RE generation in tokamaks and coupling to M3D-C1

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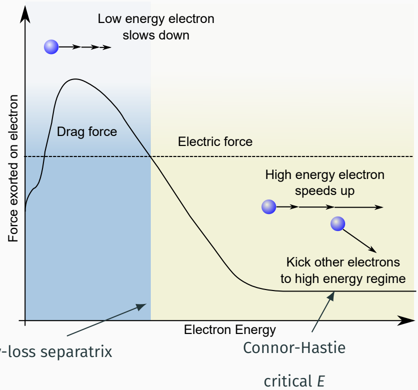
Generation of runaway electrons is an important issue in tokamak disruption studies

- In ITER disruptions, a large population of runaway electrons can be generated due to the strong inductive electric field in thermal quench.
- Unmitigated runaway electron beam can be destructive to the device and make a serious threat to ITER
- Plasma current is carried by (almost) collisionless runaway electrons, resulting in different behaviors of MHD instabilities.
    - Current profile in RE-plasma can be very different from that before disruption.
- For MHD simulations of disruptions, it is important to include runaway electron generation and its coupling to MHD equations.

Basic pictures of runaway electrons

- In plasma, drag force on electrons due to Coulomb collision is a non-monotonic function of p. For $v \gg v_{th}$, collision frequency$\sim v^{-3}$, collisional drag force decrease with p.
- With $E > E_{CH}$ (Connor-Hastie field), electrons with momentum larger than $p_{\rm crit}$ can run away to higher energy.
- Knock-on collision of high energy electron with thermal electron can lead to avalanche growth of RE.
- Runaway electrons can be generated in flattop experiments (low density) and disruption scenarios (large E field).

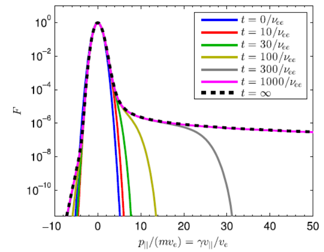


Dreicer generation: sliding-away of Maxwellian tail

$$E_D = \frac{n_e e^3 \ln \Lambda}{4\pi \epsilon_0^2 T_e}$$

- For $E \gtrsim E_D$, all the electrons will run away.
- For $E \ll E_D$, the high-energy electrons have E field exceeding collisional drag, and will slide away to higher energy, while lower energy electrons will fill the gap due to diffusion.
- The quasi-steady state distribution function (non-relativistc part) can be solved as

$$-\frac{eE}{m} \cdot \frac{\partial f}{\partial v} = C(f)$$



Sliding away of Maxwellian distribution (Landreman et al. 2014)

H. Dreicer, Phys. Rev. 115, 238 (1959).

M.D. Kruskal and I.B. Bernstein, Phys. Fluids 7, 407 (1964).

R.H. Cohen, Phys. Fluids 19, 239 (1976).

Dreicer generation: sliding-away of Maxwellian tail (cont'd)

Dreicer generation rate

$$\frac{dn_r}{dt} = kn_e\nu_{ee}\left(\frac{E_D}{E}\right)^{-3(1+Z)/16}\exp\left(-\frac{E_D}{4E}-\sqrt{\frac{(1+Z)E_D}{E}}\right)$$

- Connor&Hastie gave correction to the generation rate for relativistic effects. It can be important for $E \approx E_C$, where $E_C = n_e e^3 \ln\Lambda/(4\pi\epsilon_0^2 mc^2)$.
- Problems with Dreicer generation rate
  - Depends sensitively on E and T_e
  - Only valid for quasi-steady state with slow-varying E and T_e.

J.W. Connor and R.J. Hastie, Nucl. Fusion 15, 415 (1975).

Hot-tail generation

- For disruptions, the runaway electrons are not generated with fixed T_e and E.
- In a rapid cooling process of electrons, while the low-energy population will be slowed down due to collisions, the hot population will be dragged by the inductive electric field and become runaway electrons.
- A simple way to calculate hot-tail generation is to separate the cooling of electrons and running away.
    - Assuming cooling is caused by collisional drag

$$\frac{\partial f}{\partial t} = \frac{\nu v_T^3}{v^2}\frac{\partial f}{\partial v}$$

$$f = \frac{n_0}{\pi^{3/2} v_{T0}^2} \exp\left[-\left(\frac{v^3}{v_{T0}^3} + 3\tau\right)^{2/3}\right]$$

$$\tau = \nu_0 \int_0^t \frac{n(t)}{n_0} dt$$

H.M. Smith and E. Verwichte, Phys. Plasmas 15, 072502 (2008).

Hot-tail generation (cont'd)

- The runaway electron generation is measured by counting the number of electrons beyond runaway boundary

$$n_r = \int_{v_c}^{\infty} f 4\pi (v^2 - v_c^2) dv$$

$$\frac{dn_r}{dt} = \frac{4n_0}{\sqrt{\pi}} \frac{2u_c^2 H(-du_c/dt)}{(u_c^3 - 3\tau)^{1/3}} \left(-\frac{du_c}{dt}\right) \int_{u_c}^{\infty} \frac{e^{-u^2} u^2 du}{(u^3 - 3\tau)^{2/3}}$$

$$u_c = (v_c^3/v_{T0}^3 + 3\tau)^{1/3}, \qquad v_c = \frac{TE_D}{mE}$$

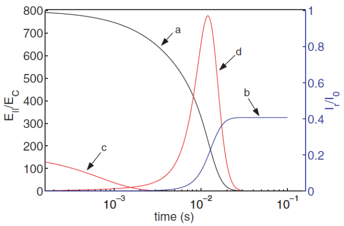
- Problems with Smith&Verwichte hot-tail model
  - The electric field is not taken into account in the distribution function evolution.
  - The cooling of bulk plasma is simplified.
  - Only the drag force is included for the collisions. Note that during thermal quench, the electron distribution function deviates from Maxwellian and the linearized collision operator is not valid.

Secondary runaway electron generation (avalanche)

- Large-angle collisions between a high-energy runaway electron and a thermal electron can transfer significant energy to the thermal electron and make it a new runaway electron.
- Although large-angle collisions happen pretty rarely, it leads to an exponential growth and can dominate RE generation during current quench.
- Rosenbluth&Putvinski calculated the source term from the secondary generation using Moller collision cross section.
    - It is assumed that all the existing runaway electrons are in high energy regime with very small pitch angle ($\xi \approx 1$).

$$S = \frac{n_r \nu_{rel}}{4\pi \ln \Lambda} \delta(\xi - \xi_2) \frac{1}{p^2} \frac{\partial}{\partial p} \left(\frac{1}{1 - \sqrt{1+p^2}} \right)$$

$$\xi_2 = \frac{p}{1 + \sqrt{1+p^2}}$$



(a) electric field (b) RE current (c) Dreicer generation (d) secondary generation

M.N. Rosenbluth and S.V. Putvinski, Nucl. Fusion 37, 1355 (1997).

Secondary runaway electron generation (avalanche)

The source term is divergent for $p \to 0$, thus a cutoff boundary is needed for generation rate.

- Integrating S above $v > v_c$ can get a simple expression of secondary generation rate.

$$\frac{dn_r}{dt} = \frac{n_r}{2\tau \ln \Lambda}\left(\frac{E}{E_c} - 1\right)$$

- Considering the pitch angle scattering and trapping effects, the expression

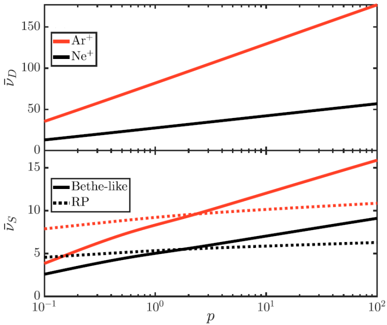
$$\frac{dn_r}{dt} = \frac{n_r \nu_{rel}}{\ln \Lambda}\left(\frac{E}{E_c} - 1\right)\sqrt{\frac{\pi\phi}{3(Z_{eff}+5)}}\left[1 - \frac{E_c}{E} + \frac{4\pi(Z_{eff}+1)^2}{3\phi(Z_{eff}+5)(E^2/E_c^2 + 4/\phi^2 - 1)}\right]^{-1/2}$$

$$\phi = (1 + 1.46\epsilon^{1/2} + 1.72\epsilon)^{-1}$$

- Other effects including finite energy of seed runaway electrons, radiation damping and kinetic instabilities can also change the avalanche rate.

Partially-screening of high-Z nuclei can affect avalanche growth

- With high-Z impurities injection, the slowing-down and the pitch-angle scattering for REs in high energy regime is significantly enhanced due to partially-screening.
  - High-energy electron can penetrate into electron cloud and get closer to the nuclei, interacting with bounded electrons and naked nuclei.
  - Slowing-down enhanced by factor of Z, and scattering is enhanced by Z^2.



- Critical electric field for avalanche increases due to enhancement of collisions.
- Bounded electrons can also collide with runaway electrons and participate avalanche, meaning an enhancement of avalanche growth rate by a factor of Z/Z_{eff}.

L. Hesslow et al., Phys. Rev. Lett. 118, 255001 (2017).
L. Hesslow et al., Plasma Phys. Control. Fusion 60, 074010 (2018).

Roadmap of adding runaway electron physics in M3D-C1 simulation

- We plan to implement a fluid model for runaway electrons, including their generation, and couple to current VDE simulation of M3D-C1.
  - Runaway density (n_{RE}) follows an advection equation with a large velocity along **B** field, and $\mathbf{E} \times \mathbf{B}$ drift velocity.
  - Runaway current is simply calculated by $\mathbf{J} = -n_{RE}ec\mathbf{b}$, and coupled to Ohm's law.
- Dreicer, avalanche and hot-tail generation will be included.
- This model has been successfully tested in M3D-K code, Strauss' M3D code and EXTREM code.

$$\frac{\partial n_{RE}}{\partial t} + \nabla \cdot (\frac{n_{RE}c}{B}\mathbf{B}) = S_{RE} - \nabla \cdot (n_{RE}\frac{\mathbf{E} \times \mathbf{B}}{B^2})$$

$$\mathbf{J}_{RE} = -n_{RE}ec\frac{\mathbf{B}}{B}$$

$$\rho\frac{d\mathbf{V}}{dt} = (\mathbf{J} - \cancel{\mathbf{J}_{RE}}) \times \mathbf{B} - \nabla p + \rho\mathbf{g}$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta(\mathbf{J} - \mathbf{J}_{RE})$$

Future plan:Fluid-kinetic framework for runaway electron simulation

A self-consistent fluid-kinetic framework for runaway electrons is needed for disruption simulation.

- Current coupling is used in previous studies of energetic ions, but for REs the generation from the bulk population is important.
- Analytical form of RE source (Dreicer, hot-tail) can be inaccurate for fast-evolving scenarios.

Method: one-fluid equations and kinetic equation for runaway electrons coupled in terms of transition probability between the kinetic tail and bulk fluid.

$$\partial_t \varrho + \nabla \cdot (\varrho \boldsymbol{u}) = -\int m_e I_e d\boldsymbol{v} \qquad \boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B} = \eta(\mu_0^{-1} \nabla \times \boldsymbol{B} + e n_{e1} \boldsymbol{v}_{e1})$$

$$\varrho(\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}) = -\nabla \cdot \mathbf{p} + \mu_0^{-1} \nabla \times \boldsymbol{B} \times \boldsymbol{B} - \sum_\alpha \boldsymbol{F}_{e1,\alpha} - \int m_e(\boldsymbol{v} - \boldsymbol{u}) I_e d\boldsymbol{v}$$

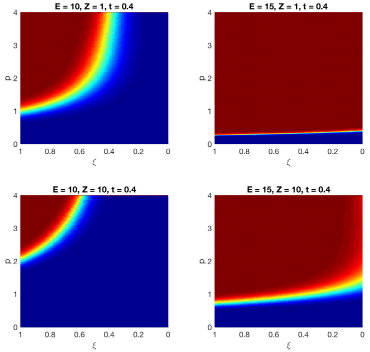
$$+ e n_{e1}(\boldsymbol{E} + \boldsymbol{v}_{e1} \times \boldsymbol{B})$$

$$\frac{df_{e1}}{dt} = \sum_\alpha C_{e\alpha}[f_{e1}, f_\alpha] + I_e$$

Compute I_e using backward Monte-Carlo method

The time-dependent transition probability (I_e) depends on the runaway probability function, which can be computed efficiently using the backward Monte-Carlo method, by following the stochastic trajectories of particles.

$$I(\boldsymbol{z}, t) = \frac{f_0}{\tau}\left(1 - \mathbb{E}[\mathbf{1}_{\Omega_0}(\boldsymbol{Z}_{t+\tau})|\boldsymbol{Z}_t = \boldsymbol{z}]\right) - \frac{f_1}{\tau}\mathbb{E}[\mathbf{1}_{\Omega_0}(\boldsymbol{Z}_{t+\tau})|\boldsymbol{Z}_t = \boldsymbol{z}]$$



G. Zhang and D. del-Castillo-Negrete, Phys. Plasmas 24, 092511 (2017).
E. Hirvijoki, C. Liu, G. Zhang, D. del-Castillo-Negrete, and D.P. Brennan, Phys. Plasmas 25, 062507 (2018).

Thank you