Reduced kinetic model of runaway electrons in NIMROD

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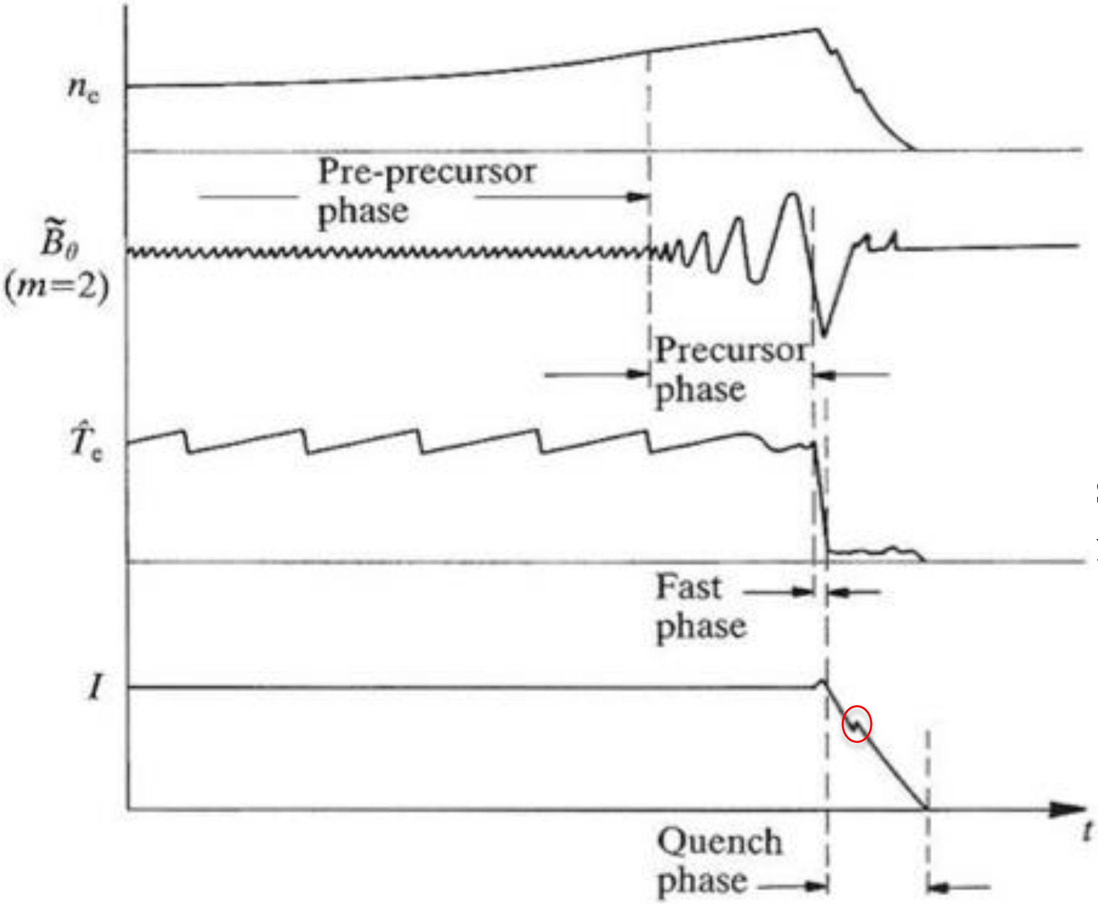
Major disruptions in tokamaks often have similar characteristics



plasma density increases

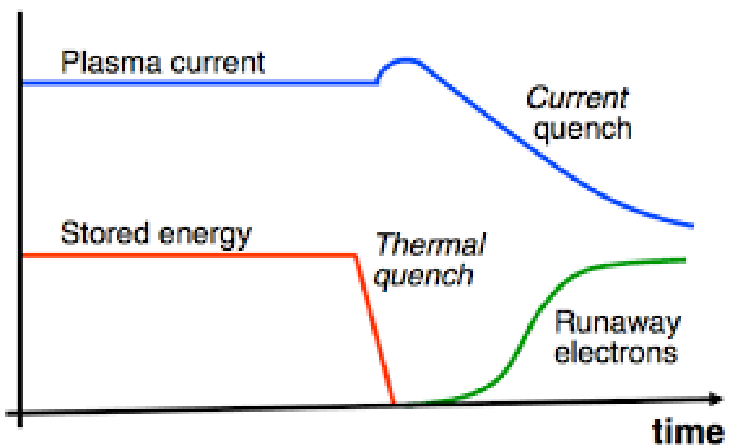
growth of m=2 NTM

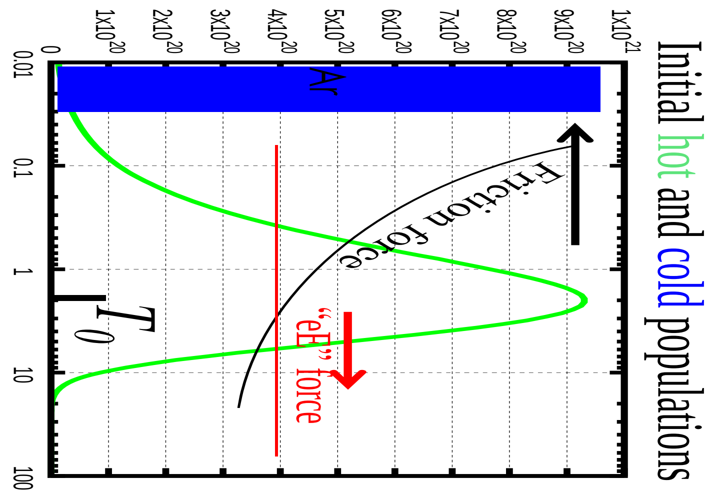
sawtooth instability followed by a fast temperature quench

slow current quench due to the generation of runaway electrons

A typical time dependence of the plasma density, m=2 magnetic fluctuations, central temperature and plasma current during a period of major disruption.
(F.C. Schuller 1995 Plasma Phys. Control. Fusion 37 A135)

Runaway electron generation is an important phenomenon of disruptions





MHD instability (tearing mode, kink mode, fishbone….) and whistler waves driven by runaway current

Thermal quench:

$$T_e, T_i \downarrow$$

$$\sigma \uparrow$$

$$E_\varphi = \sigma J_\varphi \uparrow$$

Runaway electron avalanche:

$$E_\varphi > E_D$$

Electrons are accelerated!

Current quench:

$$I_p \downarrow$$

$$I_{re} \uparrow$$

Previous work has investigated resistive MHD instabilities with runaway electrons

- The post-disruption runaway current profile could be more strongly peaked in the center of the discharge than the pre-disruption current. Plasmas with steep current profiles can be prone to tearing-mode instability. *(P.Helander, et.al. Phys.Plasma 14, 122102(2007))*

- Such a peaked runaway current can also drive other resistive magnetohydrodynamic instabilities in the plasma with high resistivity. H.S.Cai et.al. investigated the resistive internal kink mode in a tokamak plasma with runaway current using M3D. *(H.S.Cai and G.Y.Fu, Nucl. Fusion, 55 022001(2015))*

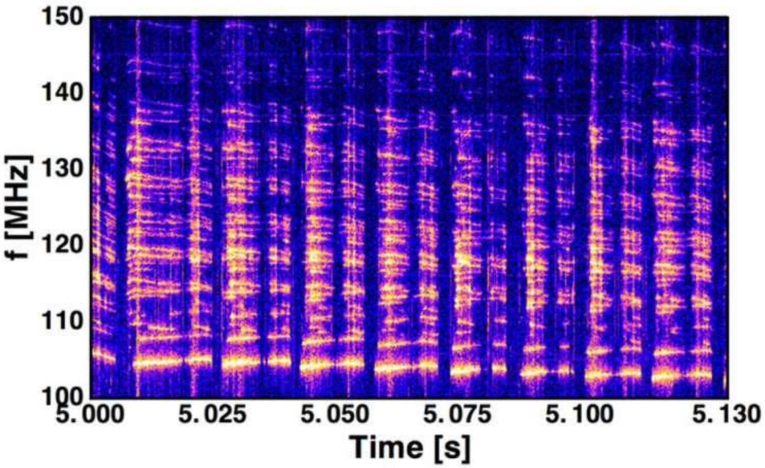
Previous work has investigated kinetic instabilities with runaway electrons

- Due to the highly relativistic speed, the runaway electrons are observed to drive the whistler wave unstable with the kinetic resonance condition,

$$\omega - k_{||}v_{||} - k_{\perp}v_d - l\omega_{ce}/\gamma = 0$$

(D.A.Spong, et.al. Phys.Rev.Lett. 120, 155002(2018)

C.Liu, et.al. Phys.Rev.Lett.120, 265001(2018))



Spectrogram of whistler wave activity measured in DIII-D.

Motivation of the research

- The linear instability of resistive MHD of the post-disruption plasma,
  1. Much shorter time scale than that of runaway electron generation.
  2. The thermal plasma current with high resistivity.
  3. The runaway electrons are highly relativistic and their velocities are mainly parallel to the magnetic field ($v_{||} \sim c$).

We adapt a runaway electron reduced model for NIMROD

Several approximations are appropriate for the RE reduced model

- Runaway electrons are collisionless and the inertia of runaway electrons is neglected;
- Runaway electrons are highly relativistic and their velocities are mainly parallel to the magnetic field as the passing particles;

$$v_{\|} \sim c \gg v_{\perp}$$

- The time scale of MHD instabilities of the post-disruption plasma is much shorter than that of runaway electron generation (the inverse runaway avalanche rate), so the generation source of runaway electrons is not considered;
- The runaway electron energy remains nearly constant with a low frequency of the resistive MHD perturbed fields,

$$\delta E = \omega \delta P_{\varphi}/n \approx 0$$

However, the constant of motion of runaways electron is applied in the whistlers,

$$\mu' = \mu - \frac{v_{\|}}{k}$$

The RE drift-kinetic equation is the basis for a fluid model

Hazeltine's drift kinetic equation with a low frequency MHD mode:

$$\frac{\partial f}{\partial t} + (\vec{v}_{\|} + \vec{v}_D) \cdot \nabla f + \left[e\frac{\partial \Phi}{\partial t} + \mu \frac{\partial B}{\partial t} - \frac{e\vec{v}_{\|}}{c} \cdot \frac{\partial \vec{A}}{\partial t} \right] \frac{\partial f}{\partial E} = C$$

$$\vec{v}_D = \boxed{\frac{\vec{E} \times \vec{B}}{B^2}} + \frac{\gamma}{\omega_{ce}} \hat{b} \times \left(\boxed{\frac{\mu}{m_e} \nabla B} + \boxed{v_{\|}^2 \vec{\kappa}} + v_{\|} \frac{\partial \hat{b}}{\partial t} \right)$$

ExB drift grad drift curvature drift

$$\frac{\gamma}{\omega_{ce}} v_{\|} \hat{b} \times \frac{\partial \hat{b}}{\partial t}$$

the electron drift due to the changing of the direction of the magnetic field, which is missed in H.S.Cai and P.Helander's work.

The reduced RE drift kinetic equation is coding in NIMROD

- Apply our model approximations on Hazeltine's drift kinetic equation with the low frequency MHD mode:

$$\frac{\partial f}{\partial E} \sim 0 \qquad C \sim 0 \qquad v_{\parallel} \sim c \qquad \frac{\mu}{m_e}\nabla B \sim \frac{v_{\perp}^2}{2}\vec{\kappa} \ll v_{\parallel}^2\vec{\kappa}$$

constant energy — collisionless — constant $v_{\parallel}$ — ignore the gradient drift of REs

where, E is the total guiding center energy:

$$E = \frac{m_e v_{\parallel}^2}{2} + e\Phi + \mu B$$

RE drift kinetic equation:

$$\frac{\partial f_{RE}}{\partial t} + c\hat{b}\cdot\nabla f_{RE} + \vec{v}_D\cdot\nabla f_{RE} = 0$$

It becomes a standard 3D convection equation and can apply the NIMROD framework to solve.

The RE drift-kinetic equation couples with the MHD field

$$\frac{\partial f_{RE}}{\partial t} + c\hat{b}\cdot\nabla f_{RE} + \vec{v}_D\cdot\nabla f_{RE} = 0$$

RE drift velocity:

$$\vec{v}_D = \frac{\vec{E}\times\vec{B}}{B^2} + \frac{\gamma}{\omega_{ce}}\hat{b}\times\left(\frac{\mu}{m_e}\nabla B + v_\parallel^2\vec{\kappa} + v_\parallel\frac{\partial\hat{b}}{\partial t}\right)$$

- A standard 3D convection equation in the fluid mechanics;
- To calculate RE drift velocity, we need update the field variables in the framework of NIMROD:

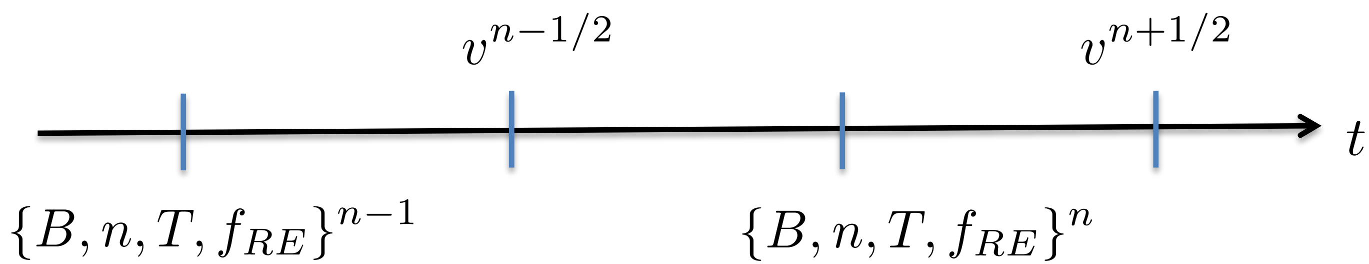
$$\vec{E} = -\vec{v}\times\vec{B} + \eta(\vec{J} - \vec{J}_{RE}) \qquad B = |\vec{B}|$$

$$\vec{J}_{RE} = -ecf_{RE}\hat{b} - \frac{ecf_{RE}}{\omega_{ce}}\hat{b}\times\left(c\vec{\kappa} + \frac{\partial\hat{b}}{\partial t}\right) \qquad \hat{b} = \frac{\vec{B}}{B}$$

$$\boxed{\frac{\partial\vec{B}}{\partial t} = -\nabla\times\vec{E} + \kappa_b\nabla\nabla\cdot\vec{B}} \qquad \vec{J} = \frac{1}{\mu_0}\nabla\times\vec{B} \qquad \vec{\kappa} = \hat{b}\cdot\nabla\hat{b}$$

$$\boxed{mn\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\vec{v} = (\vec{J} - \vec{J}_{RE})\times\vec{B} - \nabla\sum_\alpha nT_\alpha - \nabla\cdot\Pi}$$

The predict-correct scheme is used in NIMROD



Receive $\vec{v}^{n+1/2}$ from NIMROD code

$$\Delta \vec{B} = \Delta t \nabla \times \left(\vec{v}^{n+1/2} \times \vec{B}^* - \frac{\eta}{\mu_0} \nabla \times \vec{B}^* + \eta \vec{J}^*_{RE} \right)$$

where $\vec{B}^* = \vec{B}^n$(prediction), $\vec{B}^* = f\vec{B}^n + (1-f)\vec{B}^{n+1}$ (correction)

$$\Delta \hat{b} = \frac{1}{B^{n+1}} (\Delta \vec{B} - \hat{b}^* |\Delta \vec{B}|)$$

$$\Delta f_{RE} = -c\hat{b}^{n+1} \cdot \nabla f^*_{RE} - \vec{v}^{n+1}_D \cdot \nabla f^*_{RE}$$

$$\vec{J}^{n+1}_{RE} = -ecf^{n+1}_{RE} \hat{b}^{n+1} - \frac{ecf^{n+1}_{RE}}{\omega^{n+1}_{ce}} \hat{b}^{n+1} \times (c\vec{\kappa}^{n+1} + \frac{\Delta \hat{b}}{\Delta t})$$

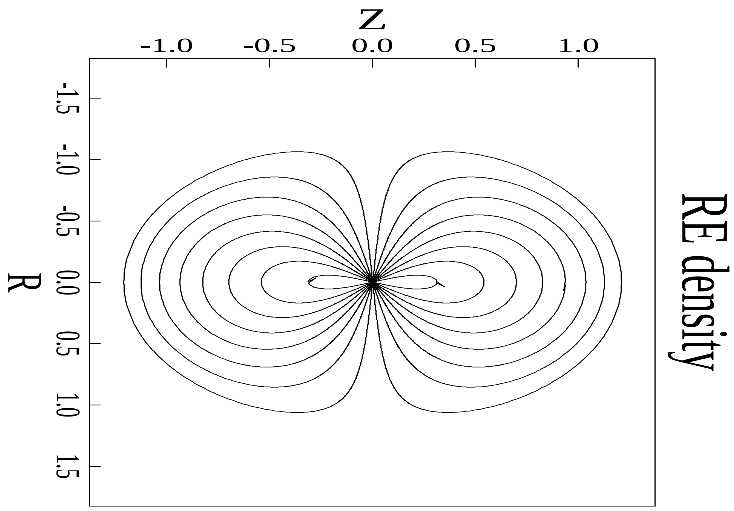
Send to NIMROD flow equation $mn \left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \vec{v} = (\vec{J} - \vec{J}_{RE}) \times \vec{B} - \nabla \sum_\alpha n T_\alpha - \nabla \cdot \Pi$

Simple case

$$\vec{v}_D = 0$$

$$\vec{J}_{RE\perp} = 0$$

$$\Delta f_{RE} = -c\hat{b}_0 \cdot \nabla f^n_{RE}$$



Conclusion

- Derivation of the drift kinetic equation in the interaction with the low frequency resistive MHD mode. The changing of the magnetic field direction is found to a component of drift velocity.

- The numerical scheme of 3D convective-like equation in the framework of NIMROD.

Reduced RE kinetic equation with whistlers

- Because of the cyclotron resonance, the magnetic momentum is not a constant of motion.

- Instead of the magnetic momentum, $E - \omega\mu$ becomes a constant of motion of runaway electrons, which leads to a 3D kinetic equation in the interaction with the whistlers.

$$\frac{\partial f}{\partial t} + c\hat{b} \cdot \nabla f + \frac{d\mu}{dt}\frac{\partial f}{\partial \mu} + \omega_c \frac{\partial f}{\partial \psi} = 0$$