Nonlinear, Extended-Magnetohydrodynamic Modeling of Disruption Mitigation

by

Brendan C. Lyons[1],

J. McClenaghan[1], C.C. Kim[2], S.C. Jardin[3], N.M. Ferraro[3], C Akçay[1], N.W. Eidietis[1], L.L. Lao[1], N. Hawkes[4], G. Szepesi[4], and JET contributors[5]

[1] General Atomics
[2] SLS2 Consulting
[3] Princeton Plasma Physics Laboratory
[4] Culham Centre for Fusion Energy
[5] J. Mailloux, Nucl. Fusion (2022) https://doi.org/10.1088/1741-4326/ac47b4

Presented at the

2022 Sherwood Fusion Theory Conference
Santa Rosa, California, USA
April 4th, 2022

Modeling of Disruption Dynamics and Mitigation Requires a Multiphysics Model

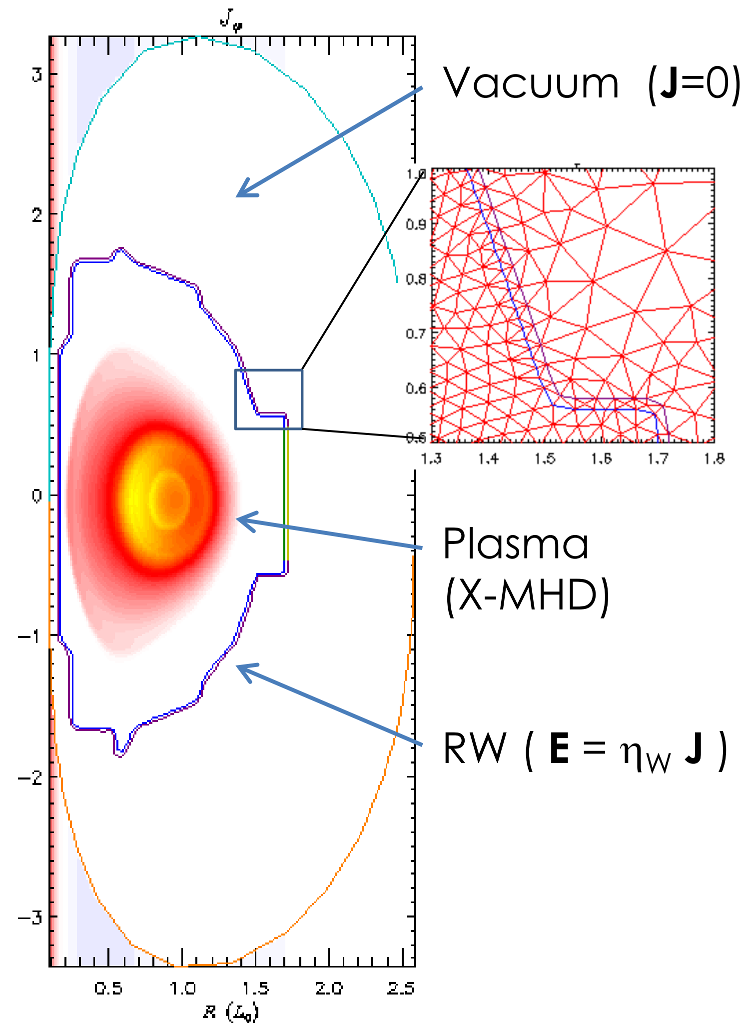
- **Disruptions pose a risk of damage to future tokamaks, necessitating robust mitigation techniques**
- **Most promising method uses pellet injection of impurities to radiate stored energy**
- **Simulations, validated against mitigation experiments, are required to project techniques to future devices**
- **Integrated model is required to capture all relevant physics**
  - Magnetohydrodynamics (MHD) for macroscopic evolution of disruption dynamics
  - Atomic physics for ionization and radiation from injected impurities
  - Drift-kinetics for phase-space evolution of runaway electron population

M3D-C1 Code Overview

GENERAL ATOMICS

M3D-C1* Solves the Extended-MHD Equations

- **Three-dimensional toroidal geometry**
- **Full (not reduced) MHD**
- **Solves for potential and stream-function fields for $\vec{A}$ & $\vec{v}$ ($\nabla \cdot \vec{B} = 0$ intrinsically)**
- **Includes resistivity, density diffusivity, viscosity, & thermal conductivity**
- **Two-fluid effects (optional)**
- **3D high-order finite elements**
  - Unstructured, triangular mesh in poloidal plane
  - Structured toroidally, but can pack planes
- **Can solve with finite-thickness resistive wall in domain****



***S. C. Jardin, et al., Comput. Sci. Discovery 5, 014002 (2012).**
****N.M. Ferraro, et al. ,Phys Plasma23 056114 (2016).**

M3D-C1 Solves the Extended-MHD Equations

Blue terms are 2-fluid

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\mathbf{V}) = \nabla \bullet D_n \nabla n + S_n$$

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla \Phi, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla_\perp \bullet \frac{1}{R^2} \nabla \Phi = -\nabla_\perp \bullet \frac{1}{R^2} \mathbf{E}$$

$$nM_i \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \bullet \nabla \mathbf{V} \right) + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \bullet \mathbf{\Pi}_i + \mathbf{S}_m$$

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne} \left(\mathbf{R}_c + \mathbf{J} \times \mathbf{B} - \nabla p_e - \nabla \bullet \mathbf{\Pi}_e \right) - \frac{m_e}{e} \left(\frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \bullet \nabla \mathbf{V}_e \right) + \mathbf{S}_{CD}$$

$$\frac{3}{2} \left[\frac{\partial p_e}{\partial t} + \nabla \bullet \left(p_e \mathbf{V} \right) \right] = -p_e \nabla \bullet \mathbf{V} + \frac{\mathbf{J}}{ne} \bullet \left[\frac{3}{2} \nabla p_e - \frac{5}{2} \frac{p_e}{n} \nabla n + \mathbf{R}_c \right] + \nabla \left(\frac{\mathbf{J}}{ne} \right) : \mathbf{\Pi}_e - \nabla \bullet \mathbf{q}_e + Q_\Delta + S_{eE}$$

$$\frac{3}{2} \left[\frac{\partial p_i}{\partial t} + \nabla \bullet \left(p_i \mathbf{V} \right) \right] = -p_i \nabla \bullet \mathbf{V} - \mathbf{\Pi}_i : \nabla \mathbf{V} - \nabla \bullet \mathbf{q}_i - Q_\Delta + S_{iE}$$

$$\mathbf{V}_e = \mathbf{V}_i - \mathbf{J} / ne$$

$$\mathbf{R}_c = \eta ne \mathbf{J}, \qquad \mathbf{\Pi}_i = -\mu \left[\nabla \mathbf{V} + \nabla \mathbf{V}^\dagger \right] - 2(\mu_c - \mu)(\nabla \bullet \mathbf{V})\mathbf{I} + \mathbf{\Pi}_i^{GV}$$

$$\mathbf{q}_{e,i} = -\kappa_{e,i} \nabla T_{e,i} - \kappa_\| \nabla_\| T_{e,i}$$

$$\mathbf{\Pi}_e = (\mathbf{B} / B^2) \nabla \bullet \left[\lambda_h \nabla \left(\mathbf{J} \bullet \mathbf{B} / B^2 \right) \right], \qquad Q_\Delta = 3 m_e (p_i - p_e) / \left(M_i \tau_e \right)$$

KPRAD* Provides Needed Atomic Physics Information

- **KPRAD solves for impurity-plasma interaction in low-density, coronal model**
  - N.B. *not* coronal equilibrium
  - Based on ADPAK rate coefficients
  - Impurity charge states and electron density evolve according to ionization and recombination

$$\frac{\partial n_z}{\partial t} + \nabla \cdot (n_z \mathbf{v}) = \nabla \cdot (D \nabla n_z) + \mathcal{I}_{z-1} n_{z-1} - (\mathcal{I}_z + \mathcal{R}_z)\, n_z + \mathcal{R}_{z+1} n_{z+1} + \mathcal{S}_z$$

  - Thermal energy lost from plasma due to
    - Ionization
    - Line radiation
    - Bremsstrahlung radiation
    - Recombination radiation
- **Subcycled much faster than typical MHD time steps**

***D.G. Whyte, et al., Proc. of the 24th Euro. Conf. on Controlled Fusion and Plasma Physics, Berchtesgaden, Germany, 1997, Vol. 21A, p. 1137.**

KPRAD Couples* to the M3D-C1 Temperature Equation(s)

- **Two temperature equations (electron & all-ions)**
  - Dilution cooling of ions and electrons
  - Electrons lose energy to ionization and radiation
  - Main ions cool on electrons

$$n_e\left[\frac{\partial T_e}{\partial t}+\mathbf{v}\cdot\nabla T_e+(\Gamma-1)\,T_e\nabla\cdot\mathbf{v}\right]+\sigma_e T_e=(\Gamma-1)\left[\eta J^2-\nabla\cdot\mathbf{q}_e-\mathcal{P}_{rad}+Q_{ei}-\Pi_e:\nabla\mathbf{v}\right]$$

$$n_*\left[\frac{\partial T_i}{\partial t}+\mathbf{v}\cdot\nabla T_i+(\Gamma-1)\,T_i\nabla\cdot\mathbf{v}\right]+\sigma_* T_i=(\Gamma-1)\left[-\nabla\cdot\mathbf{q}_*-Q_{ei}-\Pi_*:\nabla\mathbf{v}+\frac{1}{2}\varpi_* v^2\right]$$

- **Single temperature equation**
  - Evolves sum over all species
  - T_e/T_i constant throughout time, implicitly assuming
    - Instantaneous thermal equilibration
    - Split of losses between species evolves as pressure ratio changes

***N.M. Ferraro et al. Nucl. Fusion 59 016001 (2019).**

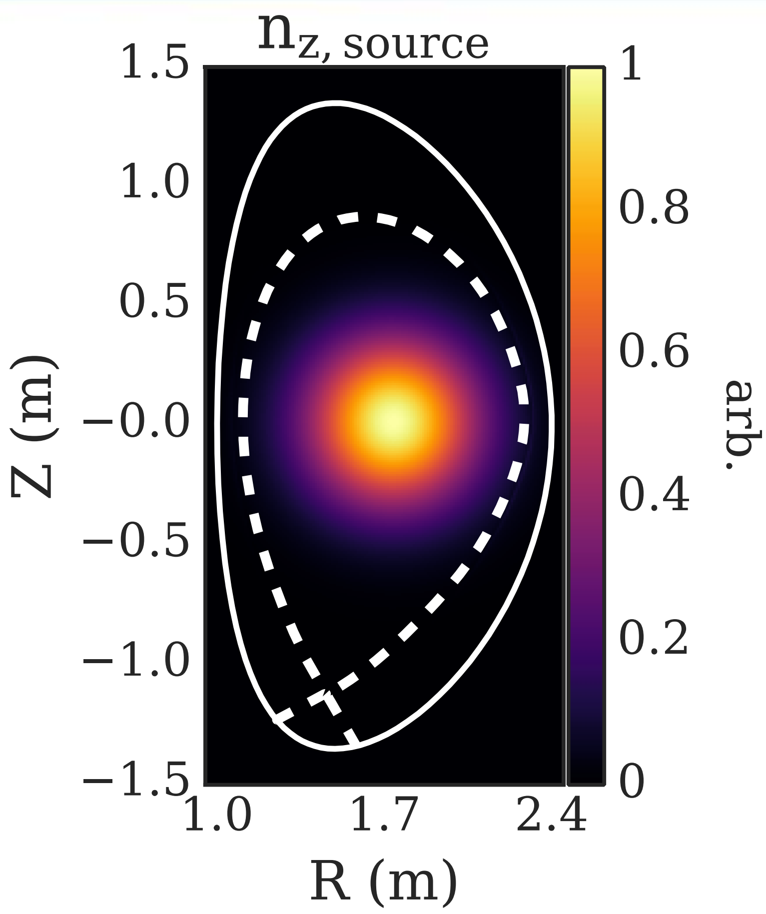
Verification Benchmarks of NIMROD & M3D-C1

Axisymmetric Benchmark Successful for Fast Impurity Injection in DIII-D Core

- **Four cases solved by both M3D-C1 and NIMROD***
  - Lyons et al., PPCF 61, 064001 (2019)
  - Shown here: argon with Spitzer resistivity
- **Simulation setup**
  - DIII-D shot 137611 @ 1950 ms
  - 2D, nonlinear, single-fluid
  - Fixed boundary
- **Continuous neutral impurity deposition**
  - No impurities to start
  - Gaussian source

$$\frac{dn_z}{dt} = \nu \frac{R_0}{R} \exp\left[-\frac{(R-R_0)^2 + (Z-Z_0)^2}{2\delta^2} \right]$$

  - Injection rate ~1 mm Ne/Ar per ms

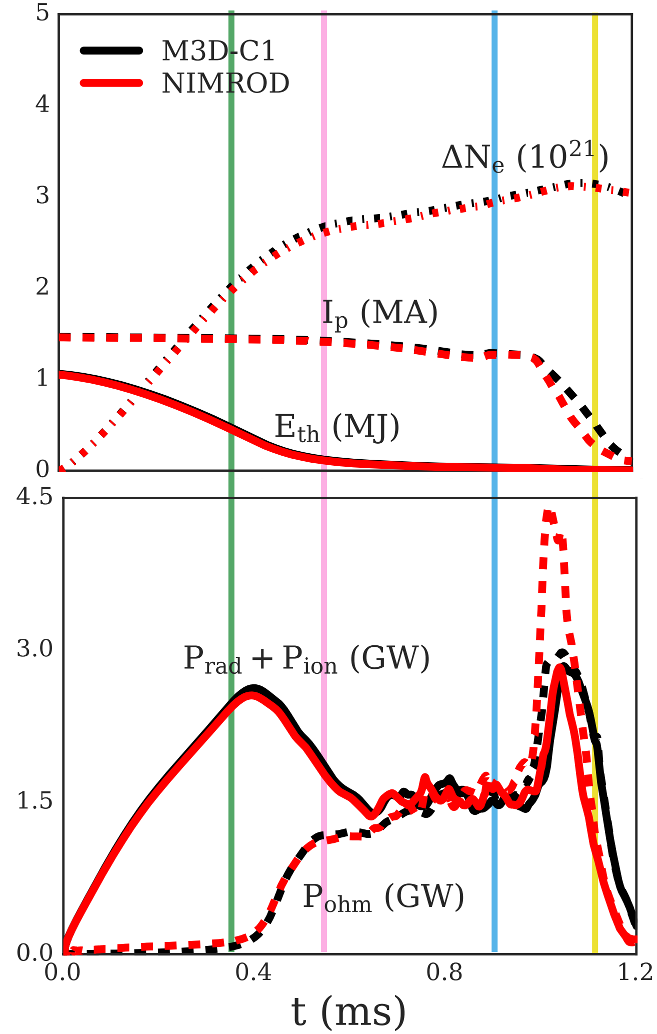


*C. R. Sovinec et al., J. Comput. Phys. 195, 355 (2004).
C. Sovinec & J. King, J. Comput. Phys. 229, 5803 (2010).

Excellent Agreement Found Between Codes in 2D

- **Quantitative agreement during thermal quench (TQ)**
- **Qualitative agreement during current quench (CQ)**
- **Low temperature in core causes resistivity to rise**
  - P_{ohm} balances P_{loss}
  - Current drops more rapidly
- **Current quench caused by contact with boundary**
- **Peak loss power when temperature on-axis falls near-zero**

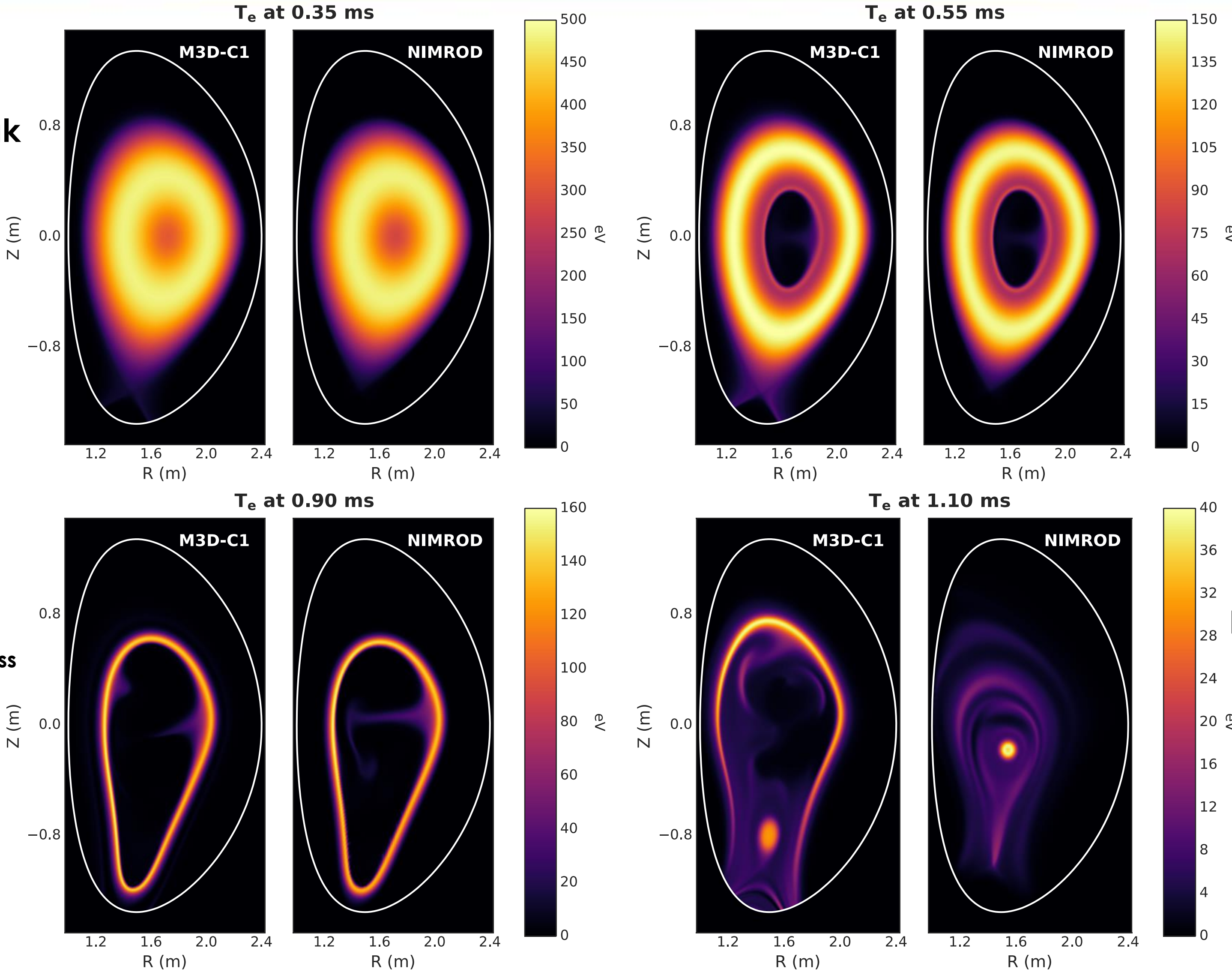
Argon injection



Impurities Induce Inside-Out Thermal Quench with Core Turbulence

Before peak radiation

After ohmic heating rise

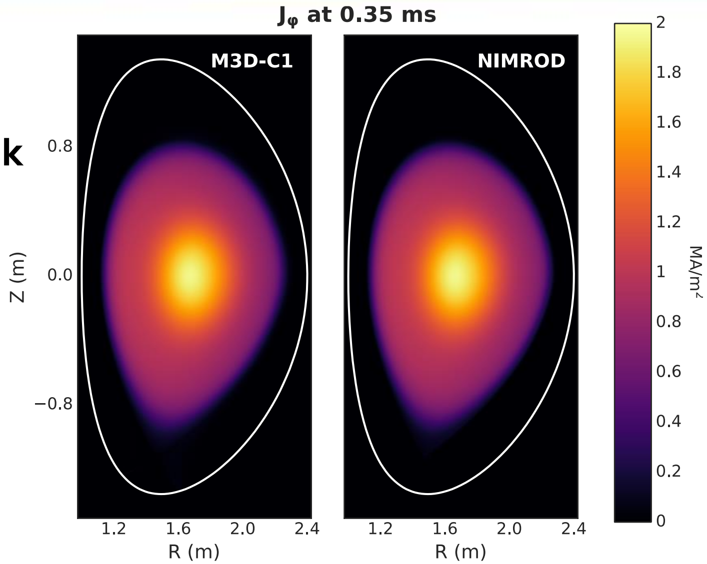


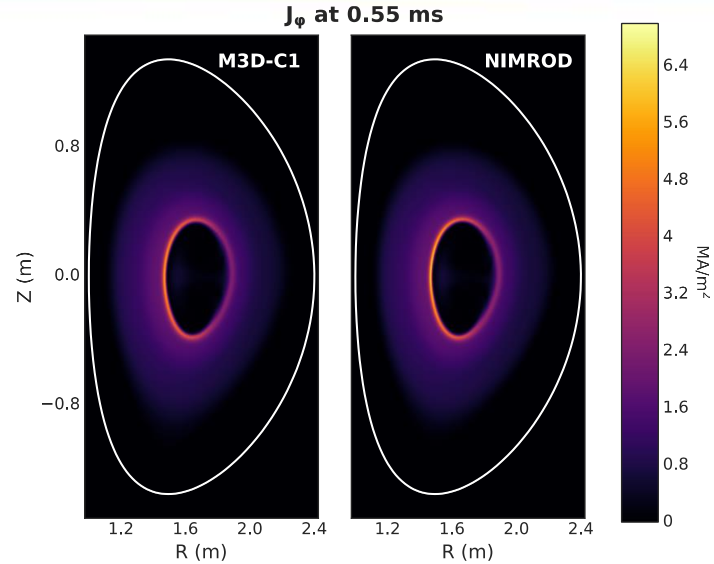
P_{ohm} = P_{loss}

Rapid current quench

Current Localizes to Thin, Expanding Shell that Contacts Domain Boundary

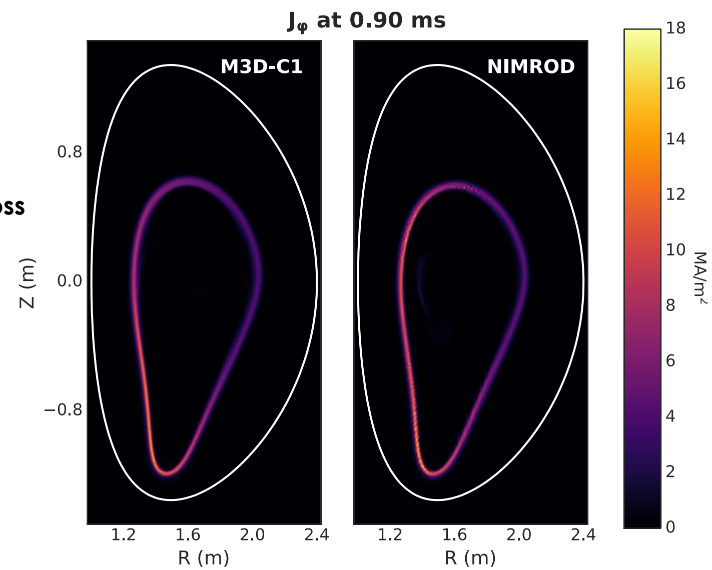
Before peak radiation

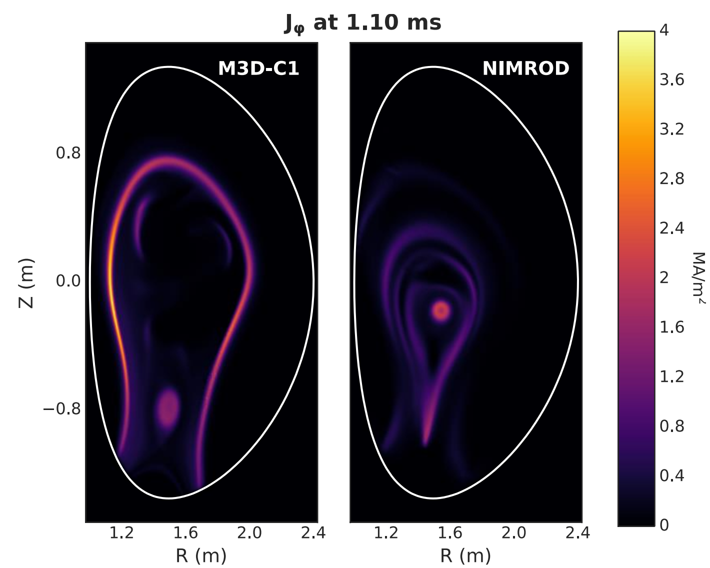




After ohmic heating rise

P_{ohm} = P_{loss}





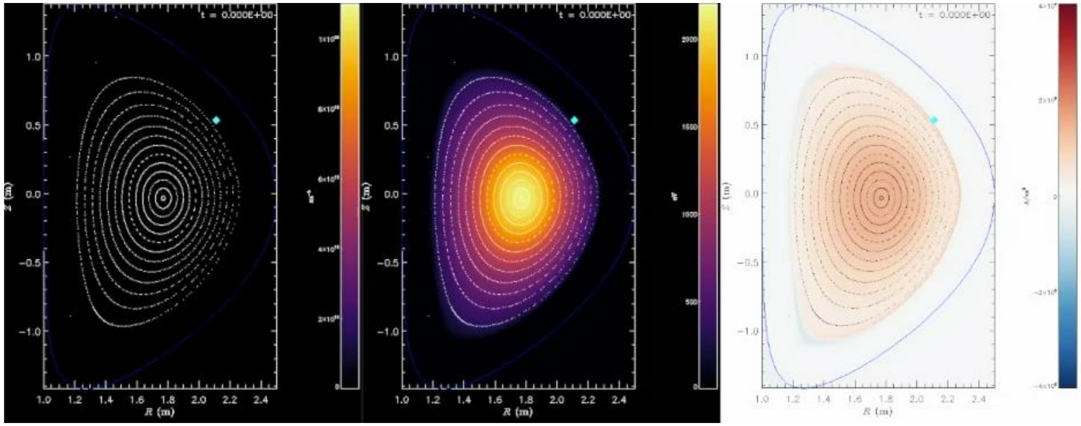
Rapid current quench

3D, Nonlinear Benchmark Between M3D-C1 & NIMROD for Realistic, Injected Pellet is Well-Underway

- **3D nonlinear MHD**
  - Fixed boundary
  - Single-temperature equation
- **Pellet/deposition parameters**
  - 3 mm radius, pure neon
  - 5 cm poloidal and 2.4 m toroidal half-width
  - 200 m/s with realistic trajectory
  - Ablation by local electron density and temperature according to model by Parks
- **Work has motivated code development and provided insight into SPI physics**

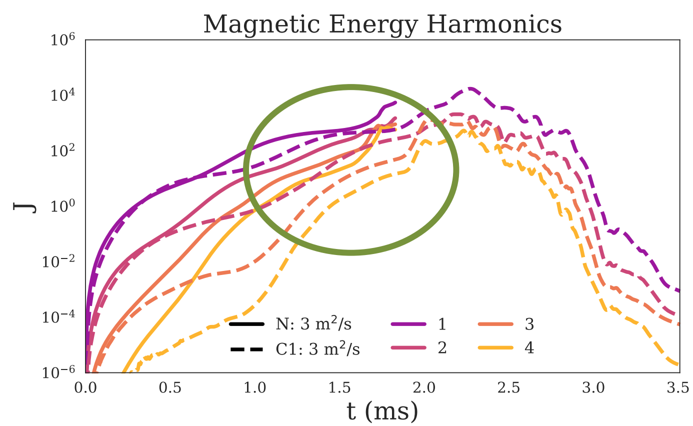
<u>M3D-C1 Modeling of DIII-D 160606 @ 2990 ms: 0.7 MJ, 1.28 MA</u>

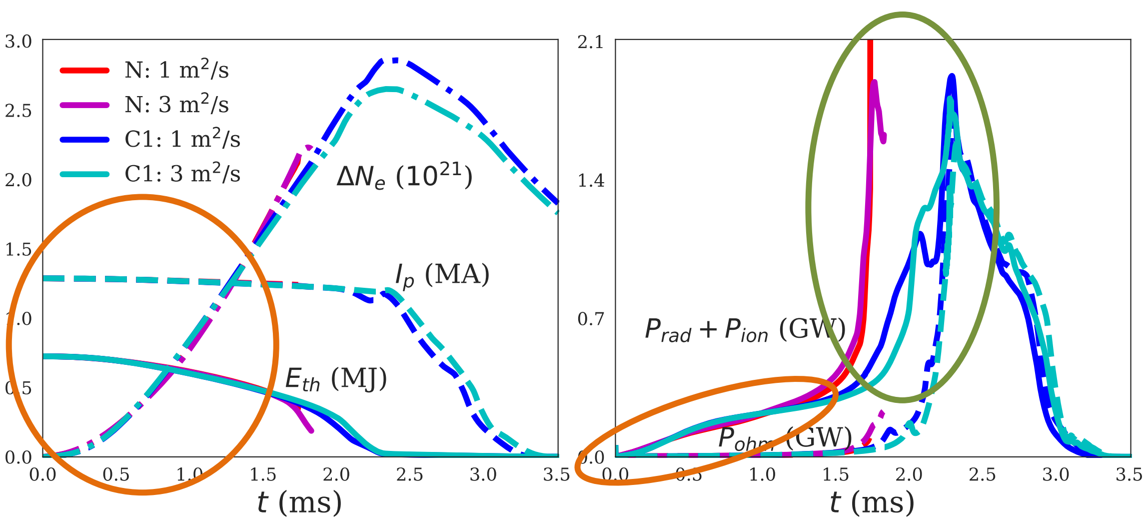
Impurity Density | **Electron Temperature** | **Toroidal Current Density**

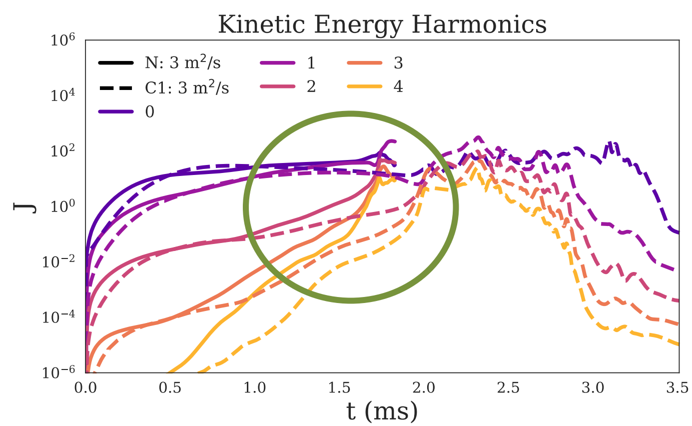


M3D-C1 & NIMROD Differ in Timing of Instability Onset

- **Early, radiation driven thermal quench in good agreement**
- **NIMROD shows earlier spike in radiation, driven by earlier MHD instability onset**
- **M3D-C1 observes stabilization from density diffusivity**

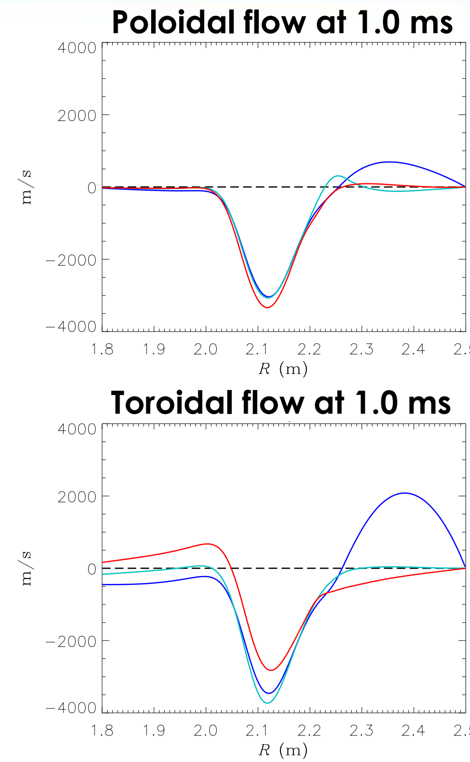




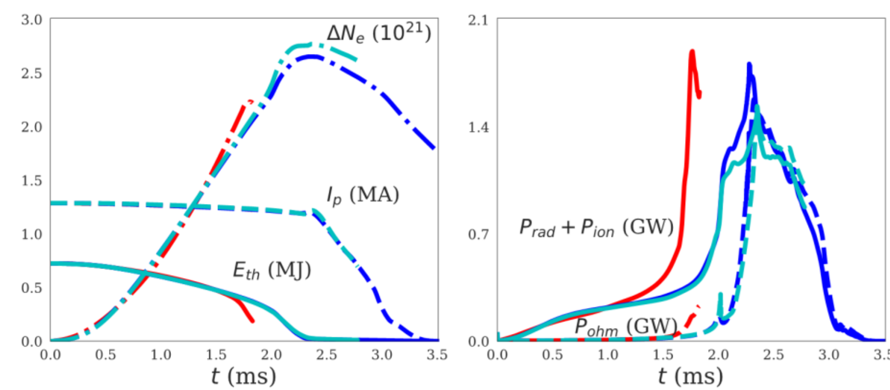


Difference in 3D Benchmark Possibly Caused by Flow Discrepancy due to M3D-C1 Boundary Condition

- **Flow differs even before time traces diverge, especially in open-field-line region**
- **Caused by M3D-C1 implementation of normal/poloidal no-flow BC**
  - M3D-C1 uses potential formulation
    $$\vec{u} = R^2 \nabla U \times \nabla\varphi + R^2 \omega \nabla\varphi + \frac{1}{R^2} \nabla_\perp \chi$$
  - BCs should be:
    - No poloidal: $R\frac{\partial U}{\partial n} + \frac{1}{R^2}\frac{\partial \chi}{\partial \tau} = 0$
    - No normal: $-R\frac{\partial U}{\partial \tau} + \frac{1}{R^2}\frac{\partial \chi}{\partial n} = 0$
  - Instead using
    $\partial U/\partial n = 0 \quad \text{and} \quad \chi = 0$
    $U = 0 \quad \text{and} \quad \partial\chi/\partial n = 0$
    which over-constrains solution
- **New BCs implemented in M3D-C1**
  - Using only new no-poloidal gives better flow agreement, but does not change 3D benchmark results
  - Maybe needs both, but that's currently numerically unstable



NIMROD
C1: Old no-pol.
C1: New no-pol.



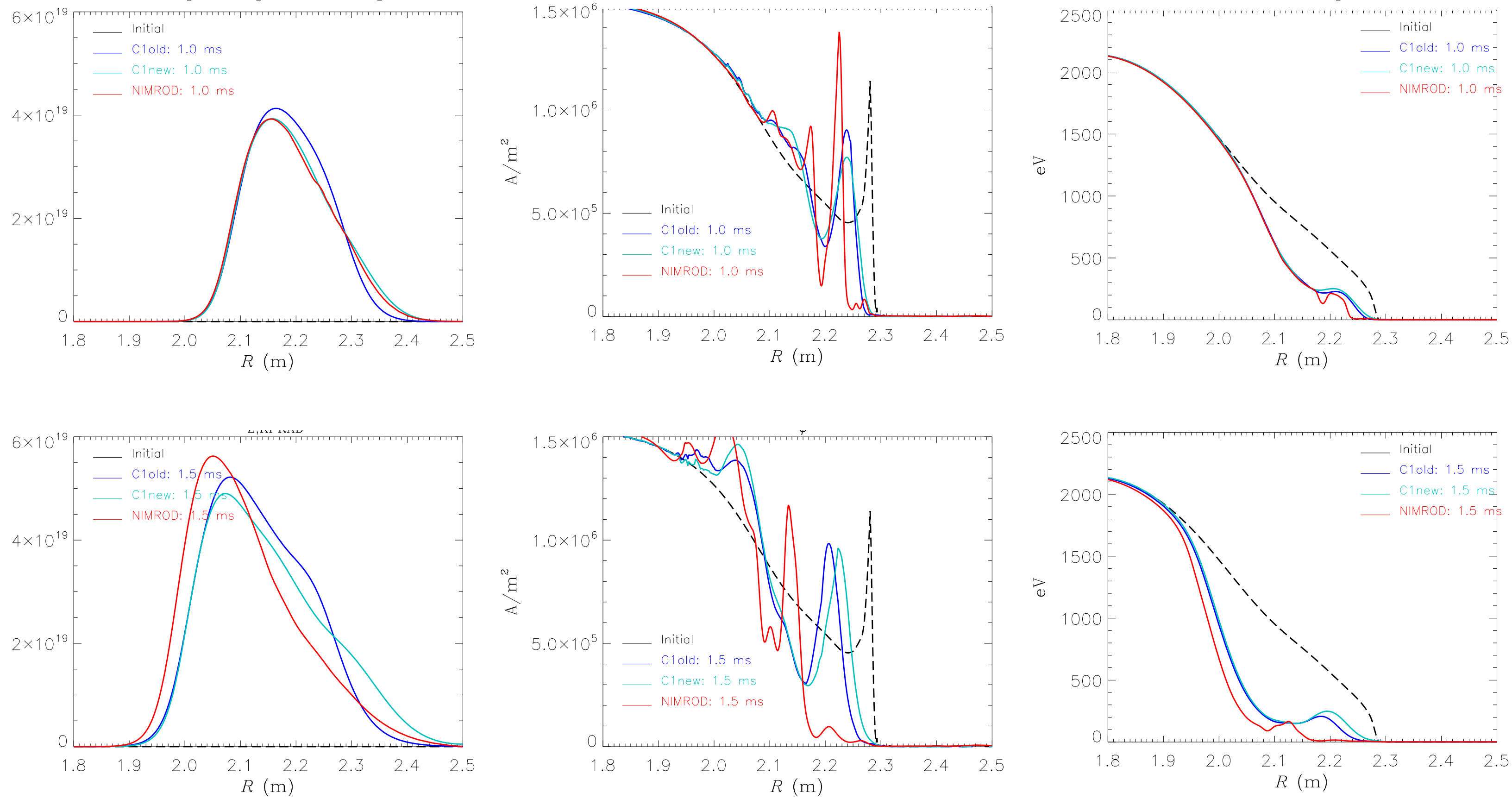
Differences in Other Key Profiles Perhaps Hint at Another Cause?

NIMROD
C1: Old no-pol.
C1: New no-pol.

Impurity Density | **Current Density** | **Electron Temperature**

1.0 ms

1.5 ms

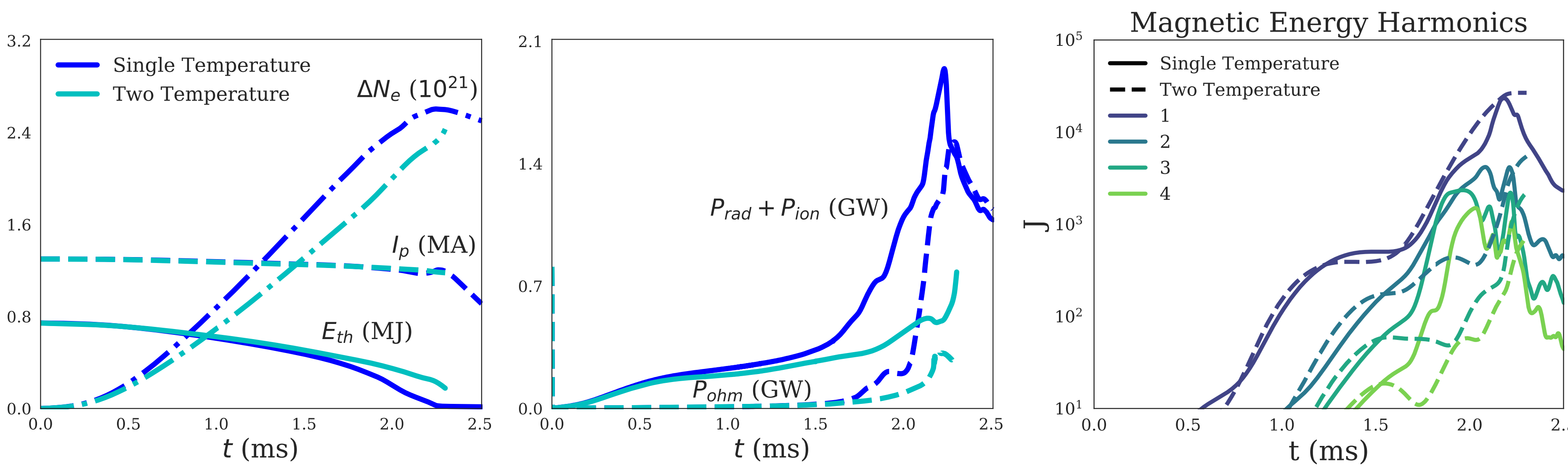


Lyons Sherwood 4-22

GENERAL ATOMICS

Two-Temperature Modeling Shows Delayed Thermal Quench

- **M3D-C1 benchmark case re-run with two-temperature model**
- **Early dynamics similar, but deviates as electrons cool faster than single-temperature model**
  - Less ablation and ionization
  - Slower thermal quench
  - Delayed instability
- **Single-temperature model may underpredict thermal-quench times and overpredict pellet assimilation**



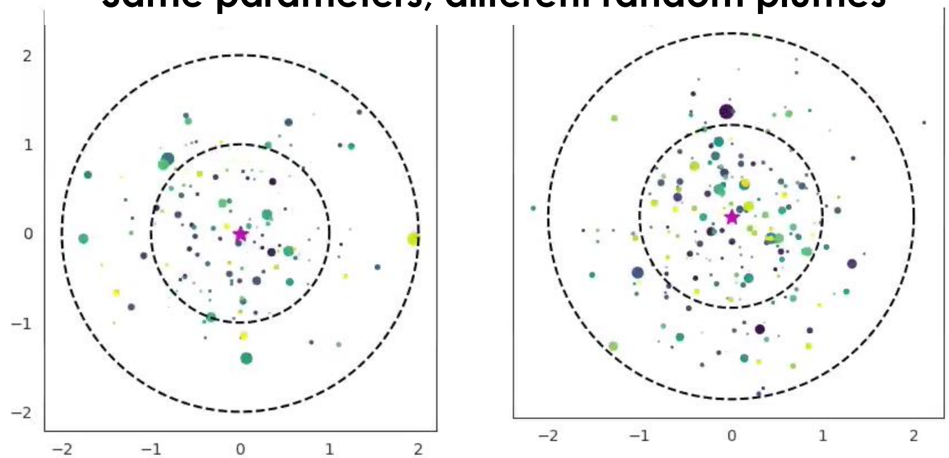
GENERAL ATOMICS

SPI Plume Modeling in JET

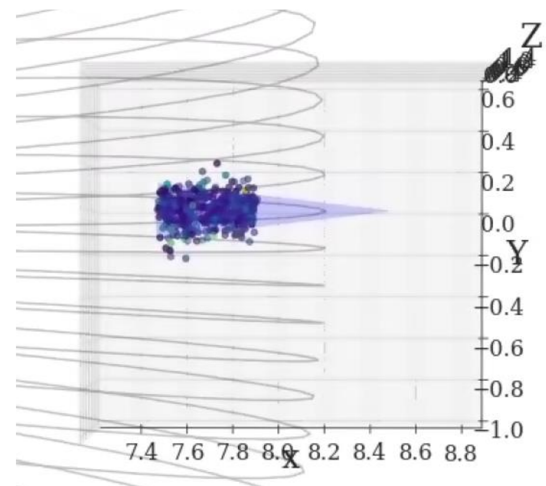
M3D-C1 Multi-Fragment Modeling Uses Realistic Model for Shattered Plumes

- **Script created to generate shatter plumes**
  - Uniform fragments
  - Fracture-threshold theory [T.E. Gebhart et al. IEEE 48, 6 (2019)]
- **Distribution options**
  - Sunflower distribution
  - 2D uniform
  - Gaussian poloidal/toroidal spread
- **Easily generate random (but reproduceable) plumes for different pellet size, speed, and composition**
- **Being used for reference plumes in JET & KSTAR modeling by M3D-C1, NIMROD, and JOREK**

Same parameters, different random plumes

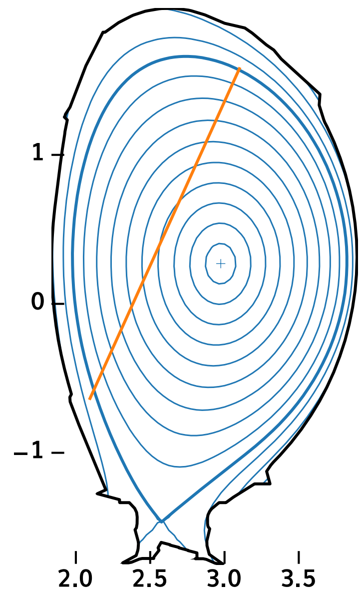


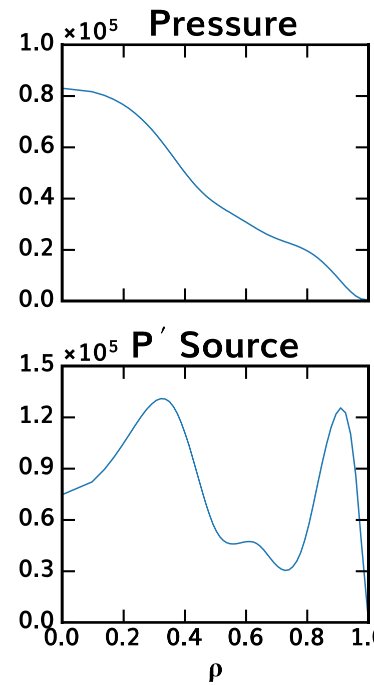
Spread in tokamak geometry



M3D-C1 JET Modeling with Realistic Plumes Performed for JET Scenario 1

- **Based on high-thermal-energy (Scenario 1) plasma with 8.1 mm cylindrical pellet**
- **Equilibria reconstructed with kinetic profiles acquired for recent experiment**
- **Two realistic pellets travel along nominal trajectory**
  - Pure Neon
    - 30 1.71-mm shards
    - 150 m/s
  - 95% D:
    - 85 1.21-mm shards
    - 300 m/s
  - Uniform shard size computed from ablation-average of cloud
- **Also consider same plumes but swapped speeds**

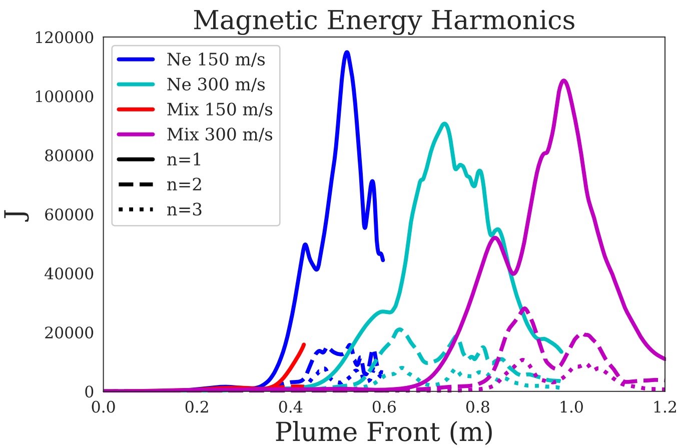


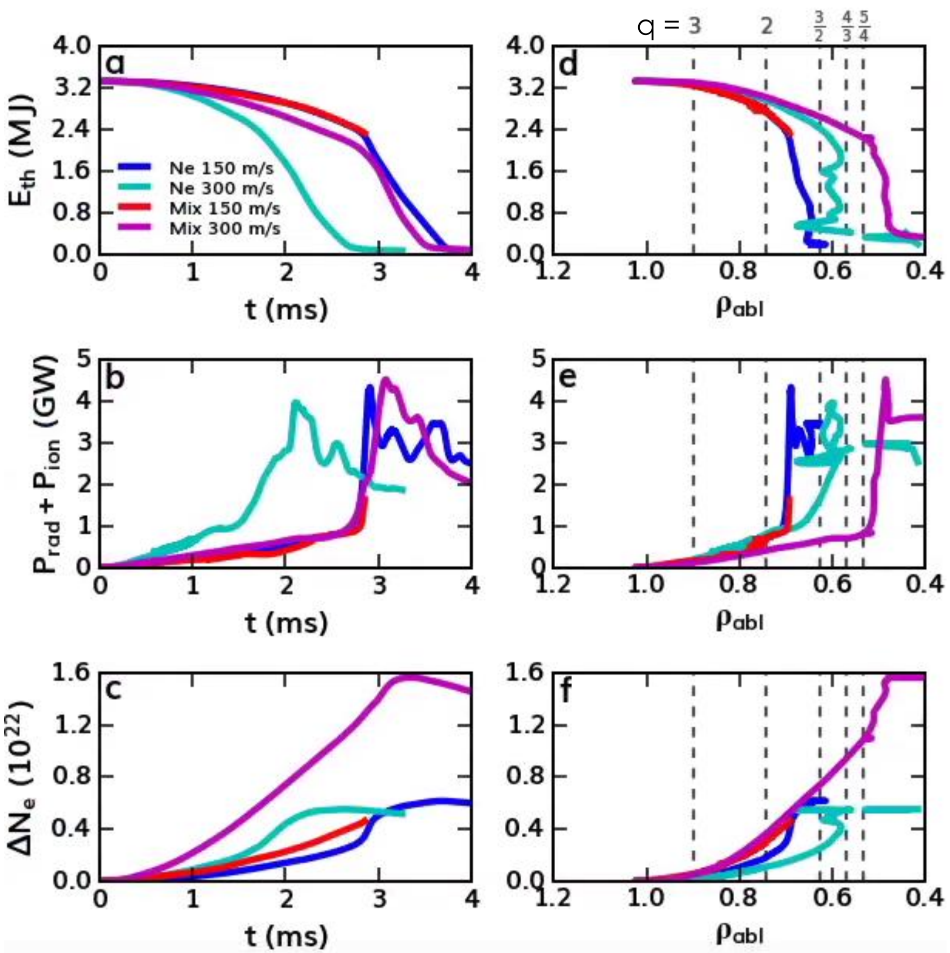


JET 95707
I_p = 2.4 MA
W_{th} = 3.4 MJ
(Scenario 1 High W_{th})

JET Modeling Shows Competition Between Rate of Travel and Rate of Radiative Dissipation

- **All plumes show similar peak radiated power**
- **Dynamics versus time**
  - Fast neon has earliest TQ
  - Others have similar TQ times
- **Dynamics versus penetration depth**
  - Slow: both travel same to same depth – *radiation dominates*
  - Fast: mixed pellet travels deeper – *doesn't radiate fast enough to induce instability*
  - Deeper penetration leads to increased mode coupling



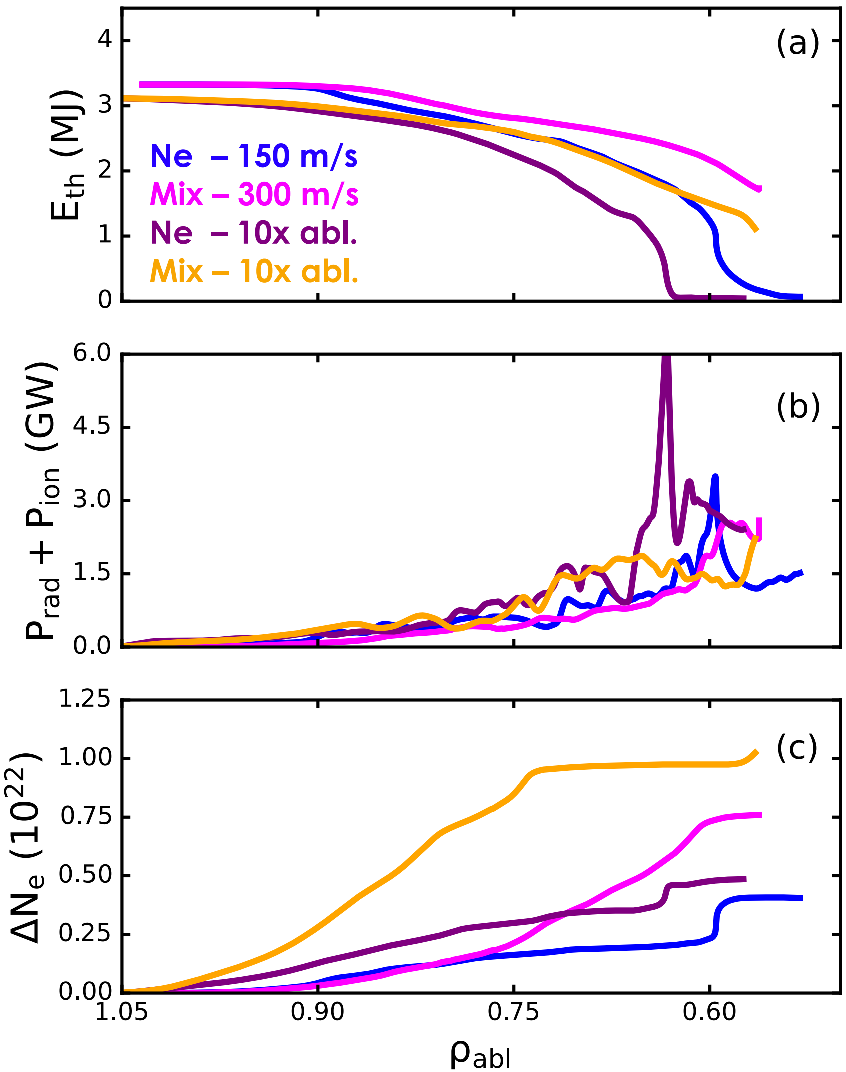


shard position, weighted by ablation rate

GENERAL ATOMICS

NIMROD JET Modeling With Same Realistic Plumes Also Underway

- **Realistic plumes**
  - Faster, mixed pellet penetrates further, as in M3D-C1
  - Generally deeper penetration than M3D-C1, possibly due to impurity deposition behind pellet?
- **Simulations with increased ablation rate show acceleration of thermal quench**
  - Longer period of similar decay perhaps shows increased dominance of radiative dissipation
  - Comparison challenging due to use of different equilibria



Future M3D-C1 Disruption Mitigation Work

- **M3D-C1 & NIMROD 3D benchmark**
  - Continue convergence studies
    - Poloidal & toroidal resolution
    - Time step
    - Diffusivities
  - Need to determine metrics for success
    - Strong nonlinearity makes exact agreement difficult
    - Chaotic evolution: small discrepancies early cause exponential deviation
    - Perhaps use physically relevant quantities, e.g., assimilation fraction, radiation fraction/peaking, TQ time
- **Validate JET modeling against experimental results**
- **Perform & validate KSTAR modeling of multiple toroidal injection**
- **Predictive modeling for ITER SPI**