Simulation of MHD instabilities with fluid runaway electron model in M3D-C^1

C. Zhao[1], C. Liu[1], S. C. Jardin[1], N. Ferraro[1]

[1]Princeton Plasma Physics Laboratory, Princeton, NJ

Outline

- Introduction to M3D-C^1
- Linear simulation of MHD instabilities with RE
- Nonlinear simulations with RE
- Summary and future work

1.Introduction to M3D-C^1

3D Extended MHD Equations in M3D-C^1

Density equation

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\mathbf{V}) = \nabla \bullet D_n \nabla n + S_n$$

Field equation

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla\Phi, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla_\perp \bullet \frac{1}{R^2}\nabla\Phi = -\nabla_\perp \bullet \frac{1}{R^2}\mathbf{E}$$

Momentum equation

$$nM_i\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \bullet \nabla\mathbf{V}\right) + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \bullet \mathbf{\Pi}_i + \mathbf{S}_m$$

Generalized Ohm's law

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne}\left(\mathbf{R}_c + \mathbf{J} \times \mathbf{B} - \nabla p_e - \nabla \bullet \mathbf{\Pi}_e\right) - \frac{m_e}{e}\left(\frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \bullet \nabla \mathbf{V}_e\right) + \mathbf{S}_{CD}$$

Pressure equations

$$\frac{3}{2}\left[\frac{\partial p_e}{\partial t} + \nabla \bullet \left(p_e \mathbf{V}\right)\right] = -p_e \nabla \bullet \mathbf{V} + \frac{\mathbf{J}}{ne} \bullet \left[\frac{3}{2}\nabla p_e - \frac{5}{2}\frac{p_e}{n}\nabla n + \mathbf{R}_c\right] + \nabla\left(\frac{\mathbf{J}}{ne}\right) : \mathbf{\Pi}_e - \nabla \bullet \mathbf{q}_e + Q_\Delta + S_{eE}$$

$$\frac{3}{2}\left[\frac{\partial p_i}{\partial t} + \nabla \bullet \left(p_i \mathbf{V}\right)\right] = -p_i \nabla \bullet \mathbf{V} - \mathbf{\Pi}_i : \nabla\mathbf{V} - \nabla \bullet \mathbf{q}_i - Q_\Delta + S_{iE}$$

$$\mathbf{R}_c = \eta n e \mathbf{J}, \qquad \mathbf{\Pi}_i = -\mu\left[\nabla\mathbf{V} + \nabla\mathbf{V}^\dagger\right] - 2(\mu_c - \mu)(\nabla \bullet \mathbf{V})\mathbf{I} + \mathbf{\Pi}_i^{GV} \qquad \mathbf{q}_{e,i} = -\kappa_{e,i}\nabla T_{e,i} - \kappa_\parallel \nabla_\parallel T_{e,i}$$

$$\mathbf{\Pi}_e = (\mathbf{B}/B^2)\nabla \bullet \left[\lambda_h \nabla\left(\mathbf{J} \bullet \mathbf{B}/B^2\right)\right], \qquad Q_\Delta = 3m_e(p_i - p_e)/\left(M_i \tau_e\right)$$

Blue terms are 2-fluid terms. Also, now have impurity and pellet models for disruption mitigation. **NOT reduced MHD.**

Fluid Runaway Electron Model

- In our model, the runaways move at a large speed c (is much higher than Alfven speed) and parallel to the magnetic field line.

- Runaway electron is coupled to bulk plasmas through the runaway current in generalized Ohm's law.

- We also add parallel diffusion term to RE on 3D nonlinear simulation to stabilize the numerical instabilities.

$$\frac{\partial n_{RE}}{\partial t} + \nabla \bullet (n_{RE} c \frac{\mathbf{B}}{B}) = S_{RE}$$

RE density equation

$$\mathbf{J}_{RE} = -e n_{RE} c \frac{\mathbf{B}}{B}$$

RE current assumption

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne}(\mathbf{R}_c - \mathbf{R}_{RE} - \nabla \bullet \Pi_e) + \mathbf{S}_{CD}$$

Single fluid Ohm's law with RE

$\mathbf{R}_{RE} = \eta n e \mathbf{J}_{RE}$, Red terms are additional runaway electron terms.

3D finite elements method in M3D-C^1

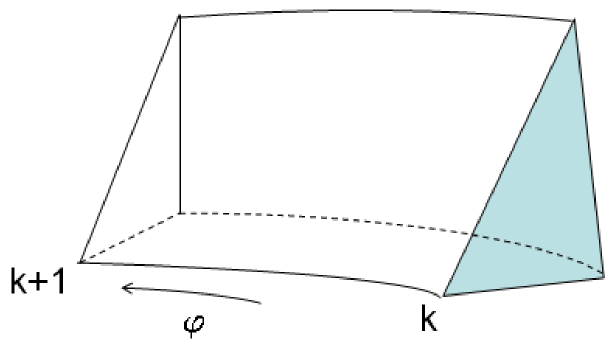
- M3D-*C^1* uses high-order curved triangular prism elements with *C^1 continuity.*

- Within each triangular prism, there is a polynomial in (R,φ,Z) with 72 coefficients.

- The solution *and 1st derivatives* are constrained to be continuous from one element to the next.

- Thus, there is much more resolution than for the same number of linear elements.

- Error ~ h^5



Also, implicit time-stepping allows for very long time simulations

2. Linear results with RE

Reduced MHD equations with RE

$$\omega\psi - k_{||}\phi = i\eta\left(\nabla_\perp^2\psi + j\right)$$

$$\omega\nabla_\perp^2\phi - k_{||}\nabla_\perp^2\psi = \frac{mj_0'}{r}\psi$$

$$\left(k_{||} + \omega v_A/c\right) j = \frac{mj_0'}{r}\left(\psi + v_A\phi/c\right)$$

P. Helander, 2007

Where $$\nabla_\perp^2 = \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) - \frac{m^2}{r^2}$$

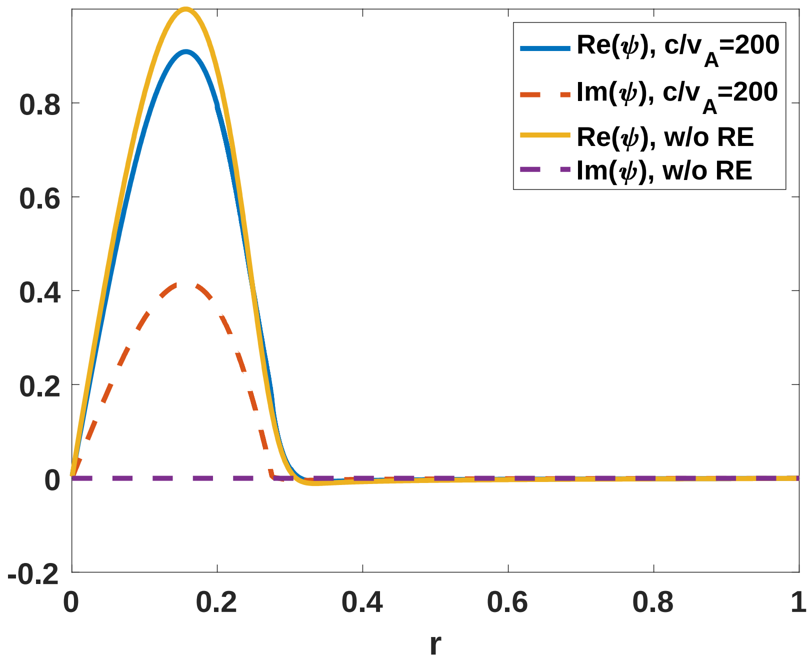
$$k_{||} = \frac{nq(r) - m}{r}$$

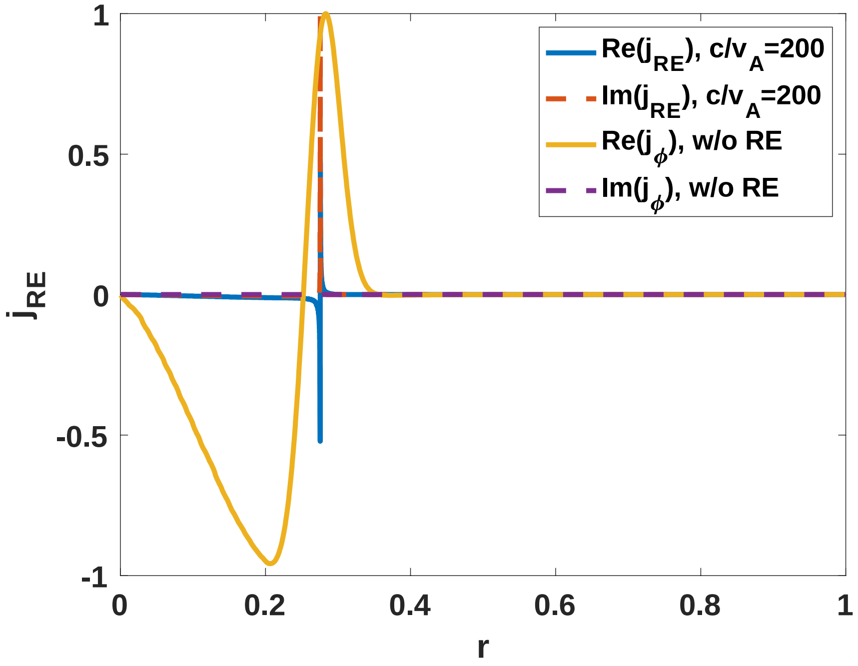
$j_0' = \dfrac{d}{dr}j_0$, and j is RE current.

- We transform the equations to the matrix and use Matlab eigenvalue solver to get the eigenvalue ω (real frequency and growth rate) and eigenvectors Ψ, ϕ, j (mode structure) of the mode.

$$\omega\begin{bmatrix} I & 0 & 0 \\ 0 & \nabla_\perp^2 & 0 \\ 0 & 0 & v_A/c \end{bmatrix}\begin{bmatrix}\psi \\ \phi \\ j\end{bmatrix} = \begin{bmatrix} i\eta\nabla_\perp^2 & k_{||} & i\eta \\ k_{||}\nabla_\perp^2 + mj_0'/r & 0 & 0 \\ mj_0'/r & mj_0'v_A/rc & -k_{||}\end{bmatrix}\begin{bmatrix}\psi \\ \phi \\ j\end{bmatrix}$$

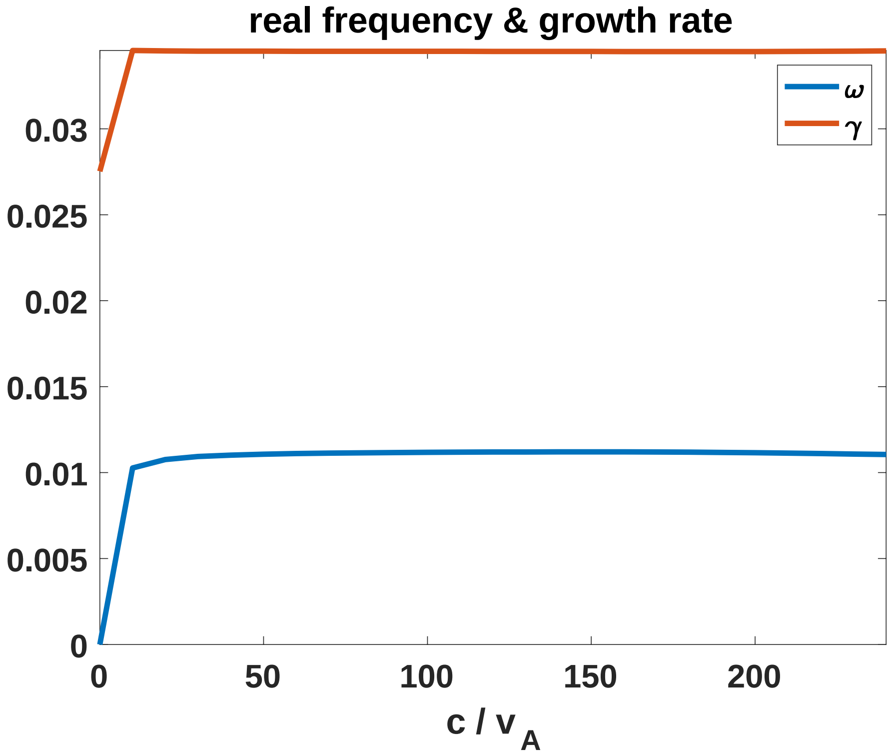
Eigenfunction solution of 1/1 resistive kink mode with RE

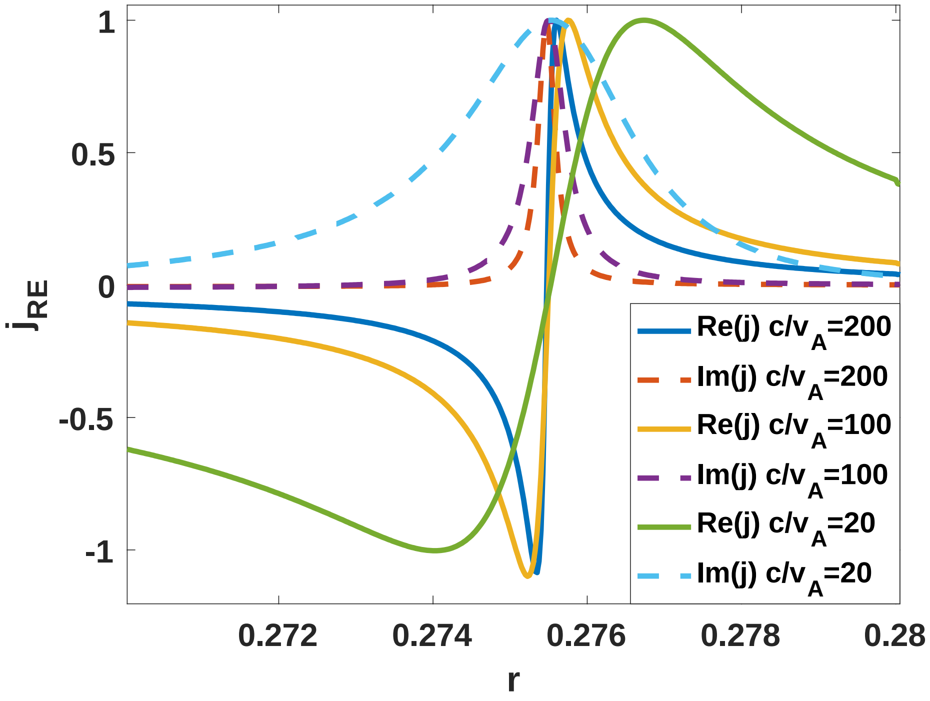




- The figures are the radial structures of magnetic flux and runaway/parallel current perturbations of 1/1 kink mode with and without runaways. The result with runaways was solved from the eigenfunction from Halender 2007.
- The RE current steeply peaked around q = 1 surface.

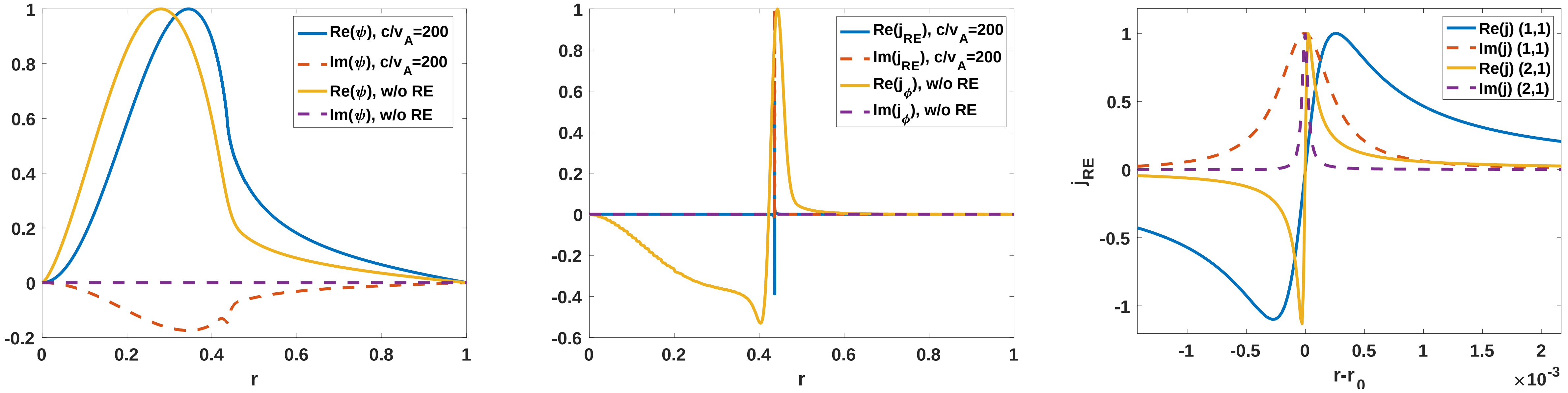
Eigenvalue and Eigenvector of 1/1 resistive kink mode with different runaway velocity





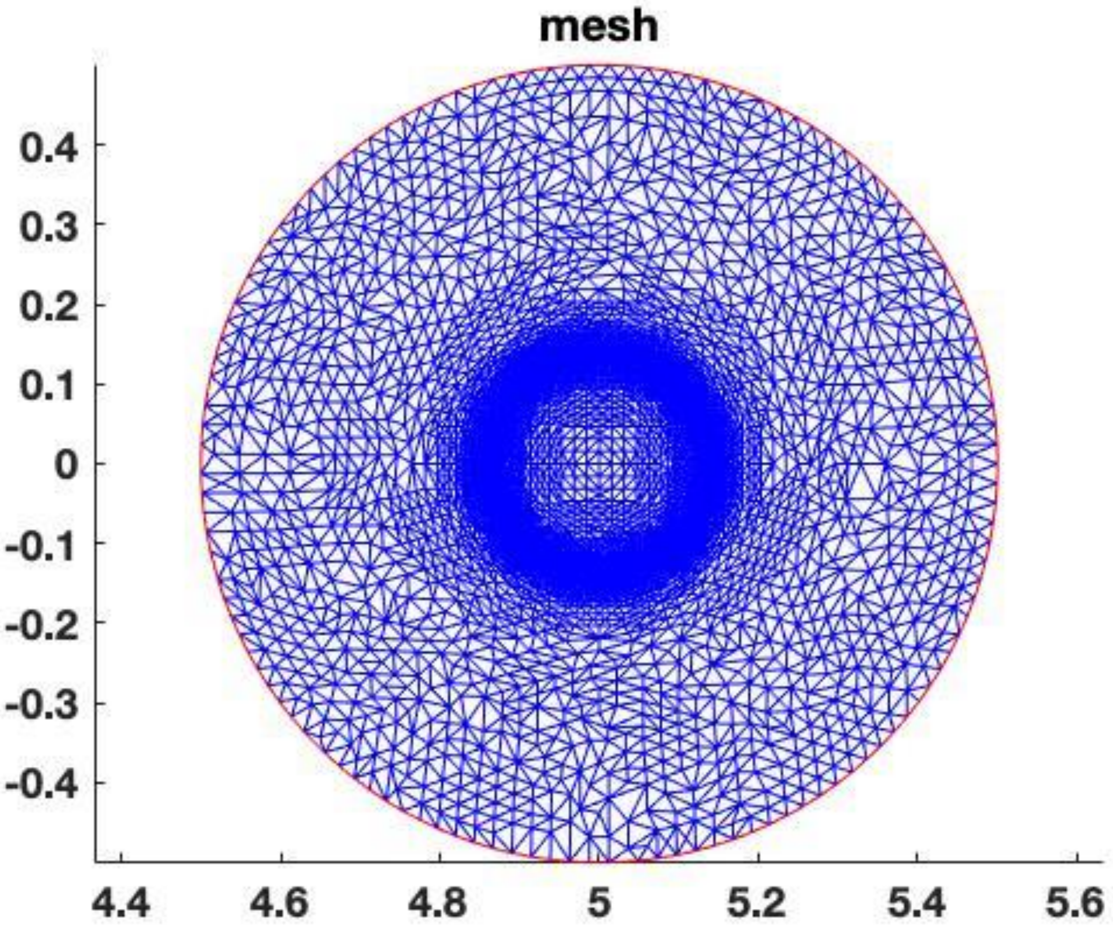
- The 1/1 mode has a non-zero and nearly constant real frequency when the RE speed lager than 10 Alfven velocity.
- The growth rate is also constant when RE speed lager than 10 Alfven velocity.
- The growth rate is about 3 times larger than real frequency.
- The runaway current scale length becomes smaller when runaway speed increasing.

Eigenfunction solution of 2/1 tearing mode with RE



- The figures are the radial structures of magnetic flux and runaway/parallel current perturbations of 2/1 tearing mode with and without runaways.
- The RE current has a very steep peak around q = 2 surface. And the scale length is also much smaller than 1/1 resistive kink mode with runaways.

Mesh and basic parameters for 1/1 resistive kink mode simulation

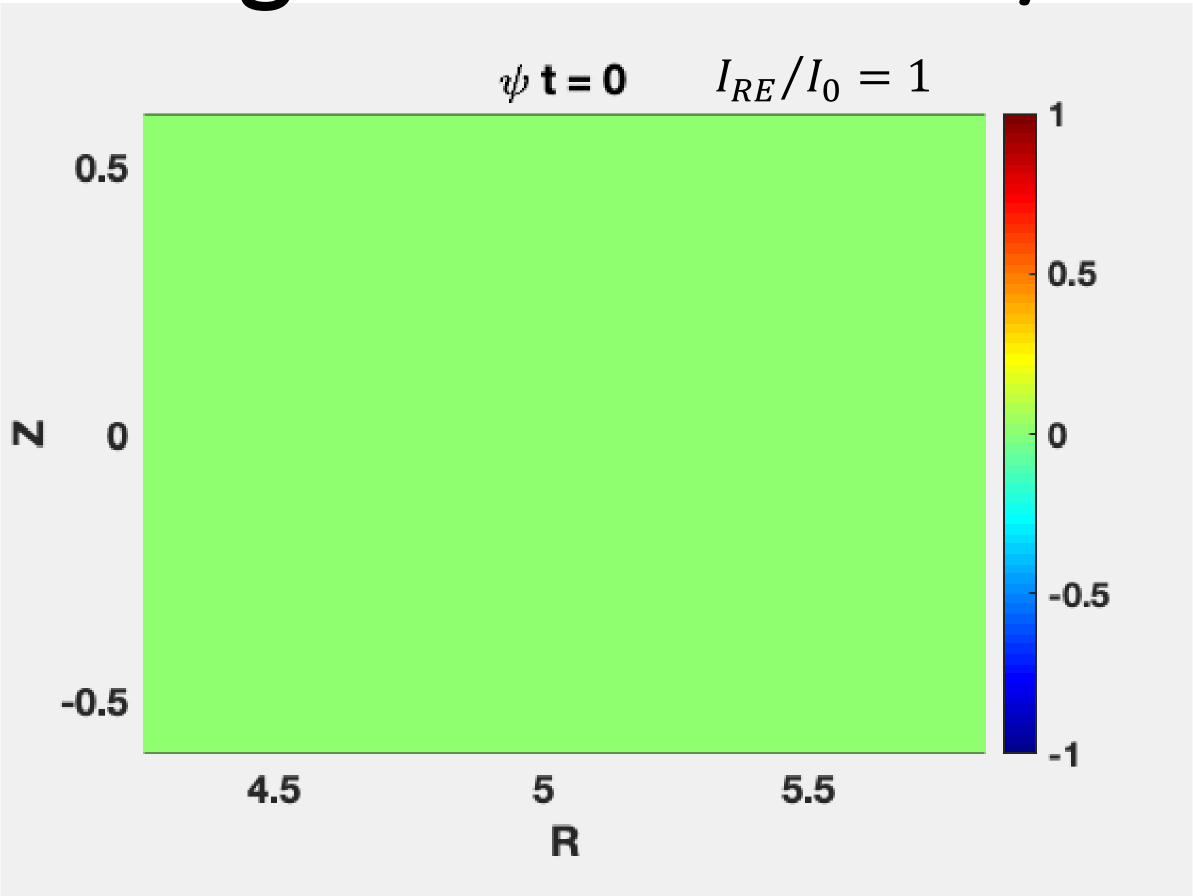


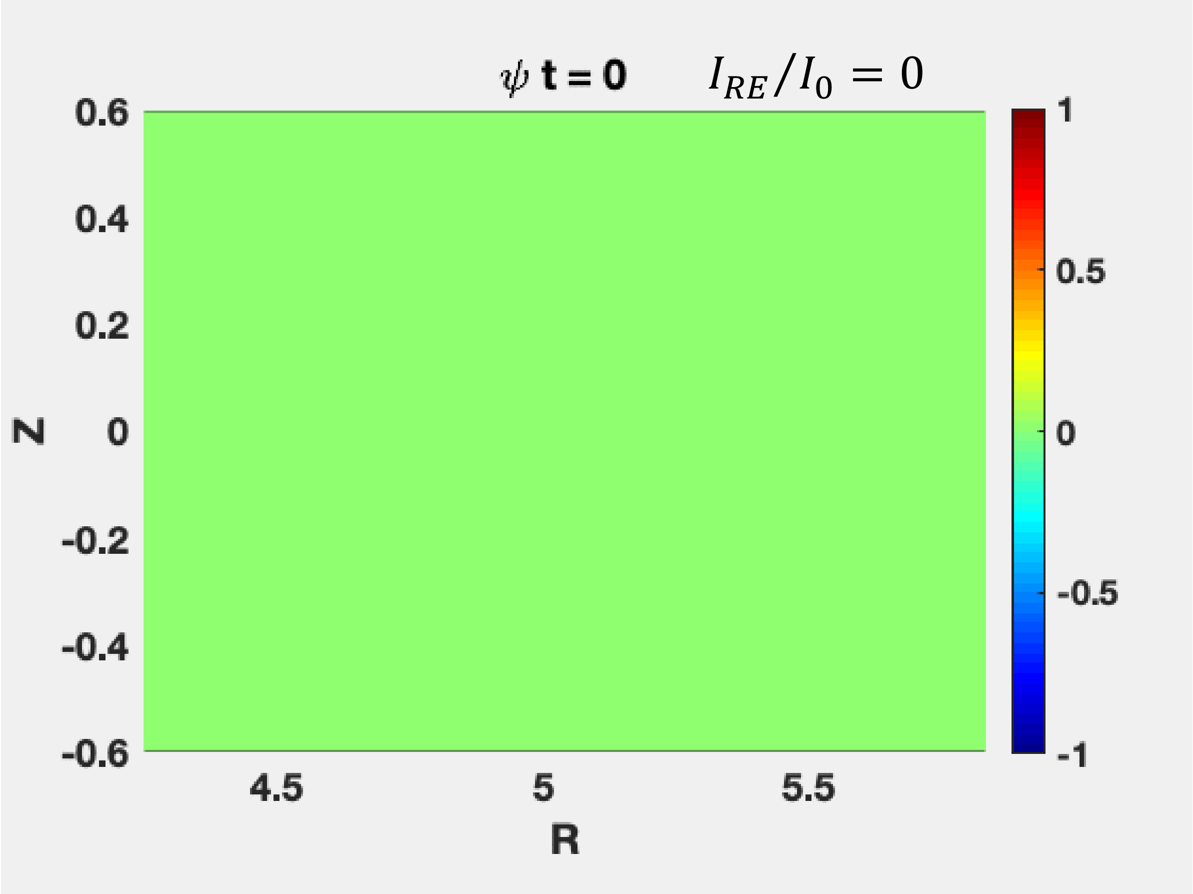
- Parameters of equilibrium

$$\beta_0 = 1.0 \times 10^{-6}$$
$$q_0 = 0.85$$
$$q = q_0 \left[1.0 + \left(\frac{r_{norm}^2}{4}\right)\right]^{1.25}, r_{norm} = \frac{r}{a}$$
$$a = 0.5m$$
$$B_0 = 4.2T$$
$$\eta = 2.0 \times 10^{-5}\ \Omega\mathrm{m}$$
$$n_0 = 1.0 \times 10^{20} m^{-3}$$
$$c = 240 v_A$$
$$N_{elements} = 1 \times 10^4$$
$$\Delta t = \tau_A$$

- In our simulations, we use an adaptive mesh which has increased resolution near the q = 1 rational surface.

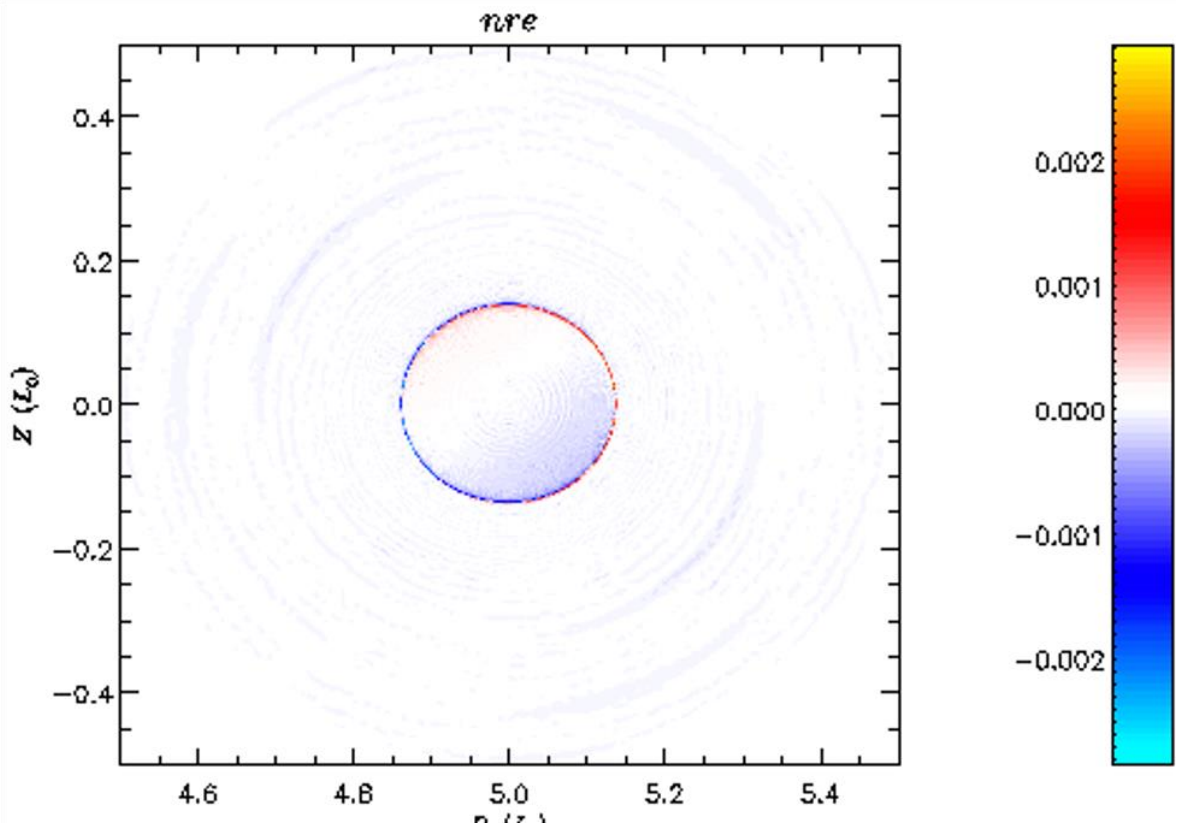
Magnetic island of 1/1 resistive kink mode

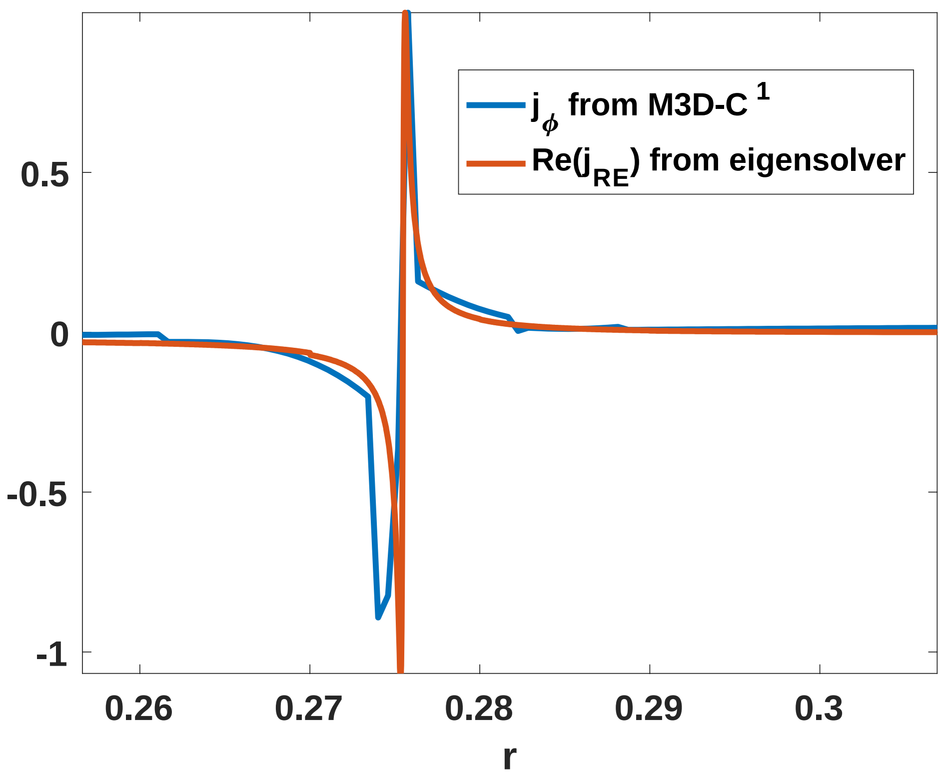




- The mode structure of 1/1 kink mode with RE is similar with 1/1 kink mode with out RE.
- The runaways drive the 1/1 kink mode islands rotate with a constant frequency.

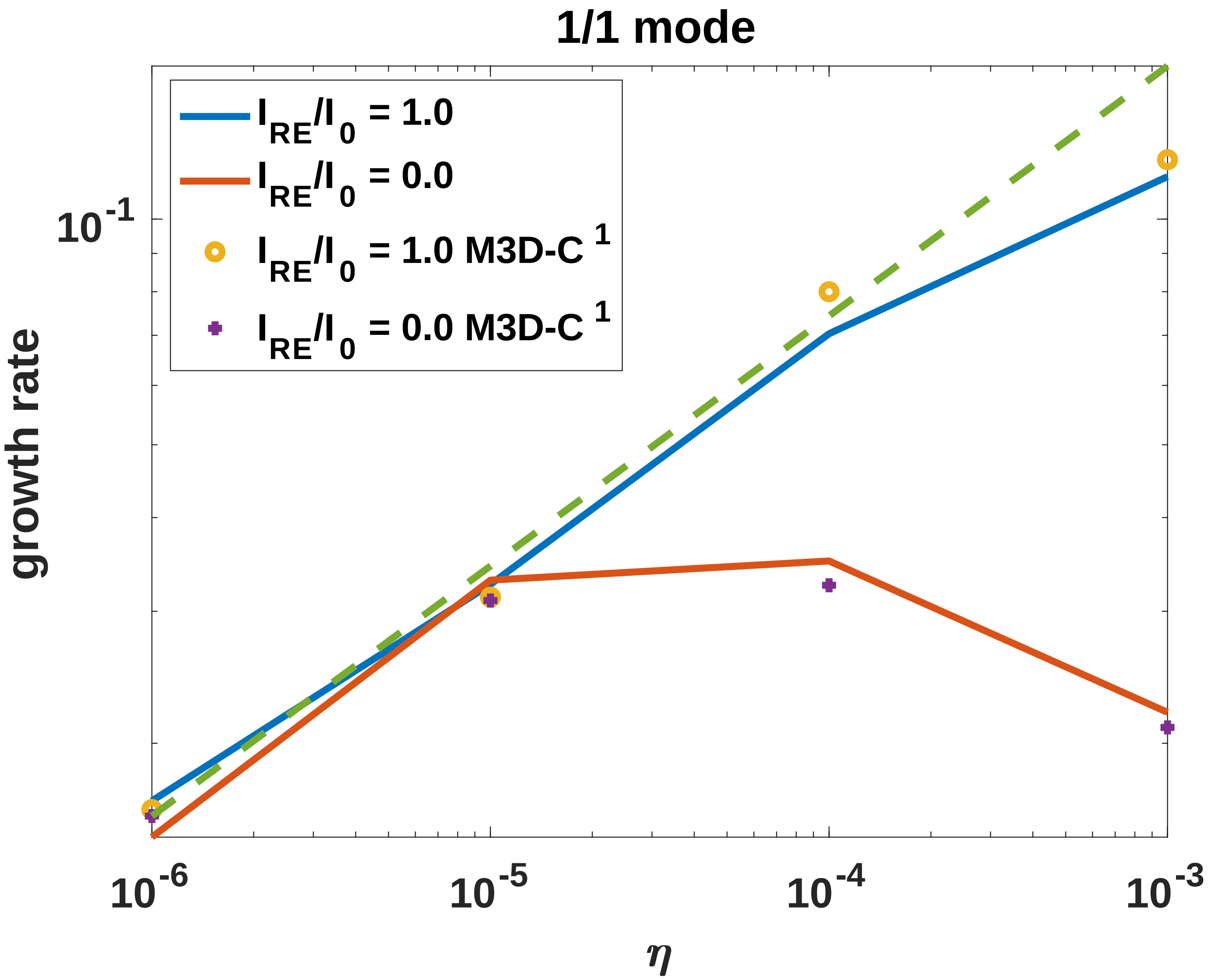
RE density and current perturbation of 1/1 resistive kink mode





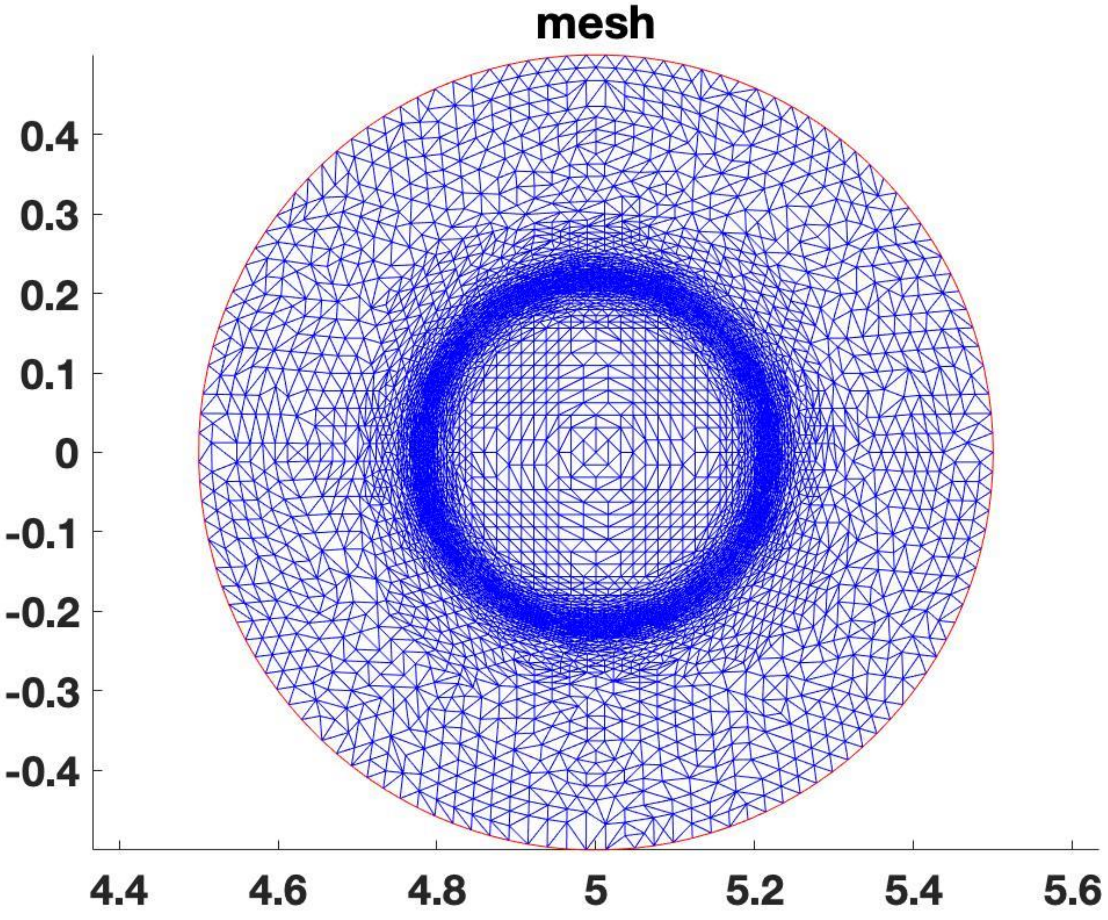
- The perturbed RE density is peaked around the q=1 surface and drives a toroidal current peaked around the rational surface. $dJ_f \sim dJ_{RE} = -\, e dn_{RE} c$
- The structure of RE current result from M3D-C^1 is consistent with eigenvalue solver.

Linear growth rate of 1/1 resistive kink mode with different resistivity



- For low resistivity cases, the growth rate of 1/1 kink mode with and without RE obeys the 1/3 law of resistivity.
- For higher resistivity cases, the runaway current has restrained the resistivity correction of kink mode.
- The result form M3D-C[1] is consistent with eigen solver.

Mesh and basic parameters for 2/1 tearing mode simulation



- Parameters of equilibrium

$$\beta_0 = 1.0 \times 10^{-6}$$
$$q_0 = 1.15$$
$$q = q_0 \left[1.0 + \left(\frac{r_{norm}^2}{5}\right)\right]^{1.2}, r_{norm} = \frac{r}{a}$$
$$a = 0.5m$$
$$B_0 = 4.2T$$
$$\eta = 2.0 \times 10^{-5}\ \Omega\mathrm{m}$$
$$n_0 = 1.0 \times 10^{20} m^{-3}$$
$$c = 240 v_A$$
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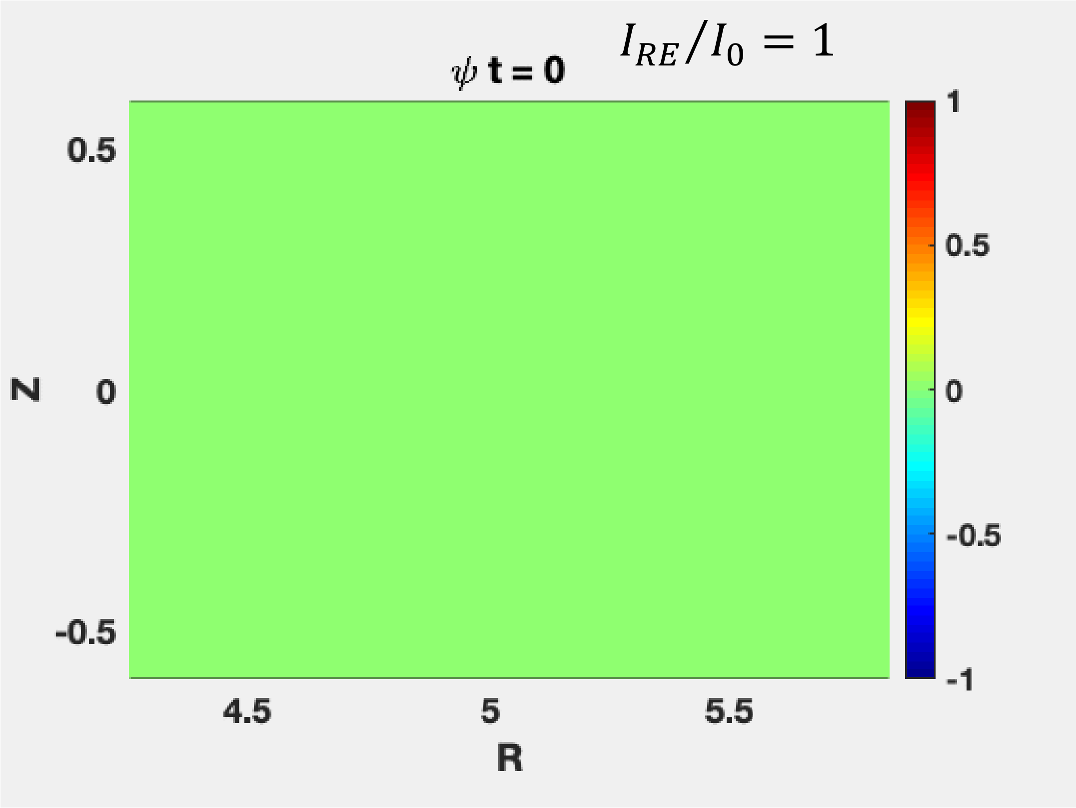

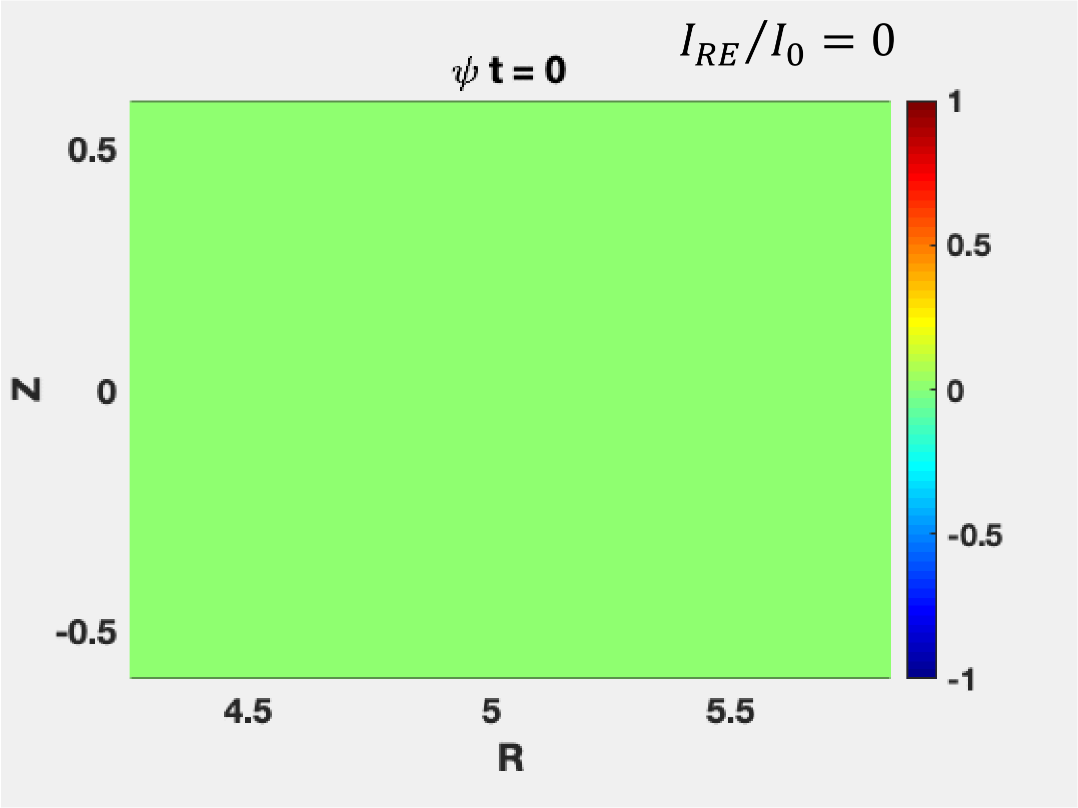

- The mode structure of 2/1 tearing mode with RE is similar with 2/1 tearing mode with out RE.
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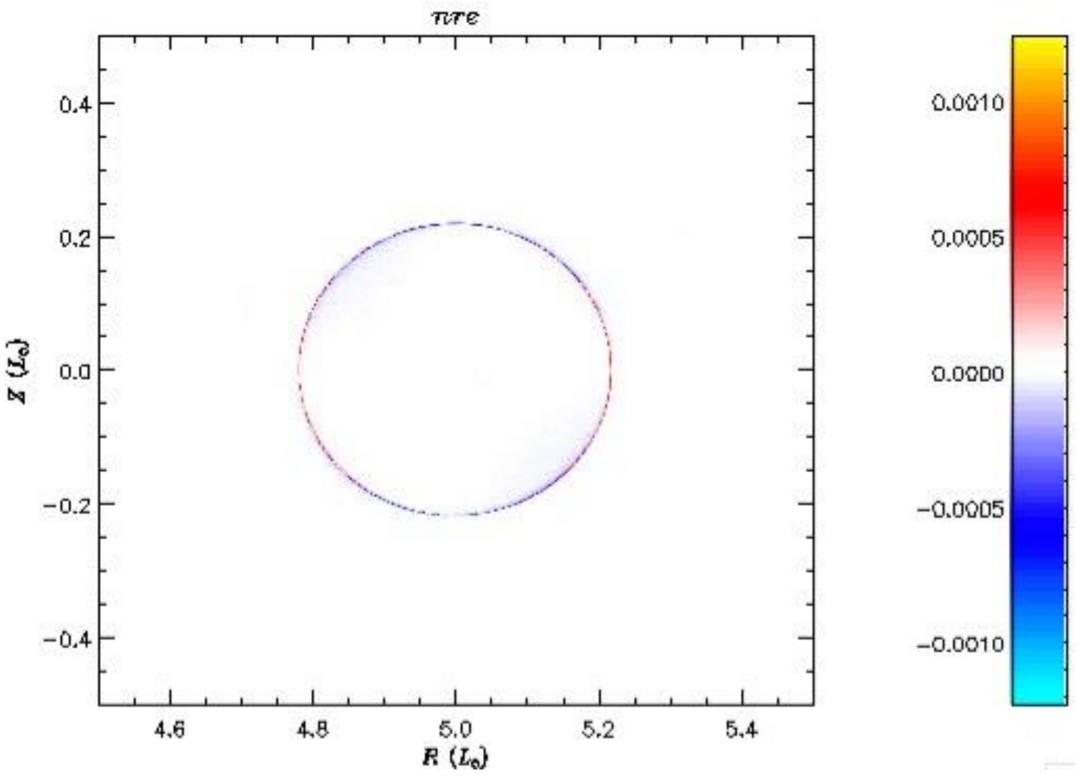
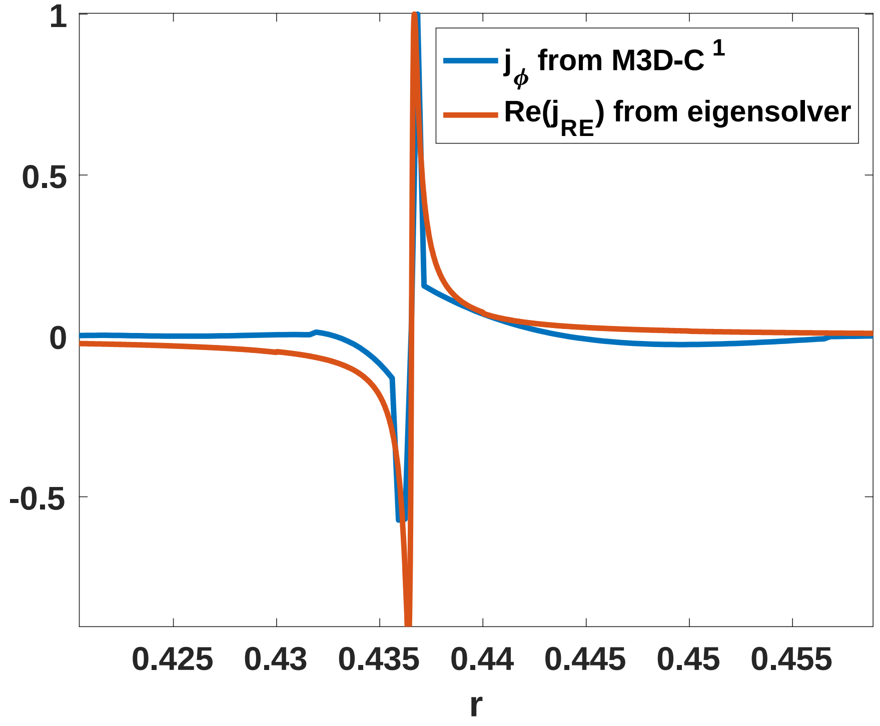
RE density and current perturbation of 1/1 resistive kink mode





- The perturbed RE density is peaked around the q=2 surface and drives a toroidal current peaked around the rational surface. $dJ_f \sim dJ_{RE} = - edn_{RE}c$
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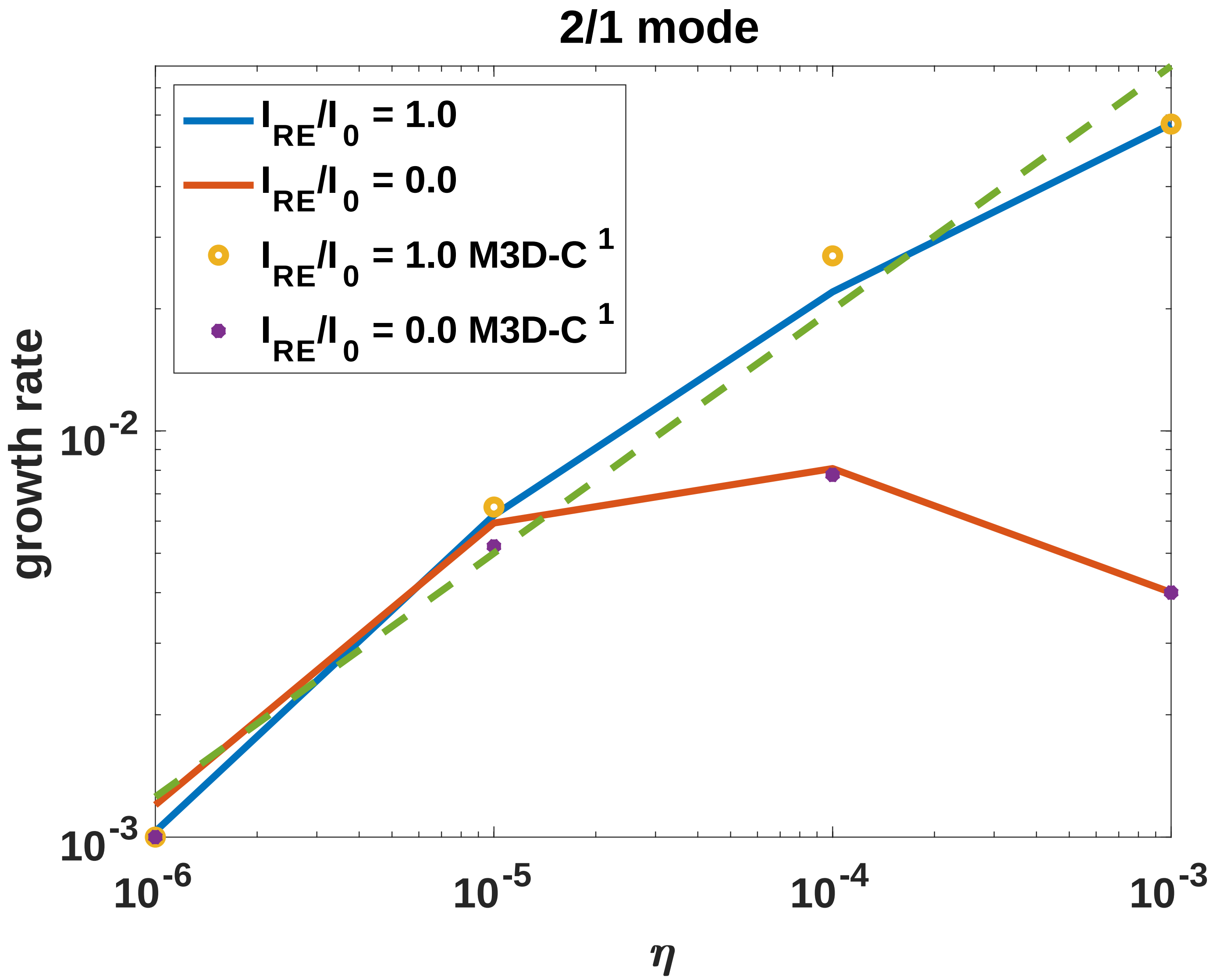
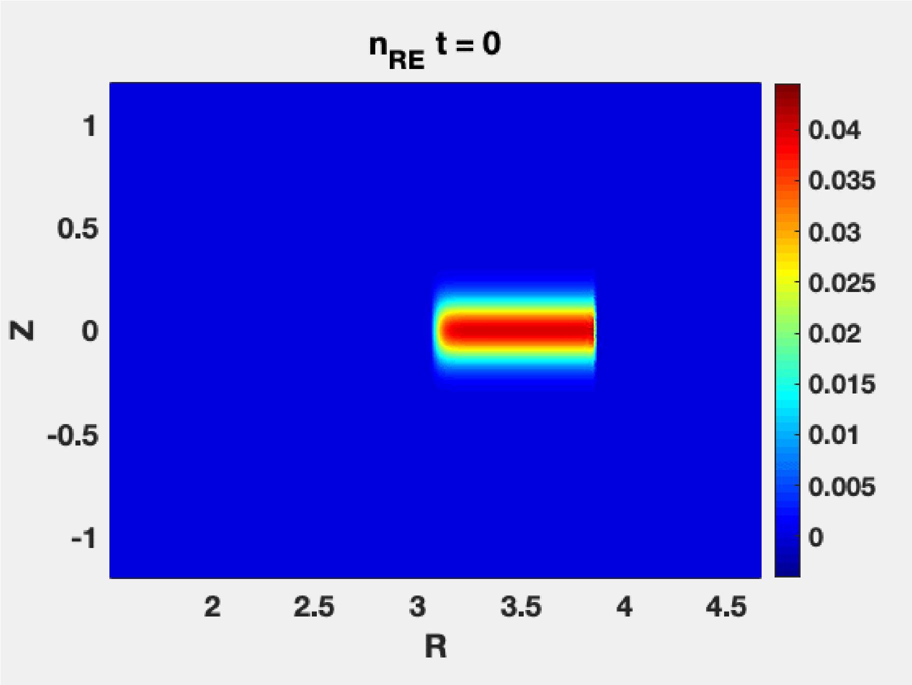


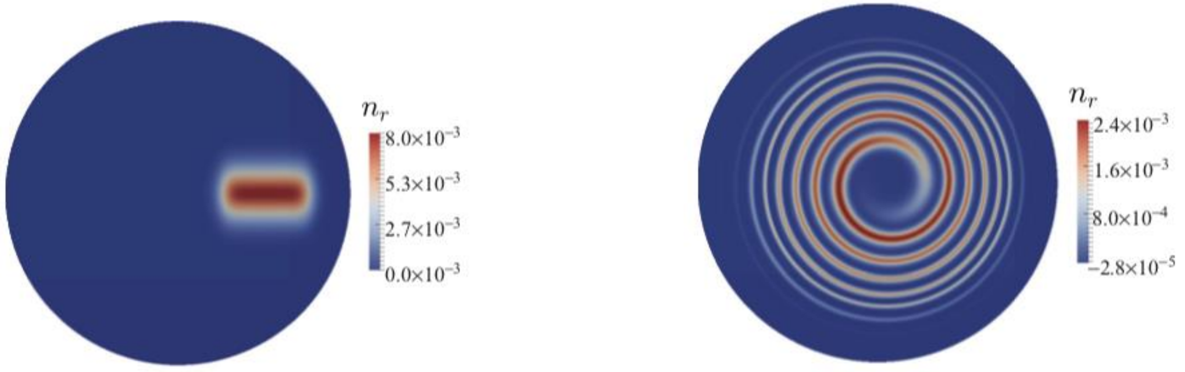
- For low resistivity cases, the growth rate of 2/1 tearing mode with and without RE obeys the 3/5 law of resistivity.
- For higher resistivity cases, the runaway current has restrained the resistivity correction of kink mode.
- The result form M3D-C[1] is consistent with eigen solver.

3. Nonlinear simulations with RE

Nonlinear poloidal convection of RE

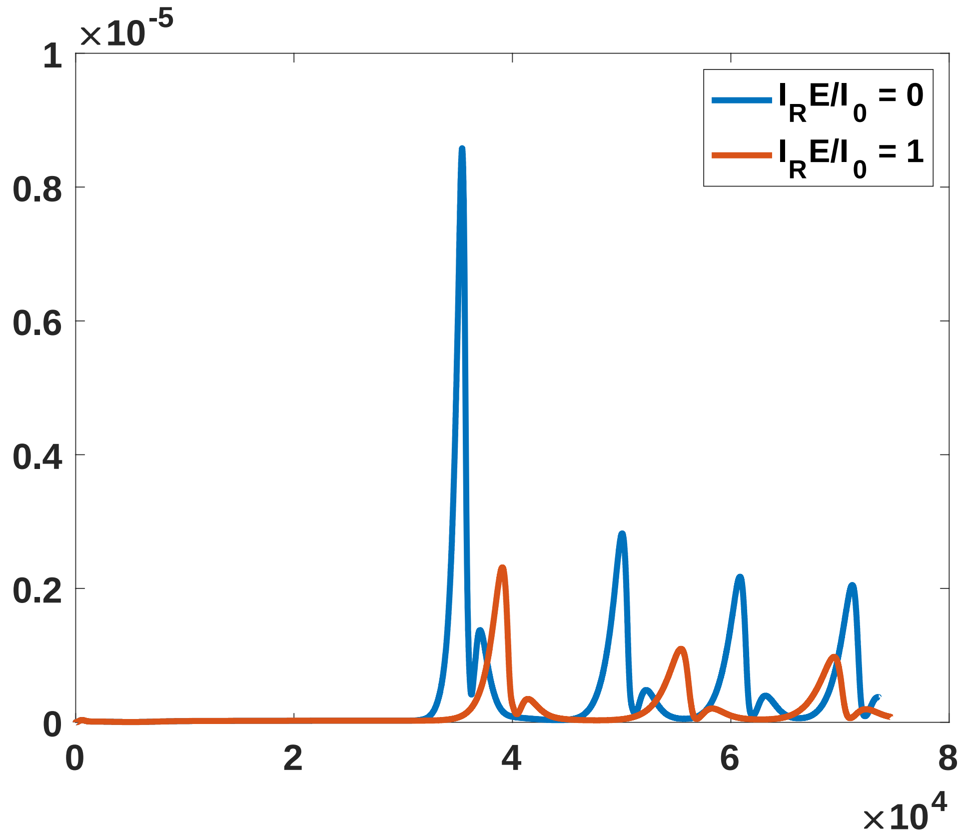
- Circular cross section of ϕ=0
- Initial runaway electron density is similar with Bandaru 2019.
- At the beginning (t < 12 Alfven time) the advection results is similar with JOREC. Then with the diffusion results the runaway density become uniform at theta direction.
- The runaway electron density moves along the magnetic field and has parallel and perpendicular diffusion, so that it become uniform at every magnetic surfaces.

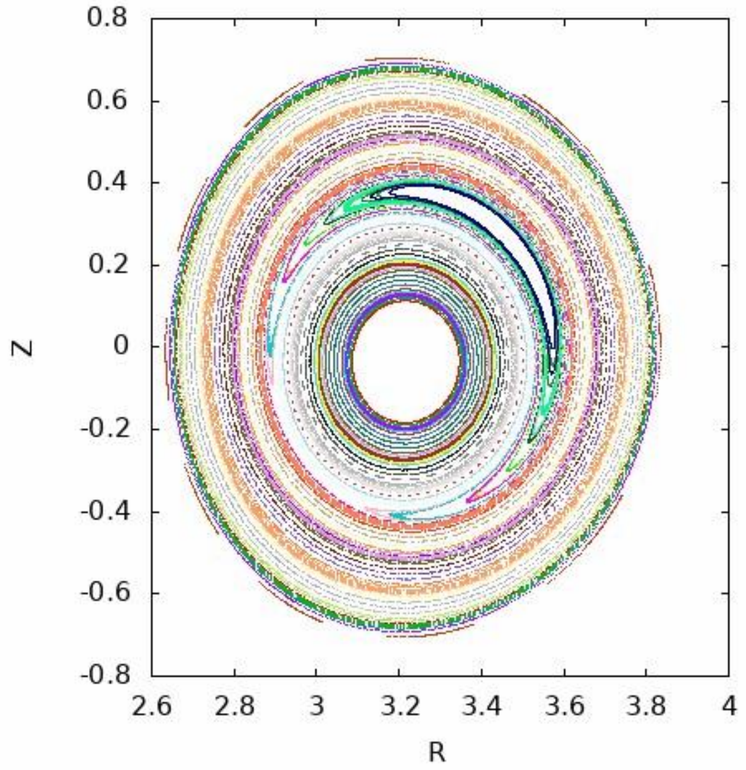




Bandaru 2019

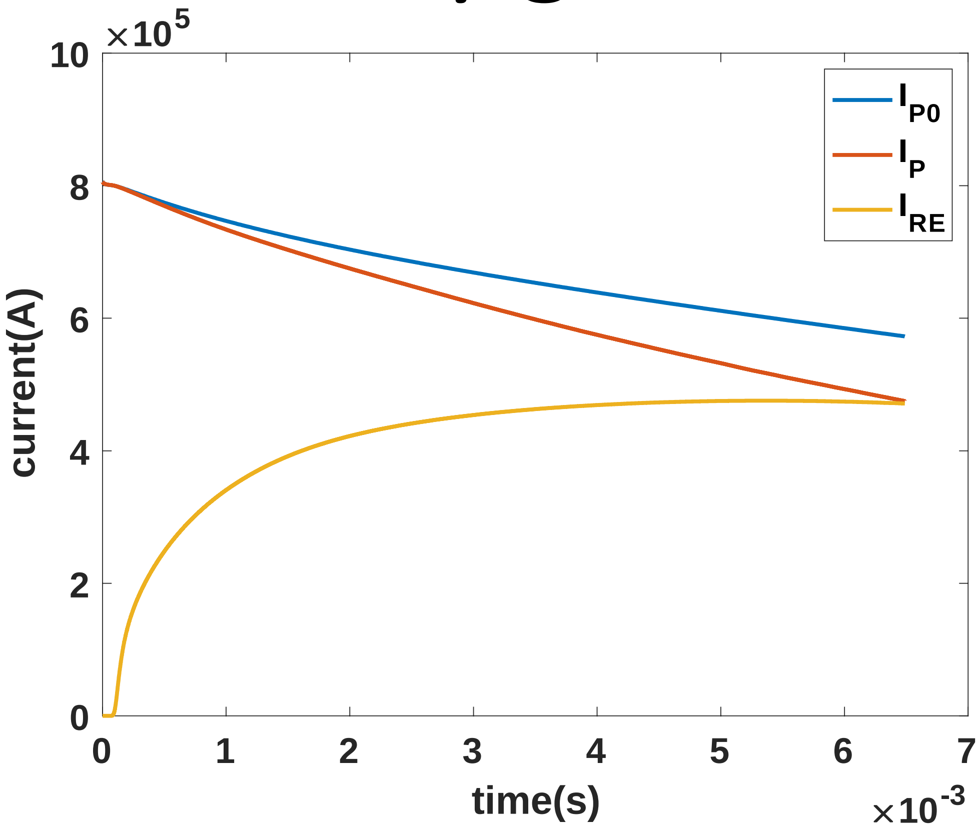
Sawtooth with RE





- The highest energy of sawtooth with runaways is lower than without runaways. And the period time of sawtooth phase with runaways is longer than without runaways. But this result is different than Cai 2015, which the sawtooth disappear after 1 sawtooth phase. We will find out the reason of the difference and do more further studies.

Runaway generation during disruption



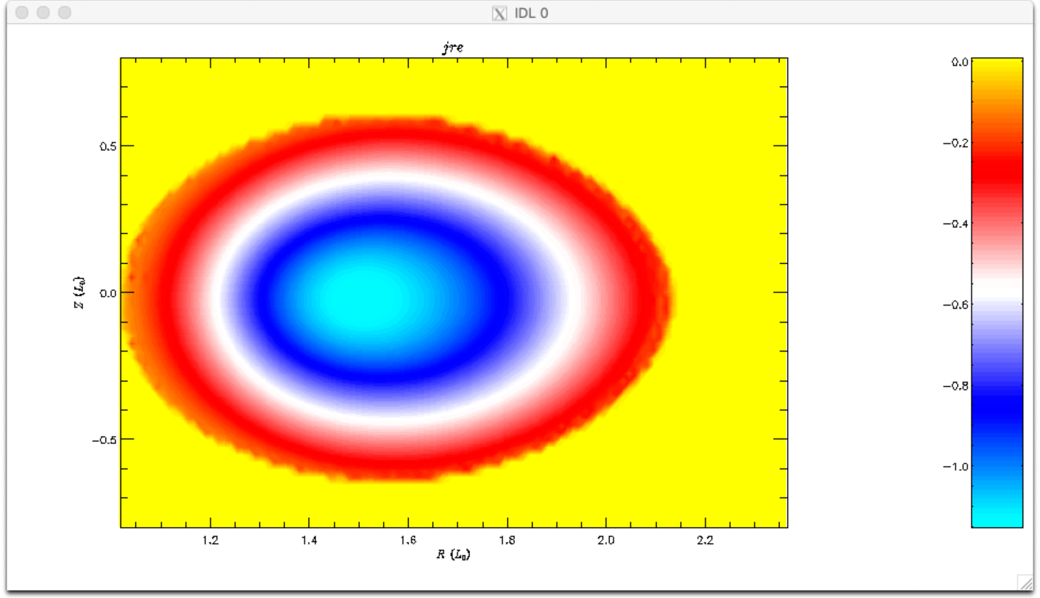
- I_P0 is the plasma current without runaways
- I_P is the plasma current with runaways

- Parameters: $\beta_0 = 0.15$

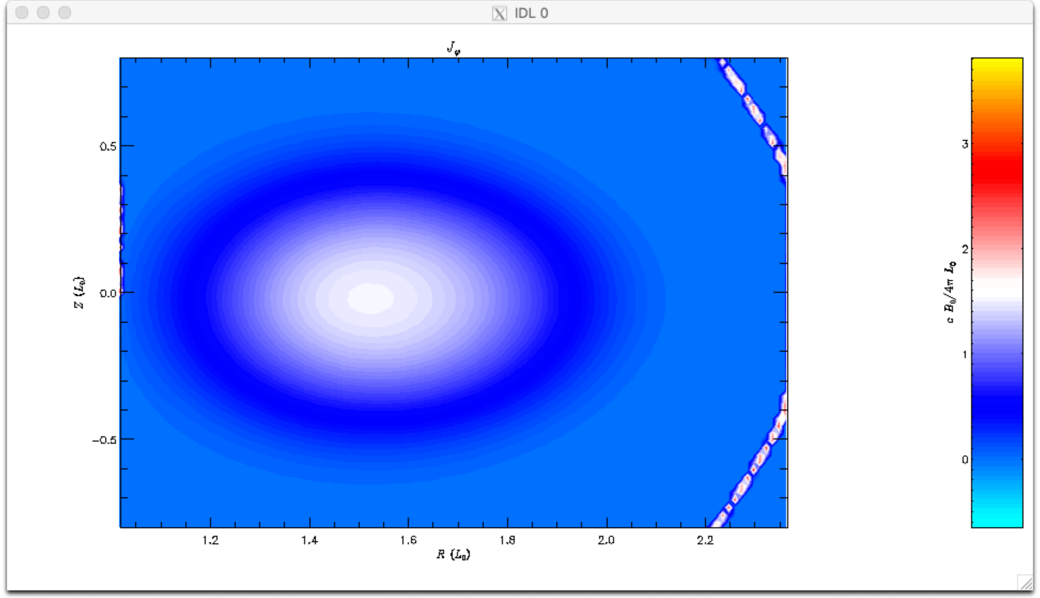
$$a = 0.65m$$
$$\mathrm{R} = 1.7m$$
$$B_0 = 1.9T$$
$$\eta = 1.0 \times 10^{-4}$$
$$n_0 = 1.0 \times 10^{20} m^{-3}$$
$$c = 150 v_A$$
$$N_{elements} = 12261$$
$$\Delta t = 0.5 \tau_A$$

- The plasma current dropped faster with runaways than without runaways.

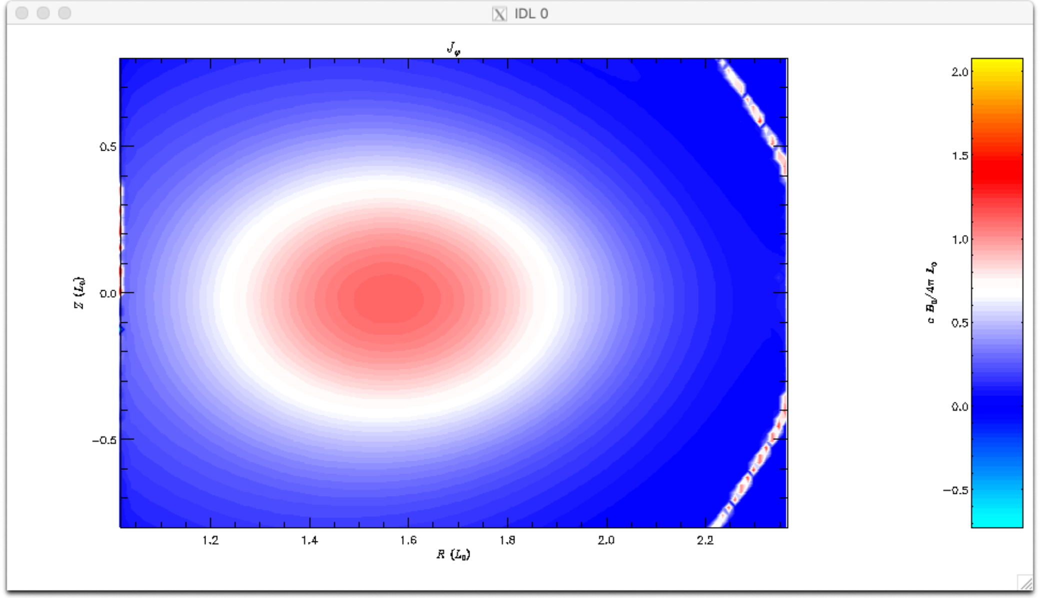
Runaway current density (t=6ms)



Plasma current with runaways (t=6ms)



Plasma current without runaways (t=6ms)



- The runaway current density structure is similar with the plasma current density when it saturates at about 6ms.
- The structure of the toroidal current density are almost the same with and without runaways. But the quantity is a little bit lower than with runaways.

4. Summary and future works

- The perturbed toroidal current of 1/1 and 2/1 mode will be peaked around the rational surface by the RE current effects. The RE current causes the 1/1 and 2/1 mode to rotate with time, and if the runaway speed is large enough, it does not affect the growth rate and real frequency when it increases. The scale length of the runaway current becomes smaller with higher runaway speed.

- The growth rate of 1/1 and 2/1 mode with RE is consistent w M3D-C^1 results. And the runaway electrons have restrained the resistive correction effect in high resistivity cases. (Chen et. al. 2020)

- The runaway electrons have convection, parallel diffusion and perpendicular diffusion in nonlinear phase and the runaways restrained the sawtooth.

- We have already developed the source term for runaways in M3D-C1 to study the runaway generation in experiment.

- More runaway sawtooth and generation studies will be carried out in future.