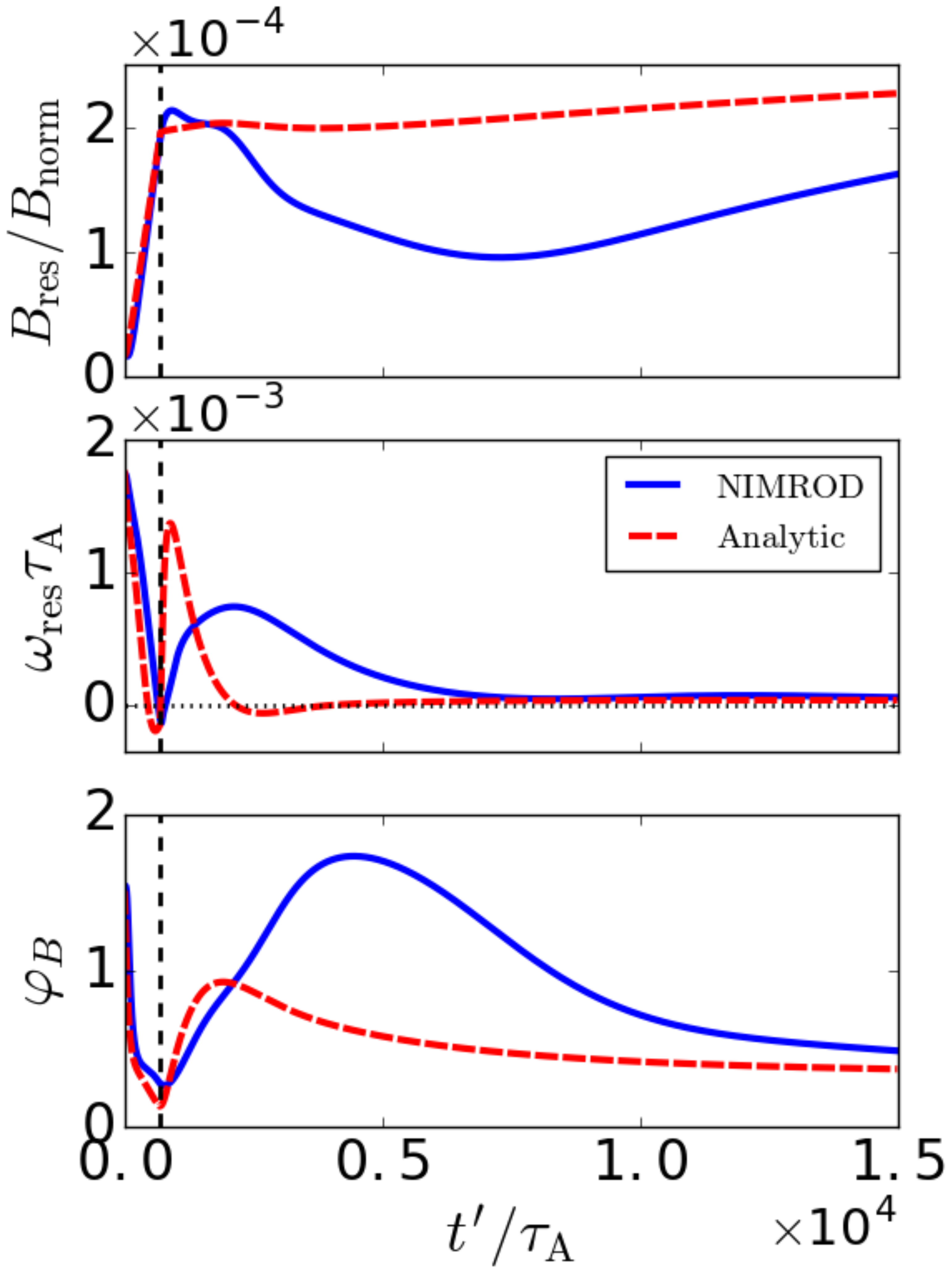
Qualitative Agreement Between Analytical Model and Computational Results

- **Agreement with NIMROD during transient**
  - Analytics yield mode penetration threshold at $B_{\text{ext,T}} = 4.75\ B_{\text{ext,0}}$ (not shown)
    - Within factor ~2 of NIMROD results
  - In line with analytics from J.D.Callen poster P3.017
- **Following transient, analytics over-predict penetrated field**
  - Correlates with difference in time history of ω_{res}
  - Linear asymptotic matching procedure inadequate during nonlinear Rutherford evolution



Qualitative Agreement Between Analytical Model and Computational Results

- **Agreement with NIMROD during transient**
  - Analytics yield mode penetration threshold at $B_{\mathrm{ext,T}} = 4.75\ B_{\mathrm{ext,0}}$ (not shown)
    - Within factor ~2 of NIMROD results
  - In line with analytics from J.D.Callen poster P3.017
- **Following transient, analytics over-predict penetrated field**
  - Correlates with difference in time history of ω_{res}
  - Linear asymptotic matching procedure inadequate during nonlinear Rutherford evolution
- **Agreement with NIMROD for island phase shift φ_B during transient**



Conclusions

- **Transient RMP can precipitate mode penetration**
  - Initial state must satisfy threshold for metastable state to exist $\omega_0 > 3\sqrt{3}/\tau'_{\rm VR}$

Conclusions

- **Transient RMP can precipitate mode penetration**
  - Initial state must satisfy threshold for metastable state to exist $\omega_0 > 3\sqrt{3}/\tau'_{\mathrm{VR}}$
- **Computational results explore mode penetration dynamics**
  - Threshold sensitive to transient parameterization
  - Sensitivity to shape suggests importance of the time history of evolution

Conclusions

- **Transient RMP can precipitate mode penetration**
  - Initial state must satisfy threshold for metastable state to exist $\omega_0 > 3\sqrt{3}/\tau'_{\mathrm{VR}}$
- **Computational results explore mode penetration dynamics**
  - Threshold sensitive to transient parameterization
  - Sensitivity to shape suggests importance of the time history of evolution
- **Qualitative agreement between analytical and computational results**

Conclusions

- **Transient RMP can precipitate mode penetration**
  - Initial state must satisfy threshold for metastable state to exist $\omega_0 > 3\sqrt{3}/\tau'_{\rm VR}$
- **Computational results explore mode penetration dynamics**
  - Threshold sensitive to transient parameterization
  - Sensitivity to shape suggests importance of the time history of evolution
- **Qualitative agreement between analytical and computational results**

Take-away: While analytic models provide rough criteria for mode penetration due to transient RMPs, computational models are necessary for accurate dynamical predictions