

NIMROD & KORC Computations of REs in MST and NIMROD Fluid RE Modeling

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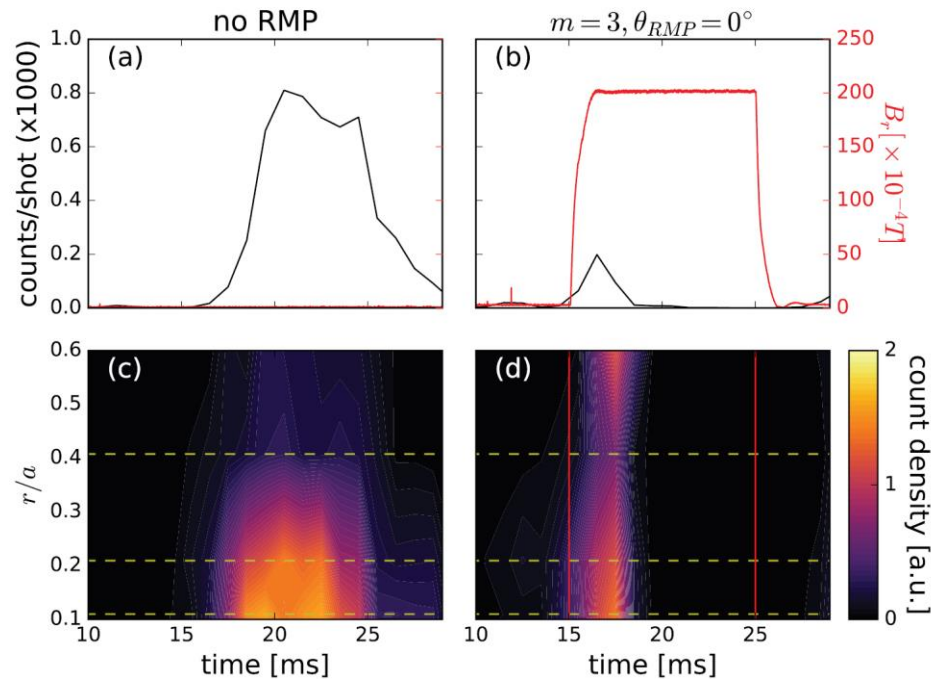
Outline

- NIMROD & KORC simulations applied to RMP studies of tokamak discharges in MST
 - Experimental summary
 - NIMROD simulations of macroscale
 - KORC computations of RE trajectories
- Fluid RE modeling
 - Equations and implementation
 - Large- R/a 2D benchmark
 - DIII-D - like case
- Conclusions



RMP deconfinement of REs is being studied in MST.

- The energetic electrons are measured through X-ray detectors.
- They are observed in low-density conditions, where E_{CH} is low.



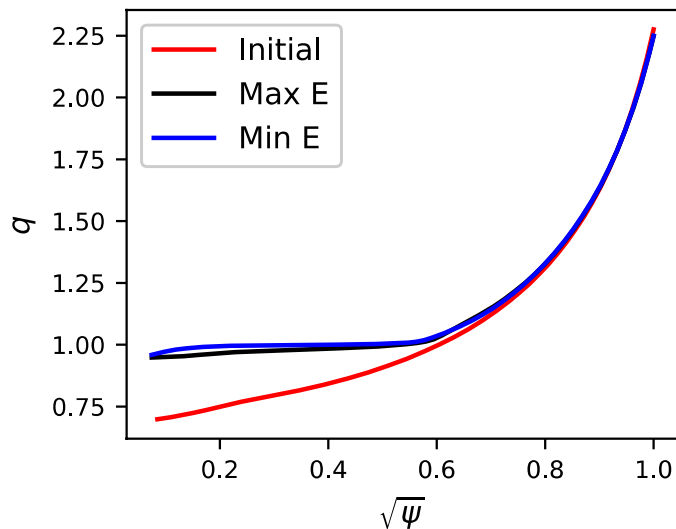
X-ray counts (black) and RMP amplitude (red) for shot with (a) and without RMP (b). Counts from an array of X-ray signals over r (c-d). [Munaretto, et al., NF 60, 046024]

RMP of poloidal harmonic $m=3$ is effective at suppressing REs, $m=1$ is not.

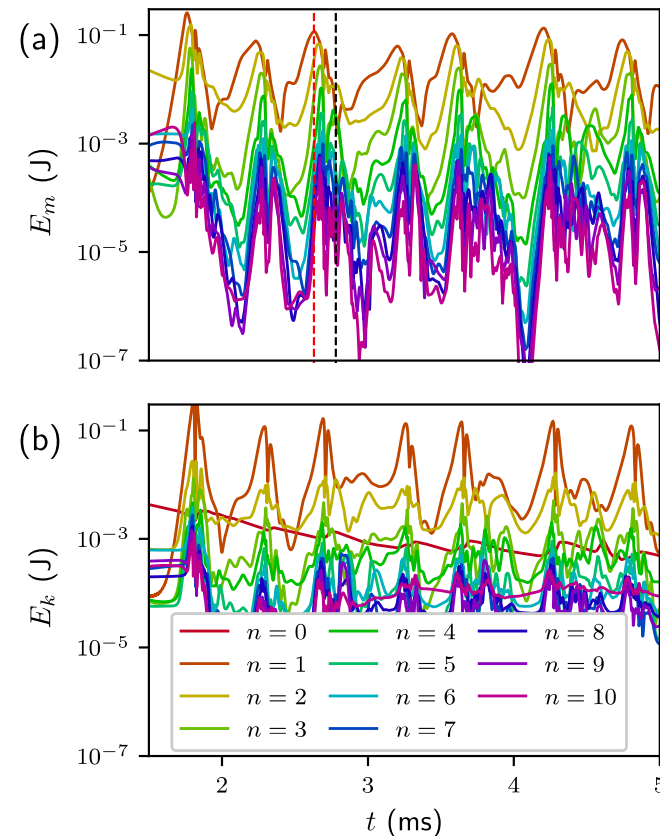


Resistive-MHD NIMROD simulations of fitted profiles show sawtoothing.

- Brian Cornille used the zero-pressure model for the low- β_{pol} conditions.



Central- q approaches unity during the initial transient, then varies only weakly over sawtoothing.

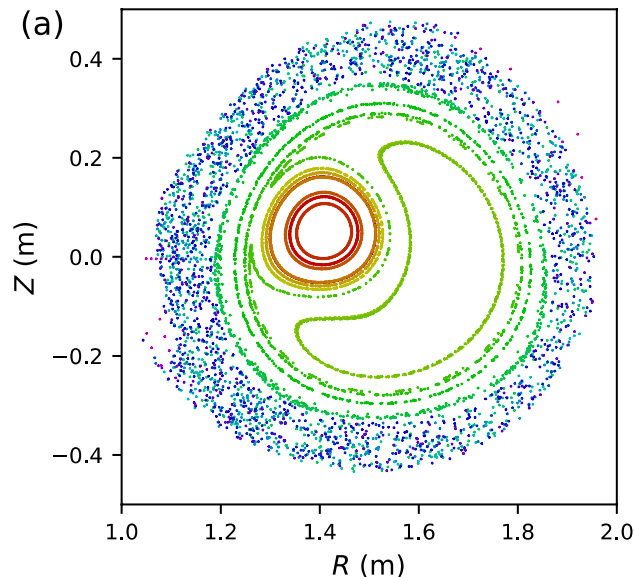


Simulated sawtooth cycle time is $\sim 1/2$ that in MST.

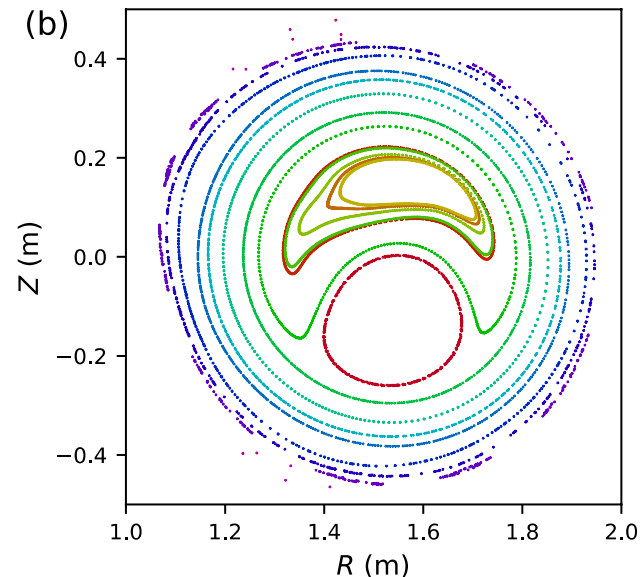


MHD simulation results suggests that sawtoothing enhances deconfinement.

- The $m=3$ RMP creates a stochastic edge, and sawtoothing reconnects the core.
- The $m=1$ RMP leaves nested flux surfaces in the edge.



Poincare plot of magnetic topology with $m=3$ RMP.

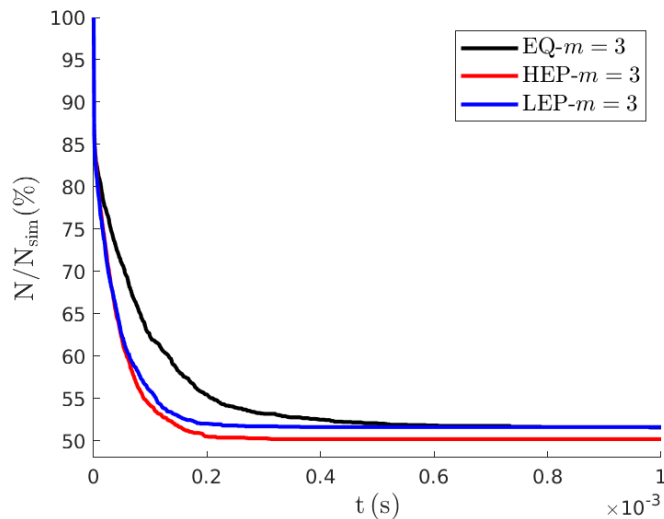


Poincare plot of magnetic topology with $m=1$ RMP.

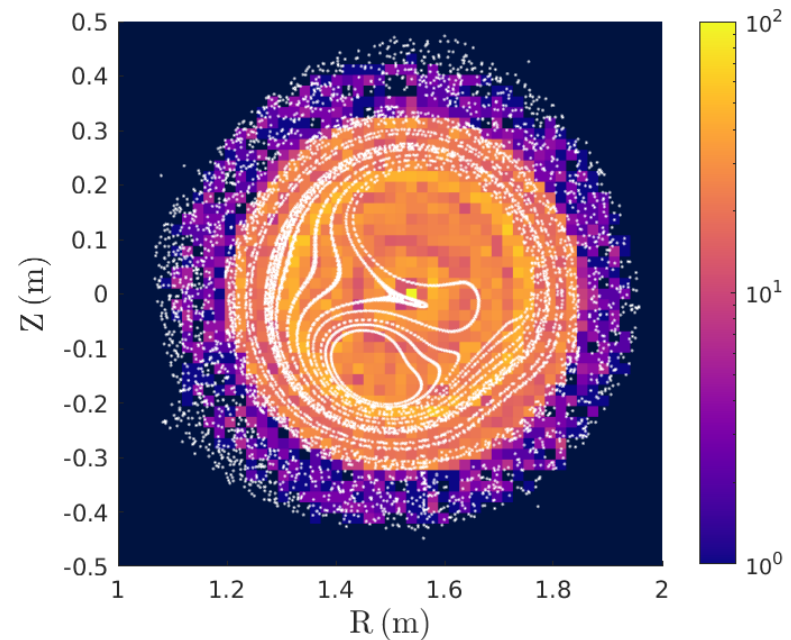


KORC has been applied to check for sawtooth reconnection related/orbit-effect deconfinement.

- KORC was developed to investigate RE behavior. [Carbajal, et al., PoP **24**, 042512]
- Brian developed NIMROD interfacing through FusionIO.
- Matt Beidler of ORNL helped with running KORC.



Fraction of particles (30 keV) that remain confined vs. time shows 50% loss with $m=3$ RMP; 85% for $m=1$.

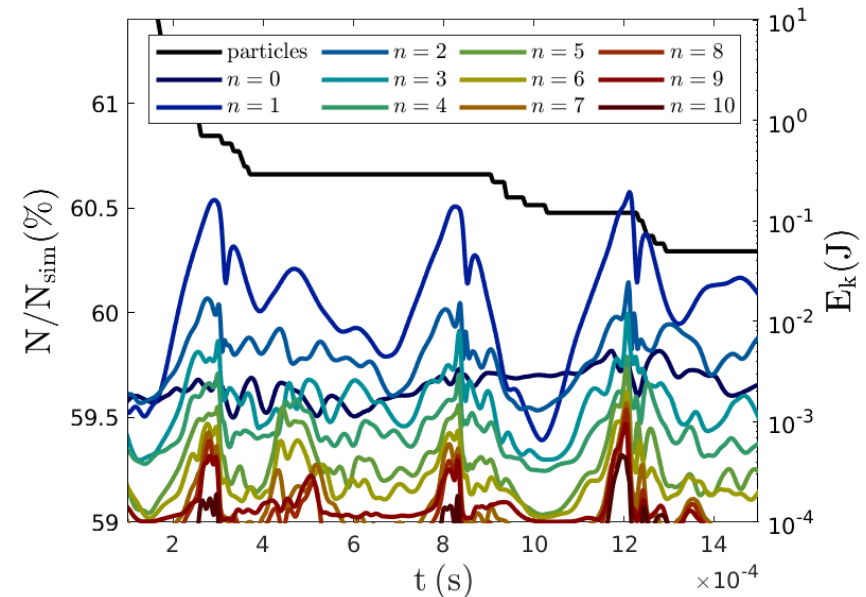


Confined particles (full-orbit) remain in the core with $m=3$ RMP.



Relevant aspects have been checked to resolve the simulation/experiment discrepancy.

- Experiment finds total deconfinement with $m=3$.
- The following have been checked:
 - Numerical resolution
 - Initial particle energy and pitch-angle variation
 - E -field acceleration
 - Sawtooth time-dependence
- Suspected causes are:
 - Simulated sawtooth too weak due to equil. fitting
 - Non-modeled RMP transient



There is correlation between sawtooth and (slight) particle deconfinement when KORC is run with time-dependent magnetic field.

B. S. Cornille PhD Thesis, UW-Madison, 2021; PoP article under review.



Fluid RE modeling in NIMROD captures interaction between RE current and B evolution.

- Model equations describe a fluid population of electrons flowing at the speed of light along magnetic field lines:

Continuity equation for runaway electron population:

$$\frac{\partial n_r}{\partial t} + \nabla \cdot (n_r \mathbf{v}_r) = S_D(\mathcal{E}_{\parallel}) + S_A(E_{\parallel}) + D_r \nabla^2 n_r \quad (1)$$

Where n_r is the number density of runaways, S_D, S_A are sources, D_r is a numerical diffusion coefficient and

$$\mathcal{E}_{\parallel} \equiv \frac{E_{\parallel}}{E_D}, \quad \mathbf{v}_r = c_r \hat{\mathbf{b}} + \frac{\mathbf{E} \times \mathbf{B}}{B^2}, \quad |c_r| = \text{const.} \gg v_{th,e} \gg v_A$$

$$E_D = \frac{n_e e^3 \ln \Lambda}{4\pi \epsilon_0 T_e}$$



RE sources are given by the Dreicer and Avalanche expressions

The model for the Dreicer source S_D is that presented by Connor and Hastie⁴

$$S_D = (n_e - n_r)^2 \frac{e^4 \ln \Lambda}{4\pi\epsilon_0^2 m_e^2 v_{th}^3} \mathcal{E}_{\parallel}^{-\frac{3}{16}(1+Z_{eff})} \exp\left\{-\frac{1}{4\mathcal{E}_{\parallel}} - \sqrt{\frac{1+Z_{eff}}{\mathcal{E}_{\parallel}}}\right\} \\ \times \exp\left\{-\frac{T_e}{mc^2} \left(\frac{1}{8\mathcal{E}_{\parallel}^2} + \frac{2}{3} \sqrt{\frac{1+Z_{eff}}{\mathcal{E}_{\parallel}^3}}\right)\right\}$$

The avalanche source S_A is given by Rosenbluth and Putvinski⁵:

$$S_A = \frac{n_r}{\tau} \sqrt{\frac{\pi a}{3(Z+5)}} \left(\frac{E_{\parallel}}{E_c} - 1\right) \left(1 - \frac{E_c}{E_{\parallel}} + \frac{4\pi(Z+1)^2}{3a(Z+5)(E_{\parallel}^2/E_c^2 + 4/a^2 - 1)}\right)^{-1/2} \\ E_c = \frac{n_e e^3 \ln \Lambda}{4\pi\epsilon_0^2 mc^2}, \quad a(\epsilon) = (1 + 1.46\sqrt{\epsilon} + 1.72\epsilon)^{-1}, \quad \tau = \frac{mc \ln \Lambda}{eE_c}$$

⁴J. W. Connor and R. J. Hastie, Nuclear Fusion **15**, 415–423 (1975)

⁵M. N. Rosenbluth and S. V. Putvinski, Nuclear Fusion **37**, 1355–1362 (1997)



RE interacts with MHD via the modified Ohms law.

In this Ohm's law, one assumes the parallel current carried by the runaways experiences no resistivity.

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \left(\mathbf{J} - en_r c \hat{\mathbf{b}} \right) \quad (2)$$

This model is valid for

$$\frac{n_r}{n_e} \ll 1, \quad m_e n_r \partial_t \mathbf{v}_r \approx 0$$

- Note that the drift velocity contribution to the runaway current is neglected.
- This model is similar the model employed in Bandaru⁶ and Matsuyama⁷.

⁶V. Bandaru et al., *Physical Review E* **99**, 1–11 (2019)

⁷A. Matsuyama et al., *Nuclear Fusion* **57**, 10.1088/1741-4326/aa6867 (2017)



Least squares projection and time-split advance stabilizes runaway advection.

$$\Delta n_r = \Delta n_{r,\text{adv}} + \Delta n_{r,\text{src}}$$

$$\begin{aligned} \int dV \left\{ \Delta n_{r,\text{adv}} + \Delta t \nabla \cdot \left(\mathbf{v}_r \frac{\Delta n_{r,\text{adv}}}{2} \right) \right\} \times \left\{ g + \Delta t \nabla \cdot \left(\mathbf{v}_r \frac{g}{2} \right) \right\} \\ = - \int dV \Delta t \nabla \cdot (n_r^k \mathbf{v}_r) \times \left\{ g + \Delta t \nabla \cdot \left(\mathbf{v}_r \frac{g}{2} \right) \right\} \end{aligned}$$

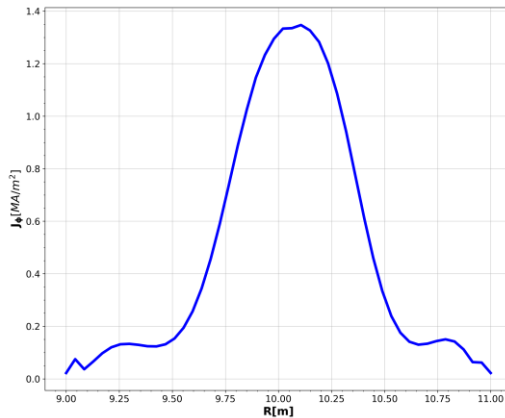
$$n_r^{k'} = n_r^k + \Delta n_{r,\text{adv}}$$

$$\begin{aligned} \int dV \left\{ \Delta n_{r,\text{src}} g + \Delta t D_r \nabla \cdot \left(\frac{\Delta n_{r,\text{src}}}{2} \right) \cdot \nabla g \right\} \\ = \Delta t \int dV \left\{ (S_D + S_A) g - \Delta t D_r \nabla n_r^{k'} \cdot \nabla g \right\} \end{aligned}$$

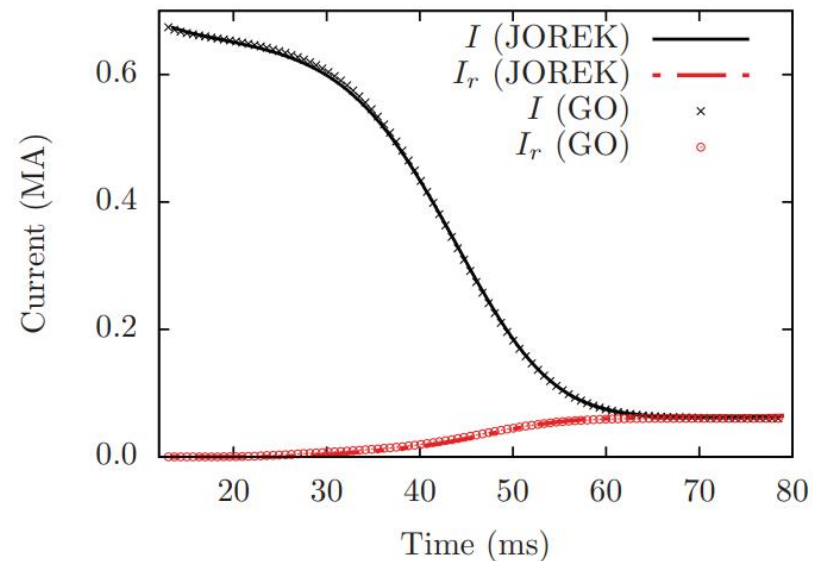
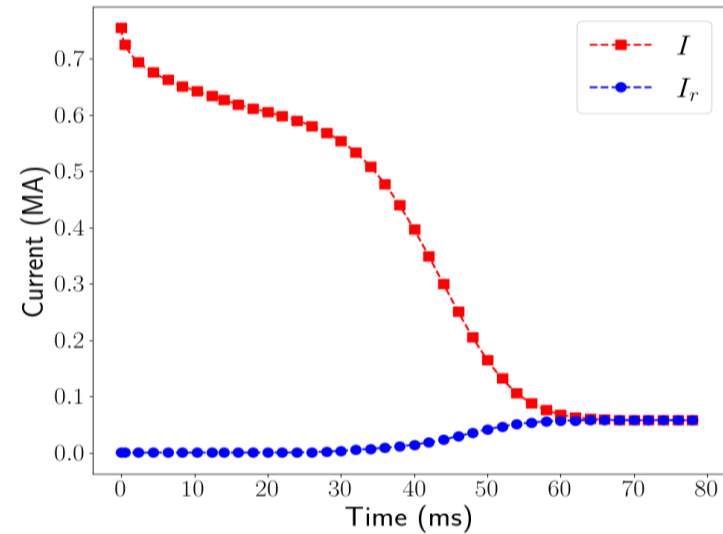
$$n_r^{k+1} = n_r^{k'} + \Delta n_{r,\text{src}}$$



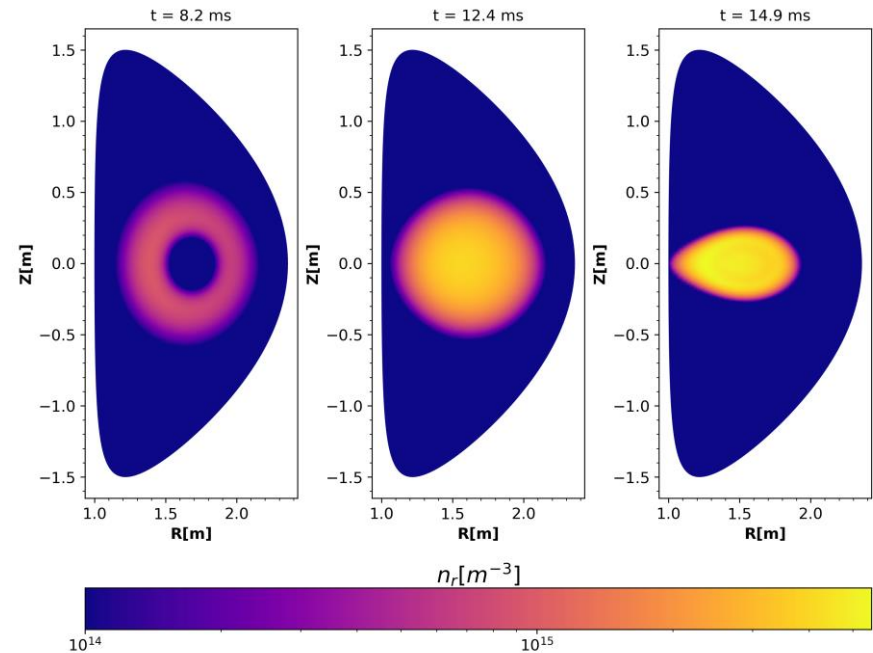
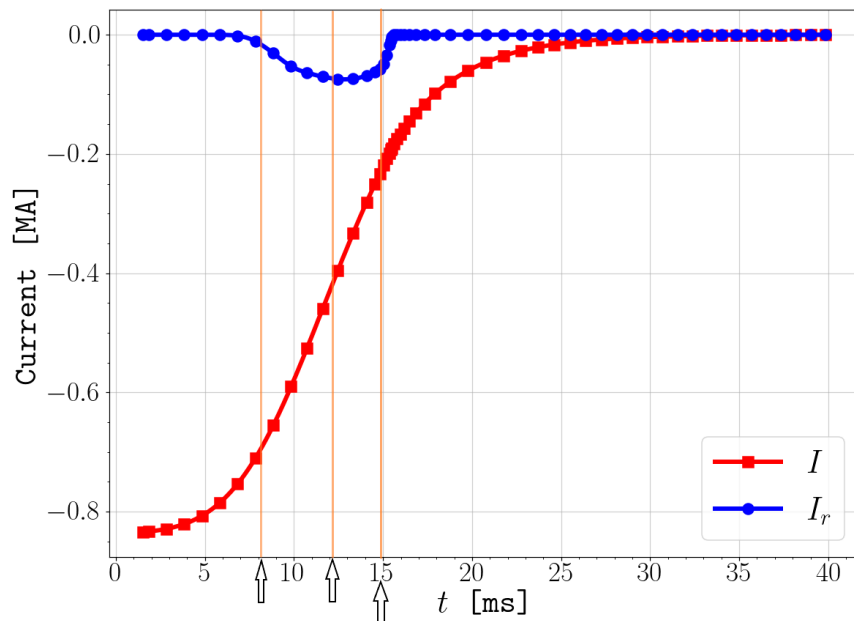
Results from 2D, high R/a benchmark between NIMROD and JOREK predict the same current quench



- Equilibrium toroidal current density profile for the benchmark case.
- Thermal quench imposed by raising the value of conductivity.
- Total current and runaway current computed in NIMROD agree with JOREK and GO [Bandaru, et al., Phys Rev E **99**, 063317]



2D results from DIII-D like case show loss to the wall disrupting formation of a current plateau.



- Contours on the right show runaway density at the three times indicated on the current traces on the left.
- The initially hollow distribution becomes more uniform and begins to be lost to the wall along open field-lines.



Conclusions

- NIMROD + KORC simulations MST find significant differences between the $m=1$ and $m=3$ RMP .
- The simulations do not predict total deconfinement of REs with the $m=3$ RMP.
- Suspected causes are:
 - Simulated sawtooth too weak due to equil. fitting
 - Non-modeled RMP transient
- Fluid RE model in NIMROD reproduces JOEREK results from 2D benchmark.
- Split-step runaway advance with least squares stabilizes the RE advection at larger time steps
- Results from DIII-D – like equilibrium case show that the model can capture the effect of RE loss to the wall along field lines.

