PRINCETON PLASMA PHYSICS LABORATORY

Exploring stellarator β-limits with nonlinear MHD modelling

Adelle Wright
Princeton Plasma Physics Laboratory

CTTS meeting
(April 3rd, 2022)

Email: awright@pppl.gov

Contents

- Motivation and existing approaches for MHD modeling in stellarators
- Examples
- M3D-C1 method and performance in stellarator geometry
- Outlook: Next steps

- **Motivation and existing approaches for MHD modeling in stellarators**
- *Examples*
- *M3D-C1 method and performance in stellarator geometry*
- *Outlook: Next steps*

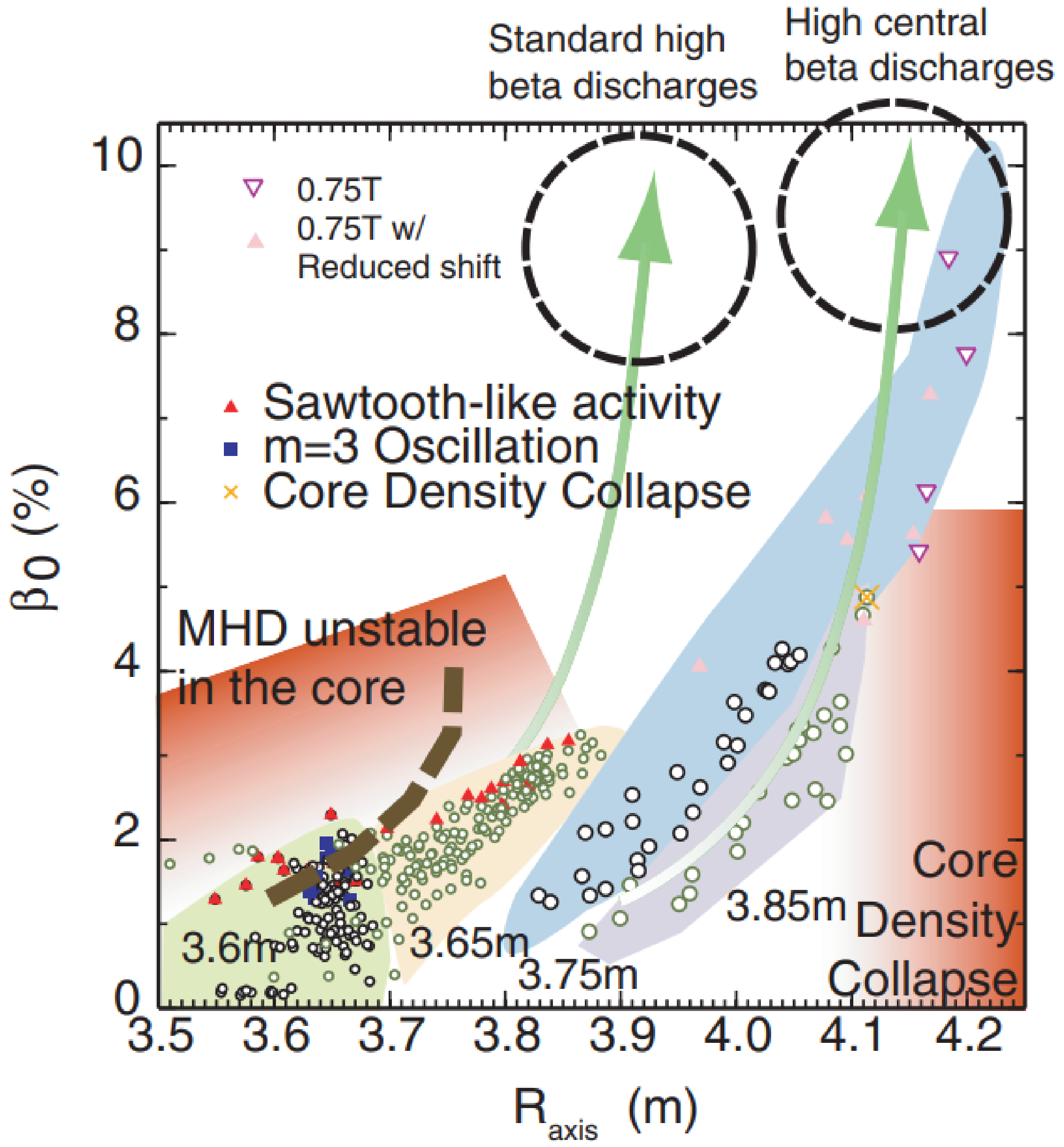
Understanding nonlinear MHD stability is important for fusion

Clarifying the role of 3D effects is critical for determining when instabilities are benign or become disruptive.

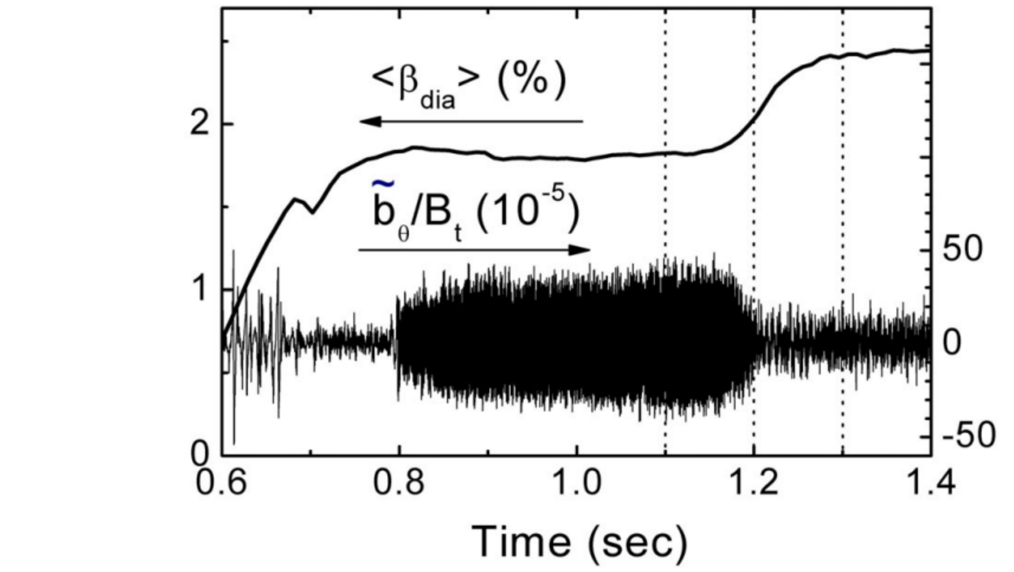
Like tokamaks, stellarators can be susceptible to (sometimes disruptive) pressure- and current-driven instabilities:

Pressure-driven MHD activity (LHD)

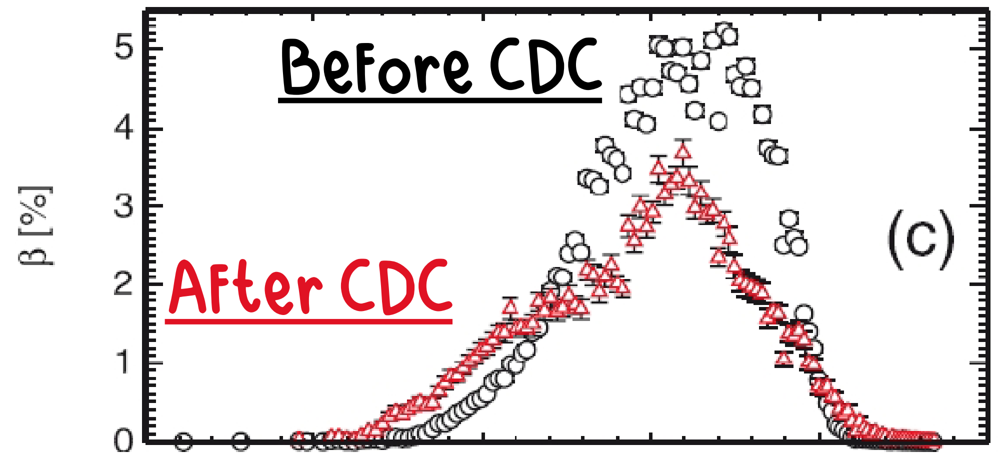
Benign MHD activity and disruptive core density collapse (CDC) events have been observed [1].



Benign mode (2,1) activity:

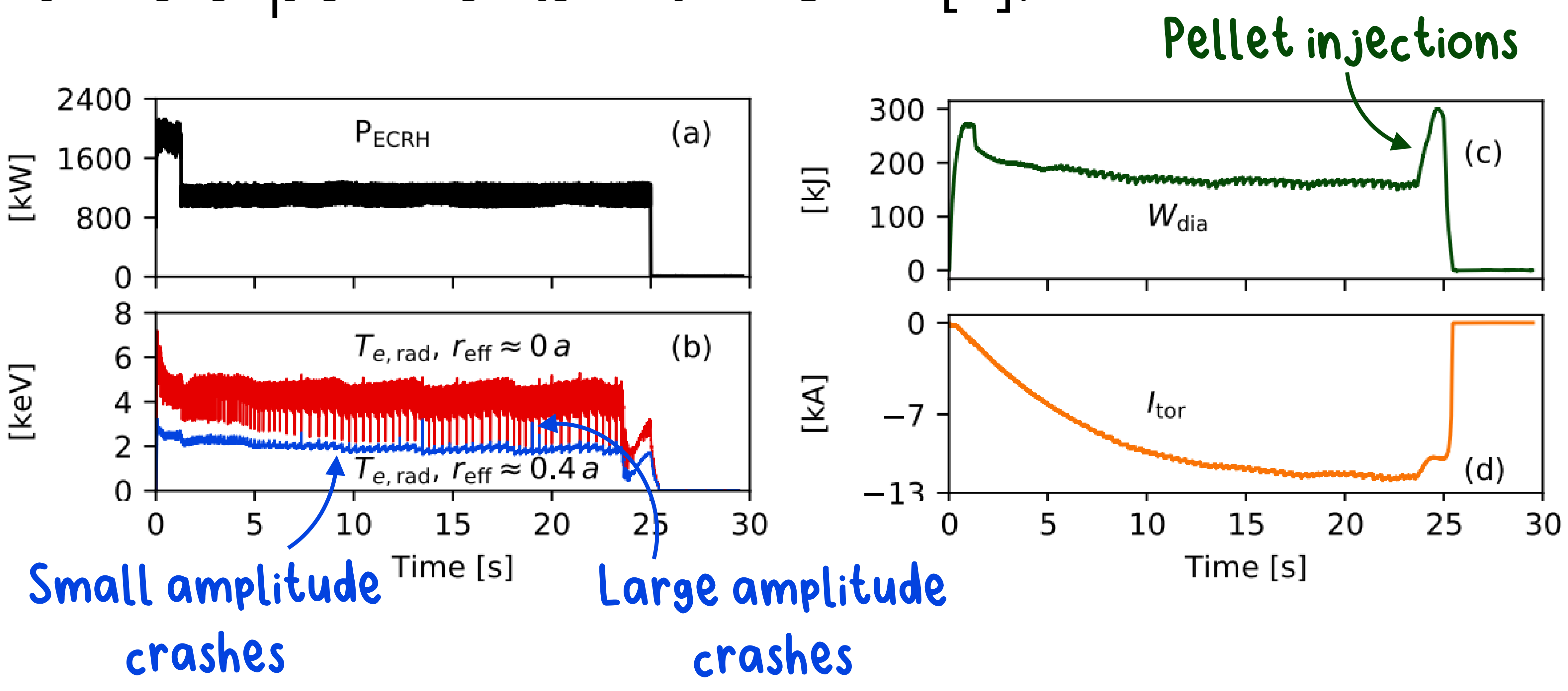


Core density collapse:



Current-driven MHD activity (W7-X)

Sawtooth-like crashes observed during current-drive experiments with ECRH [2].



[1] Ohdachi et al. FEC NIFS--890 (2008); [2] Zanini et al. Nuclear Fusion 60.10 (2020).

Tools to model nonlinear MHD in stellarator geometry have been limited

Existing approaches for understanding MHD in stellarators typically involve:

Equilibrium models ($\mathbf{J} \times \mathbf{B} = \nabla p$) + Linear MHD stability

+1 Comparatively fast and cheap to evaluate.

- Appropriate for applications requiring fast calculation of 3D fields (e.g. optimisation and reconstruction).

-1 Dynamical accessibility of solutions is not guaranteed.

If an equilibrium code predicts a finite-β equilibrium with chaotic fields and magnetic islands:

- Can the plasma actually reach this state with heating?
- What happens if the system crosses a stability boundary?

Exploration of new macroscopic physics in stellarators

Initial-value methods are needed to examine important questions:

- Evolution of pressure profiles for self-consistent equilibria, including for non-integrable fields.
- Examine dynamical accessibility of 3D equilibria (integrable and non-integrable).
- Determine nonlinear stability.

- Many concepts which may be important for future designs have not been tested experimentally, elevating the role of theory and simulation for physics understanding and to inform design.

<u>This can provide insight into questions that are relevant to tokamaks too</u>:

Current-driven instabilities

- What is the appropriate balance between current and magnetic shear?

Finite net toroidal current/high bootstrap fraction (e.g., QA)

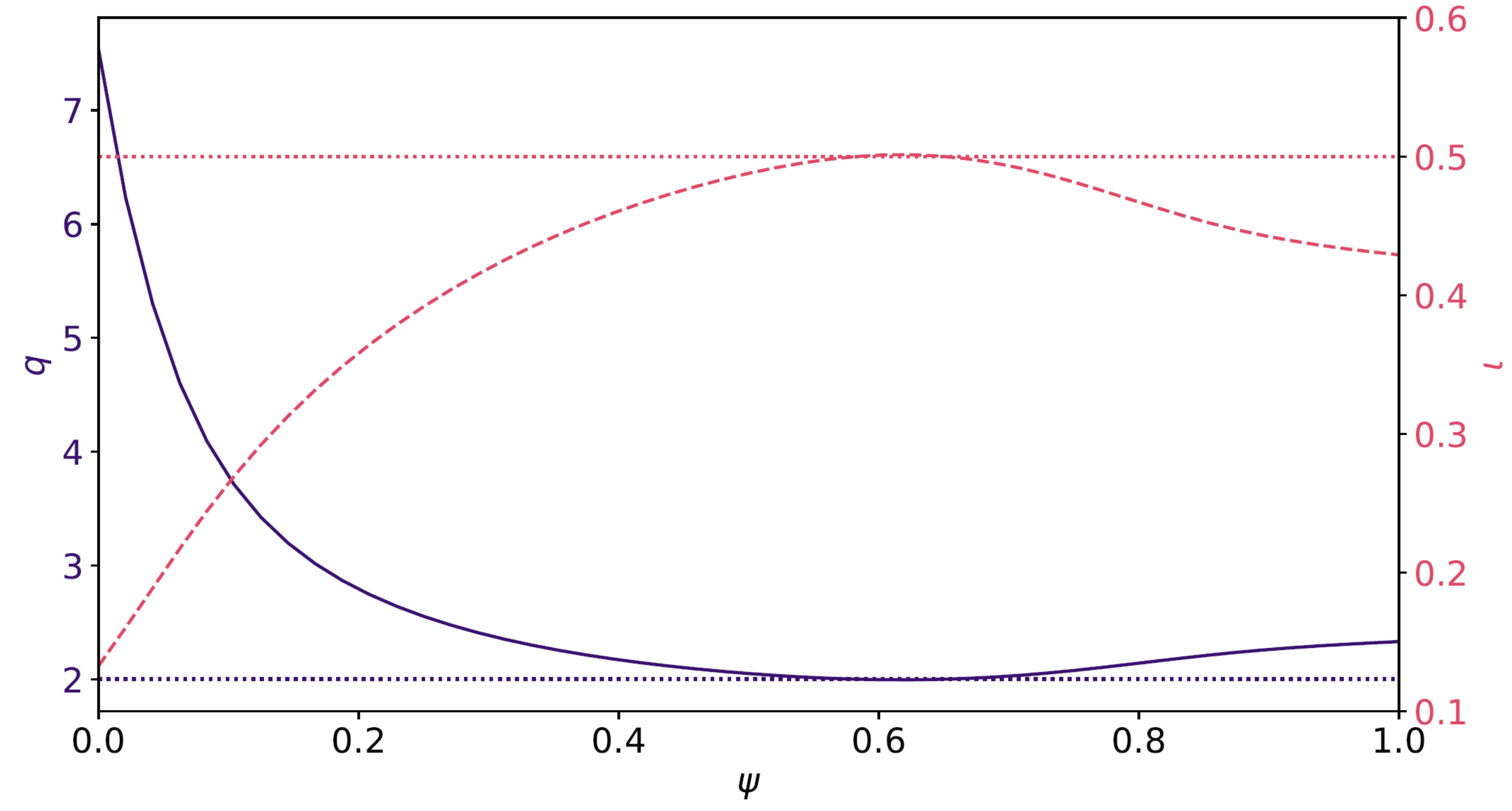
- To what extent will tokamak physics challenges be inherited (including need for more active control)?

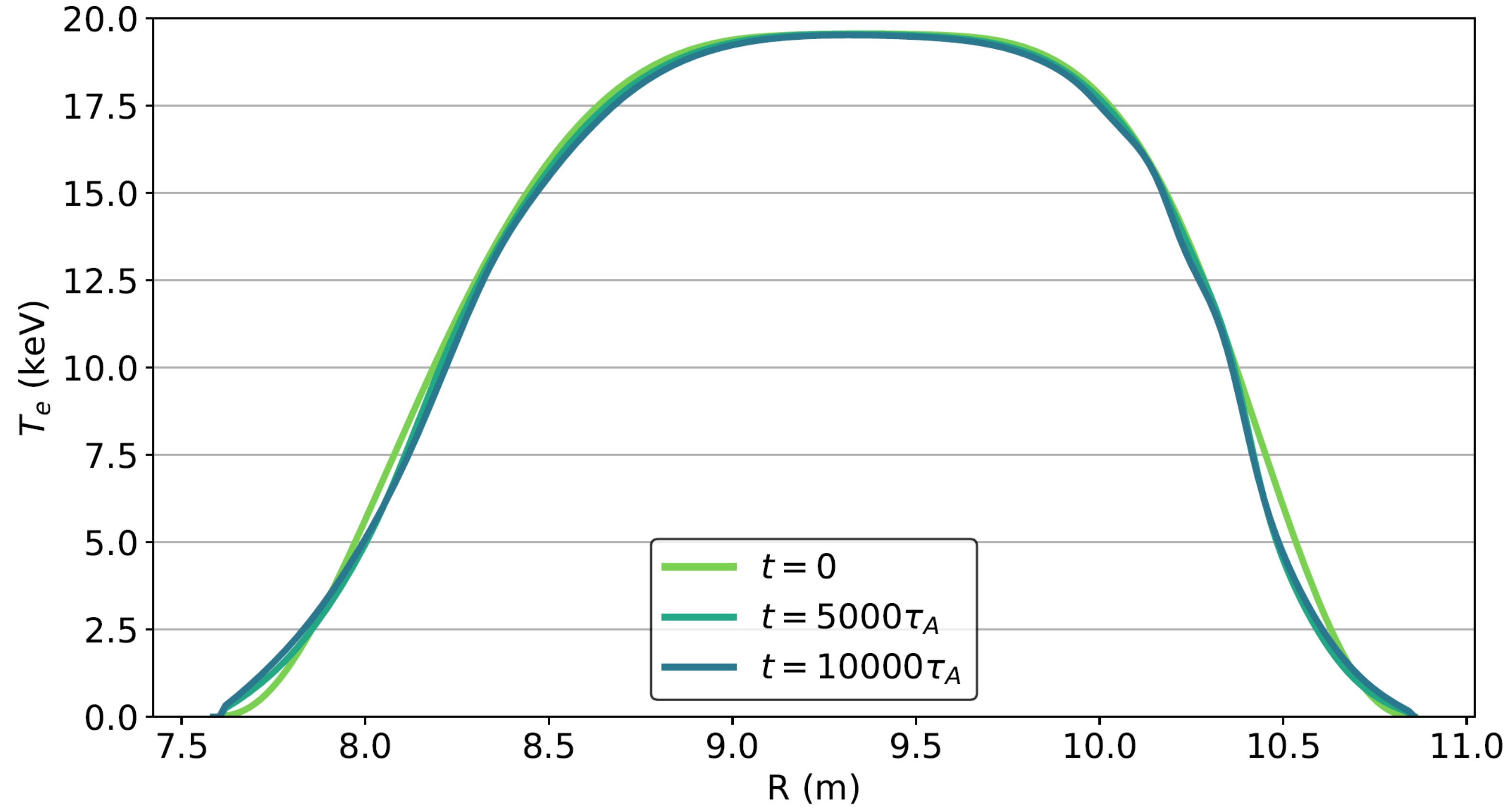
Contents

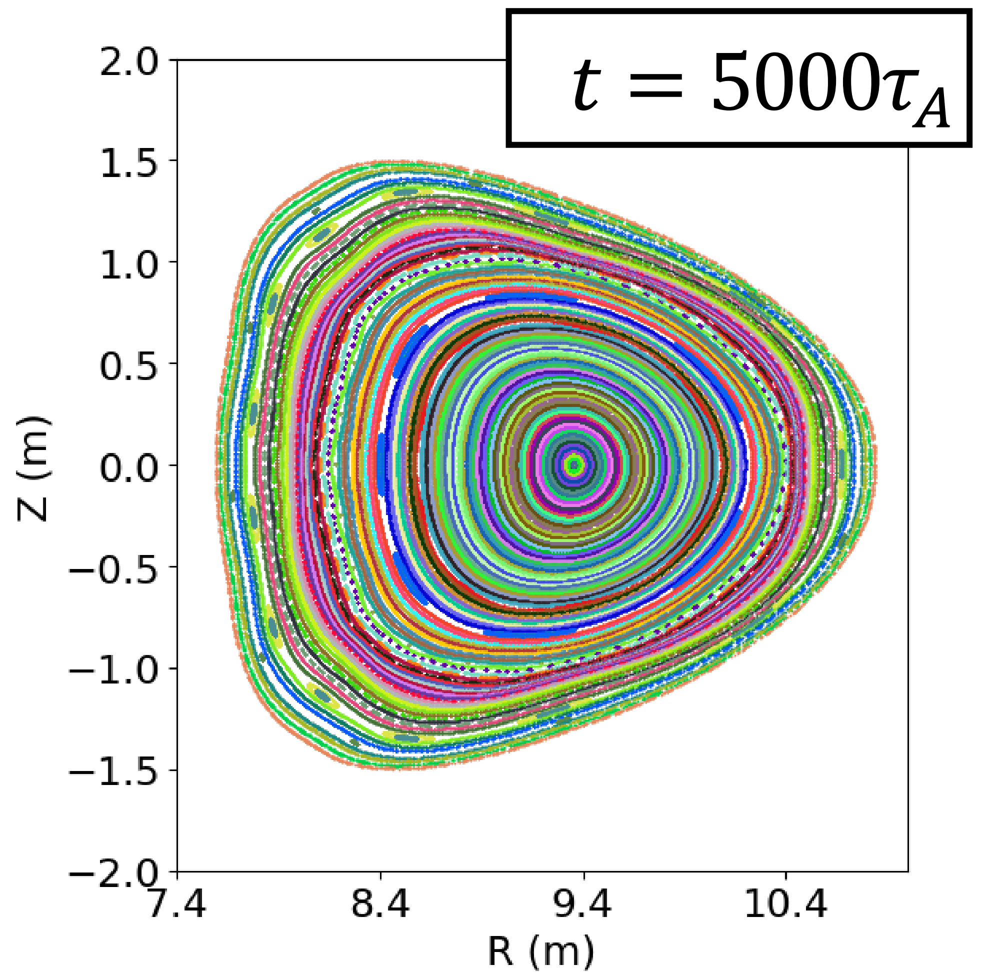
- *Motivation and existing approaches for MHD modeling in stellarators*
- **Examples**
- *M3D-C1 method and performance in stellarator geometry*
- *Outlook: Next steps*

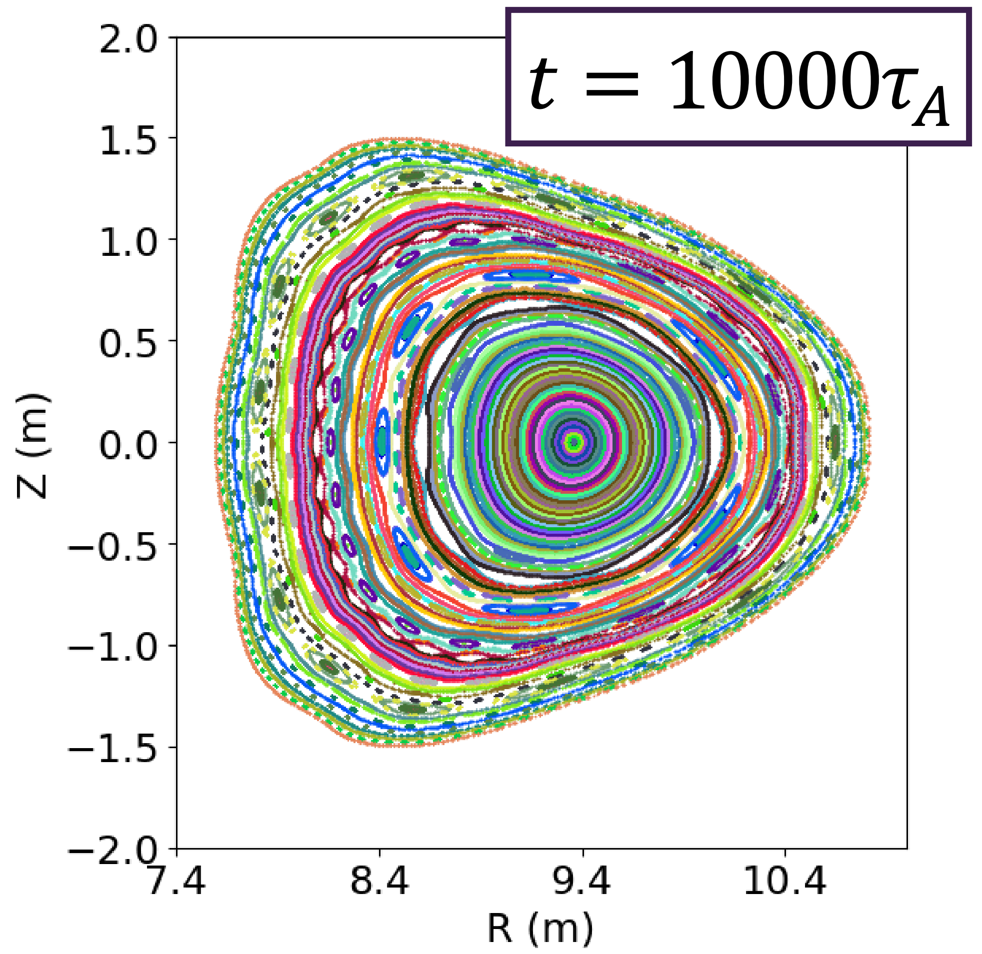
Example: Validation of ideal linear MHD stability in optimised QA configuration

For an optimised QA equilibrium (NFP=3, β=2%), we verify linear ideal MHD stability and show formation of higher-m magnetic islands on resistive instability timescales:









Parameters:

$S = 10^6$

$\nu = 5 \cdot 10^{-3}$

$\kappa_{\parallel}/\kappa_{\perp} = 5 \cdot 10^7$

Elongation=2.6

Aspect ratio=6

Volume=443m^3

Fixed boundary

Mesh=12K elements

6 nodes (96 CPUs/node)

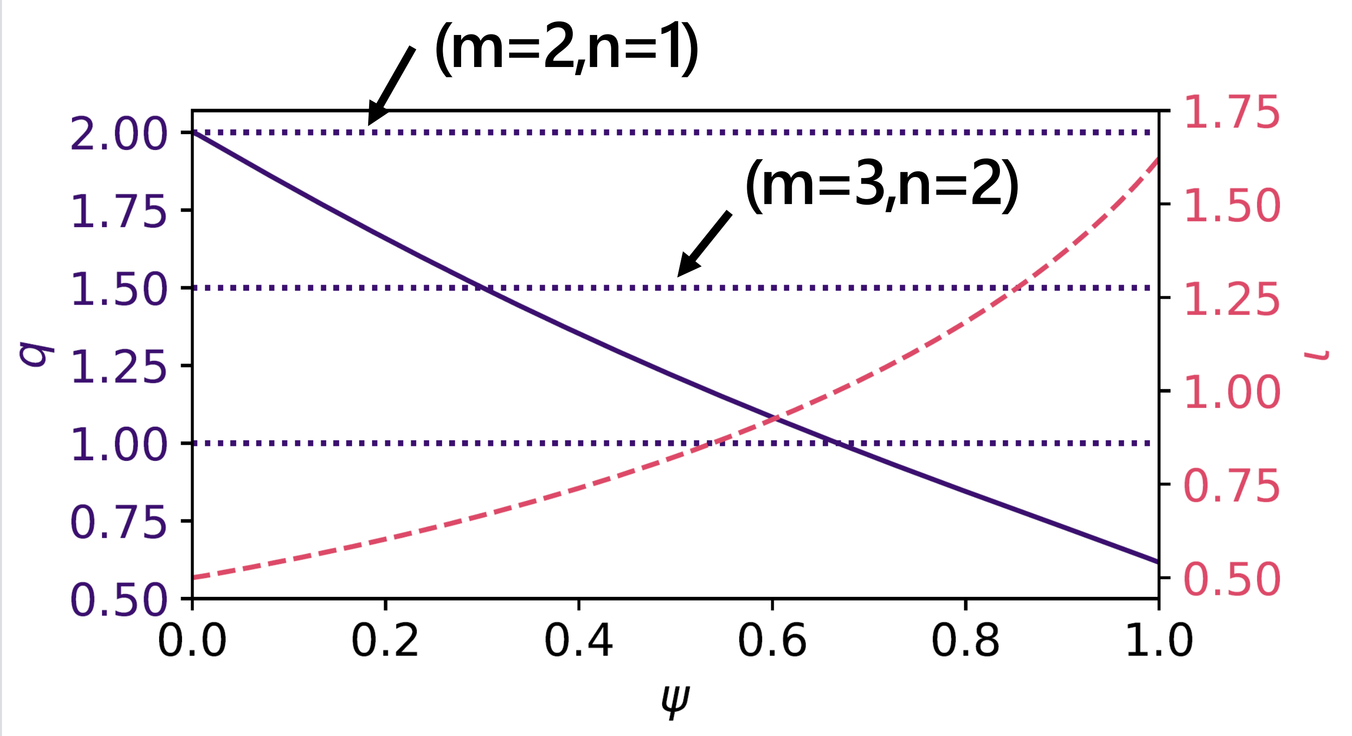
Timing: 250s/step (Δt=5τ_A)

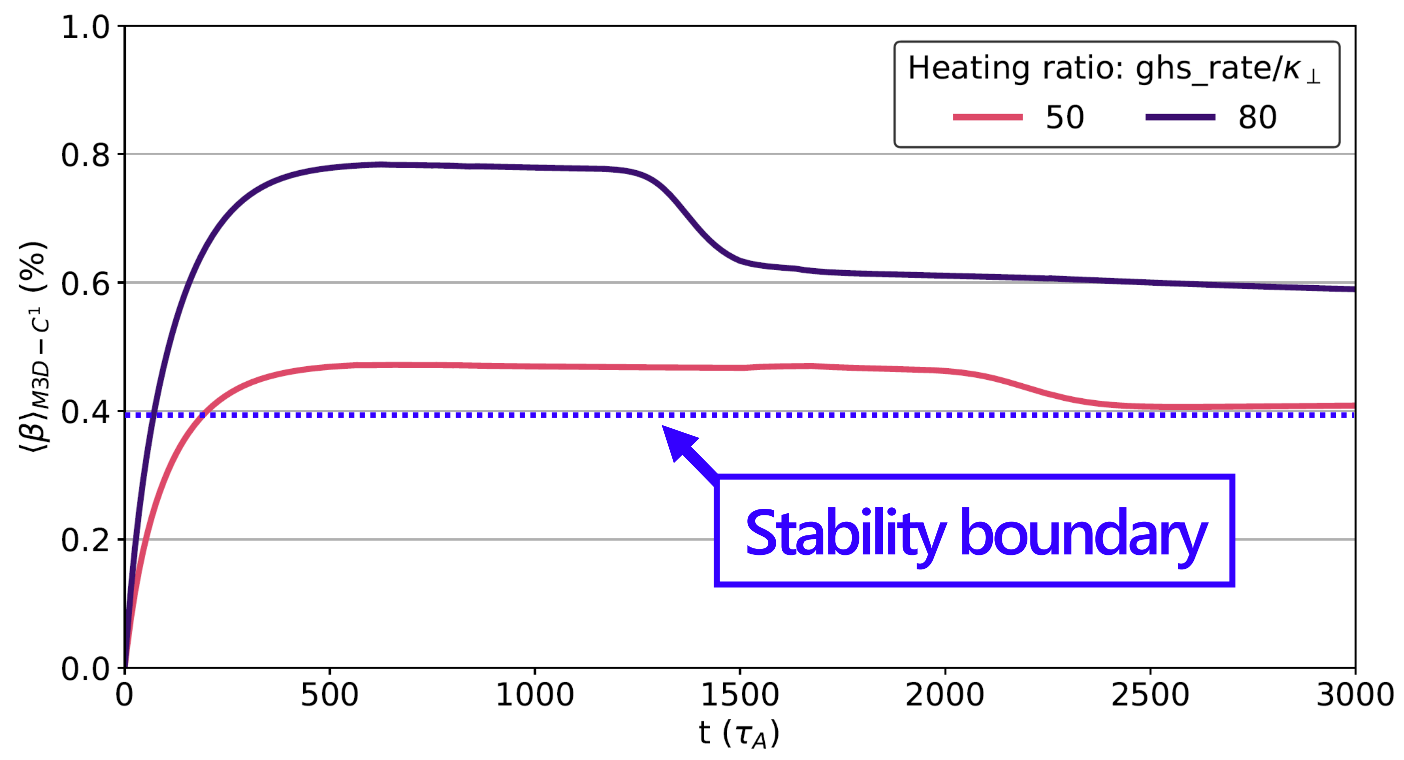
→ Ideal = 15 hrs

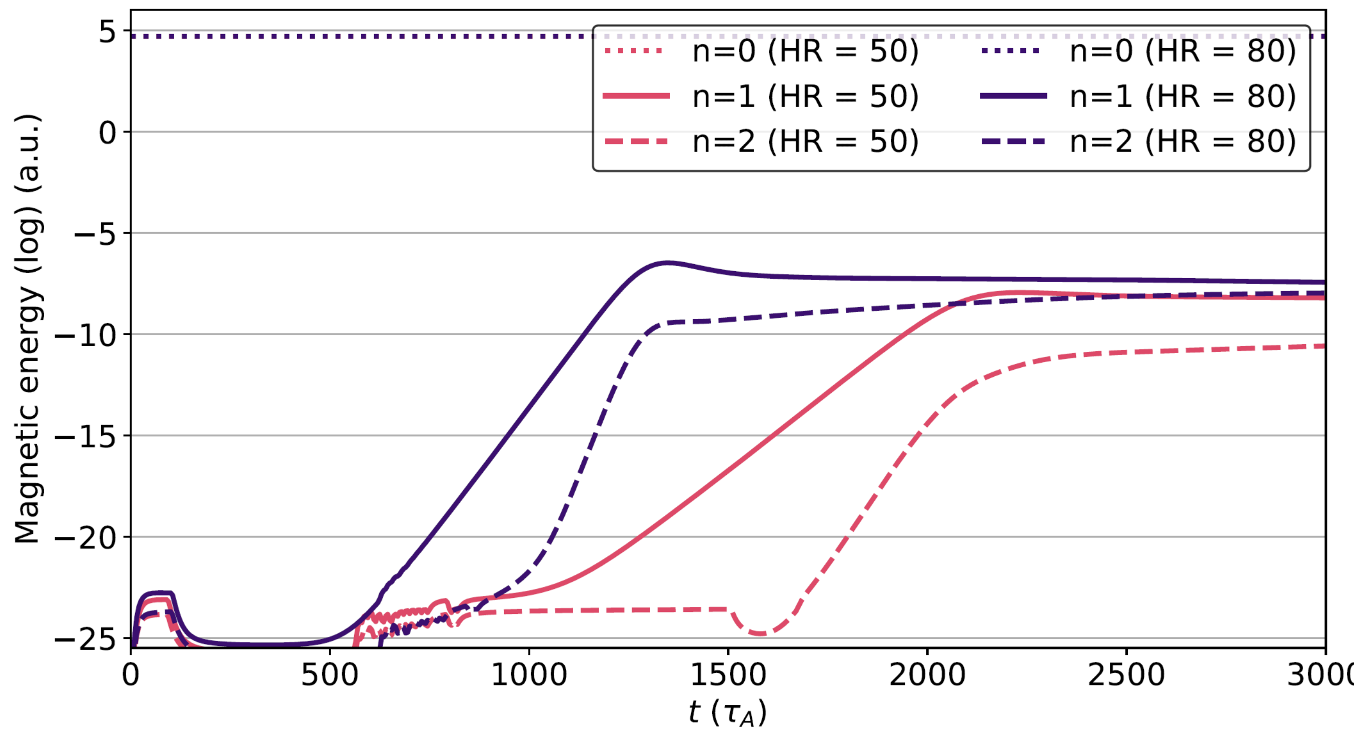
→ Resistive = 70 hrs

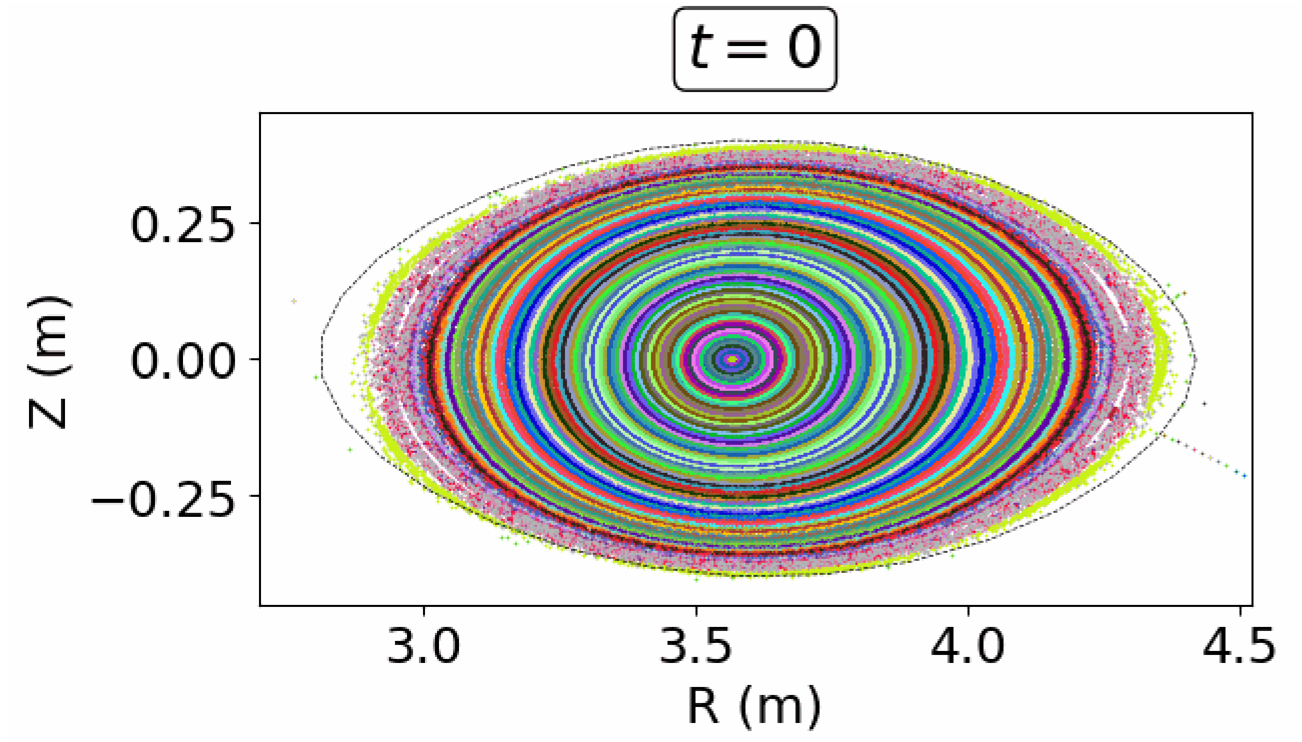
Example: Enhanced temperature flattening due to increased heating power

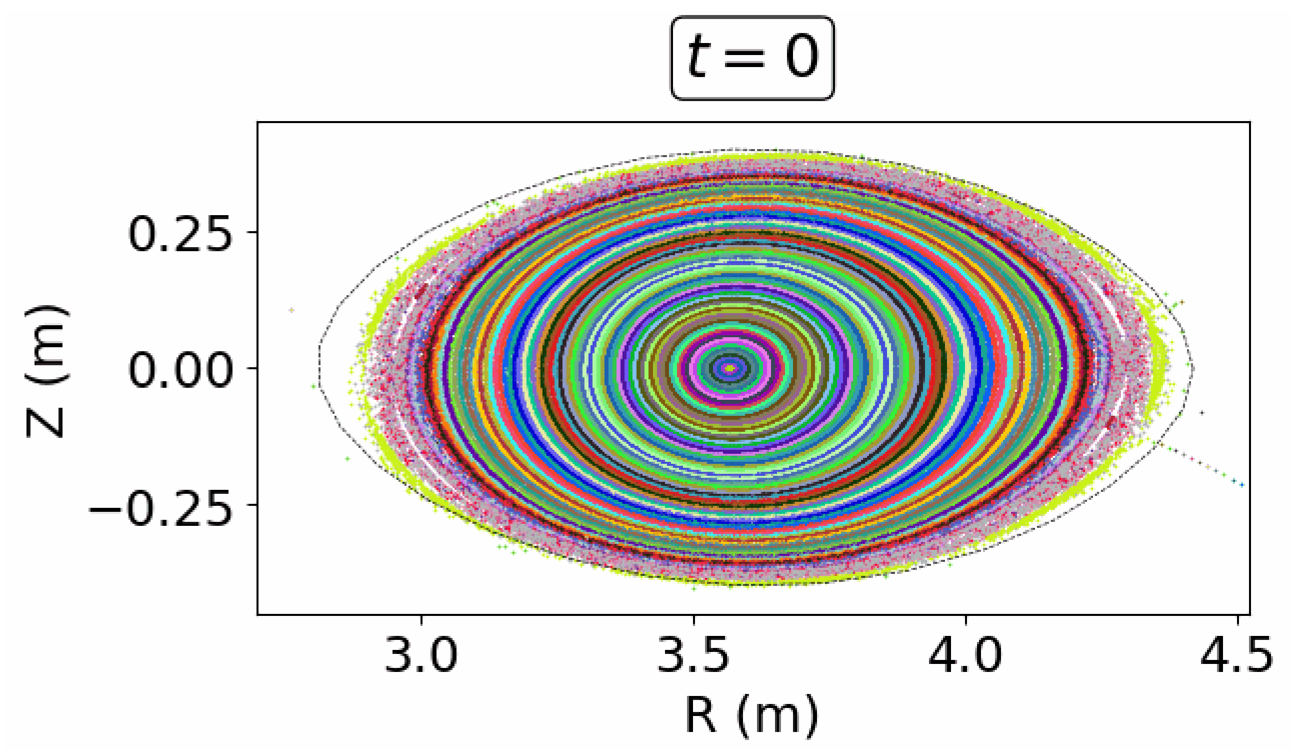
- **Free-boundary simulation of NFP=10 heliotron, with heat source applied to vacuum field.**
- **β is limited by low-n (n=1,n=2) MHD mode activity that leads to flattening of central electron temperature.**

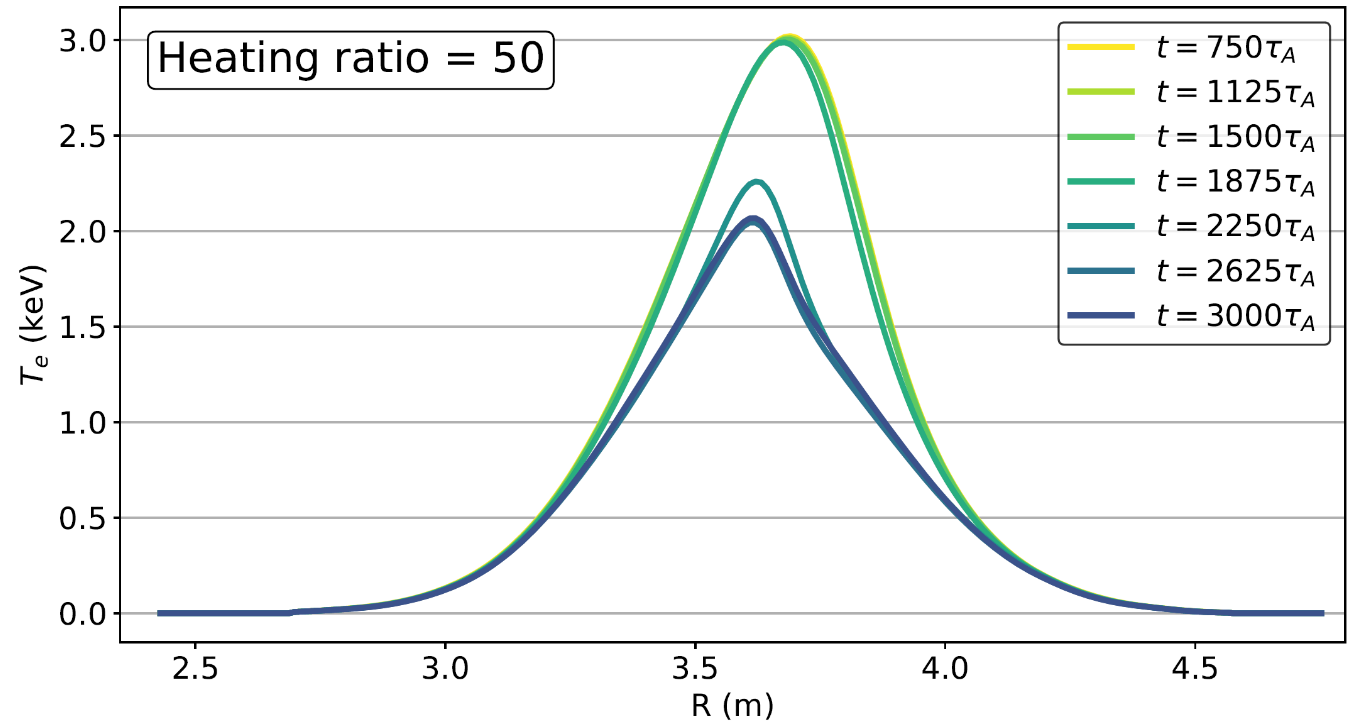


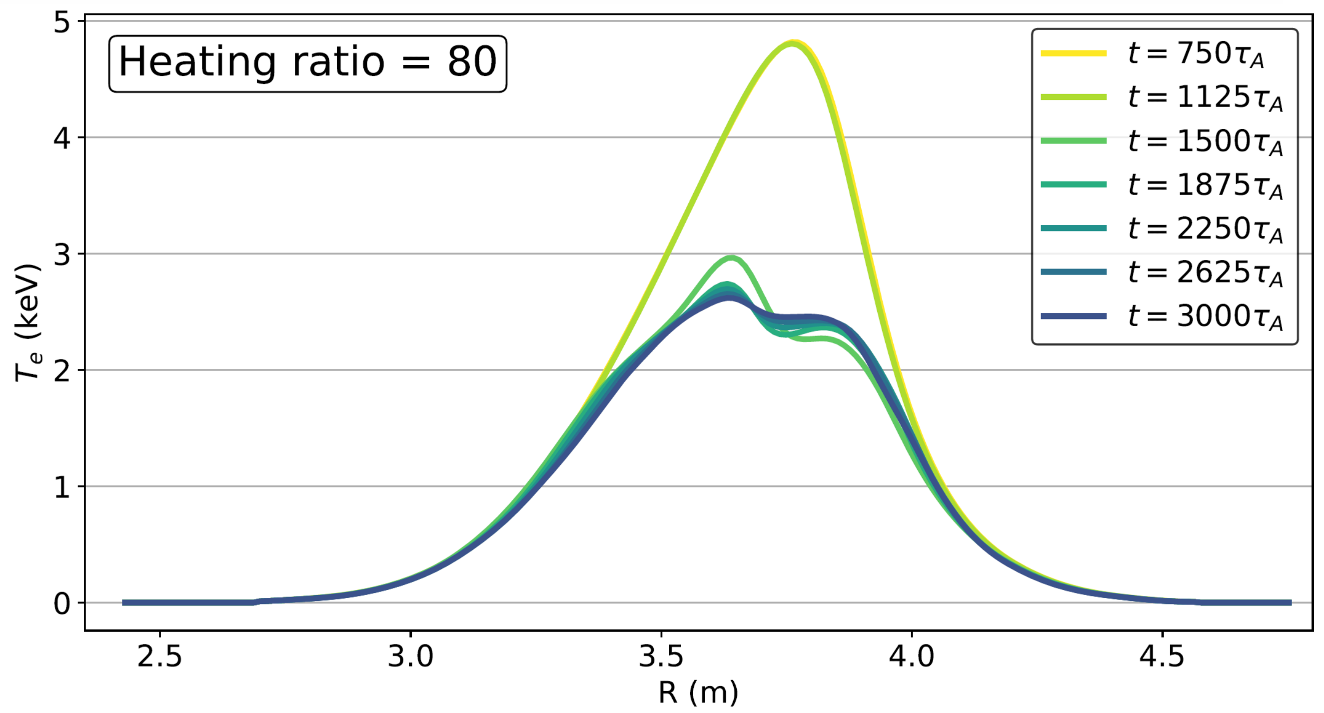












- We find an n=1 stability boundary at $\langle\beta\rangle_{M3D-C1} \sim 0.4\%$ which triggers n=1 core interchange mode, followed by an n=2 mode.
- In the MHD unstable regime, no flattening of the core electron temperature is observed for lower heating case.
- Stronger heating drives growth of the n=2 mode, leading to flattening of core electron temperature profile due to formation of chaotic magnetic fields.

Contents

M3D-C1 physics models in stellarator geometry

- Currently, the single-fluid model has been verified in stellarator geometry for 3D calculations:

M3D-C[1] model (single-fluid)

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{u}) = 0$$

$$nm_i\left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u}\right) = \vec{J} \times \vec{B} - \nabla \mathrm{p} - \nabla \cdot \Pi + \vec{F}$$

$$\frac{\partial p}{\partial t} + \vec{u} \cdot \nabla p + \Gamma p \nabla \cdot \vec{u} = (\Gamma - 1)\left[Q - \nabla \cdot \vec{q} + \eta J^2 - \vec{u} \cdot \vec{F} - \Pi : \nabla \vec{u}\right]$$

$$\vec{E} = -\vec{u} \times \vec{B} + \eta \vec{J}$$

$$\vec{J} = \mu_0^{-1} \nabla \times \vec{B}$$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$\vec{q}_{(i,e)} = -\kappa_{(i,e)}^{\perp} \nabla T_{(i.e)} - \kappa_{(i,e)}^{\parallel} B^{-2} \vec{B}\vec{B} \cdot \nabla \mathrm{T}_{(i,e)}$$

- Two-fluid model (implemented in tokamak geometry) not yet tested in stellarator geometry. Should work, in principle, but may require minor modifications to include terms that are not small when $\partial_\phi \neq 0$.

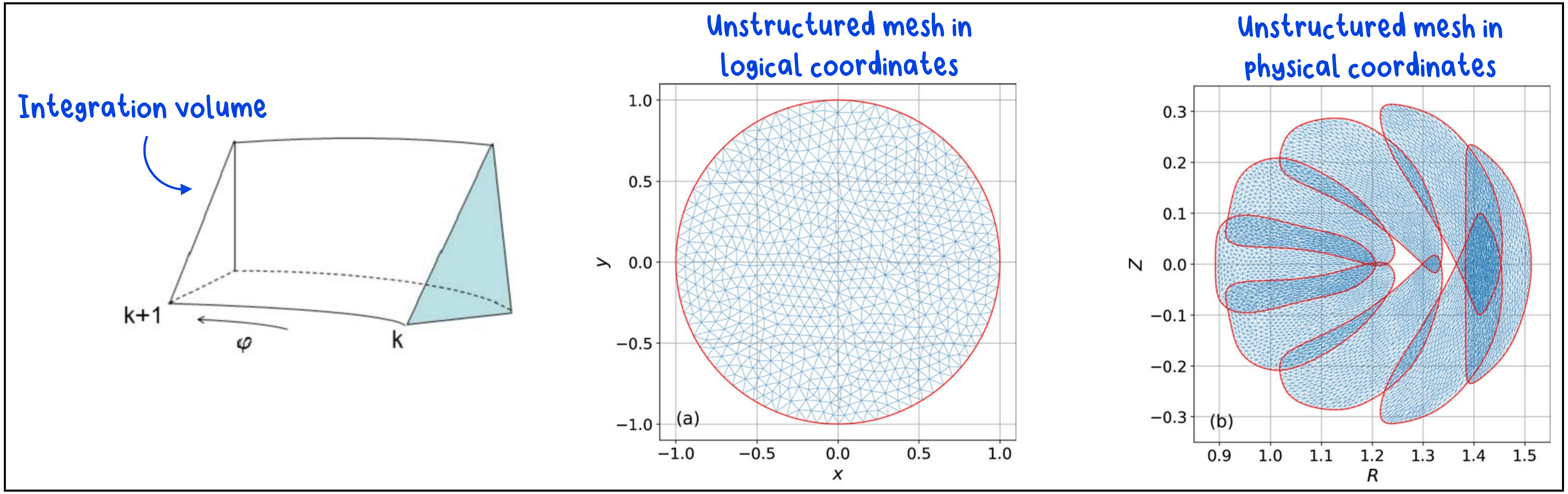
- Because of toroidal mode coupling, there is currently no linear version of the code for stellarator geometry.

- Other physics models that are implemented but not yet tested in stellarator geometry include: pellet injection, torque sources and rotation and resistive wall.

- Model equations expanded in cylindrical (R, φ, Z) coordinates.
- Stellarator geometry: FE defined and numerical integration performed in 'logical' (x, y, z) coordinates.
- Mapping $(x, y, z) \rightarrow (R, \varphi, Z)$ specified by boundary shape. The only condition is that it must preserve C^1-continuity.
- In stellarator geometry, careful treatment of higher order derivatives was required to preserve C^1 property:

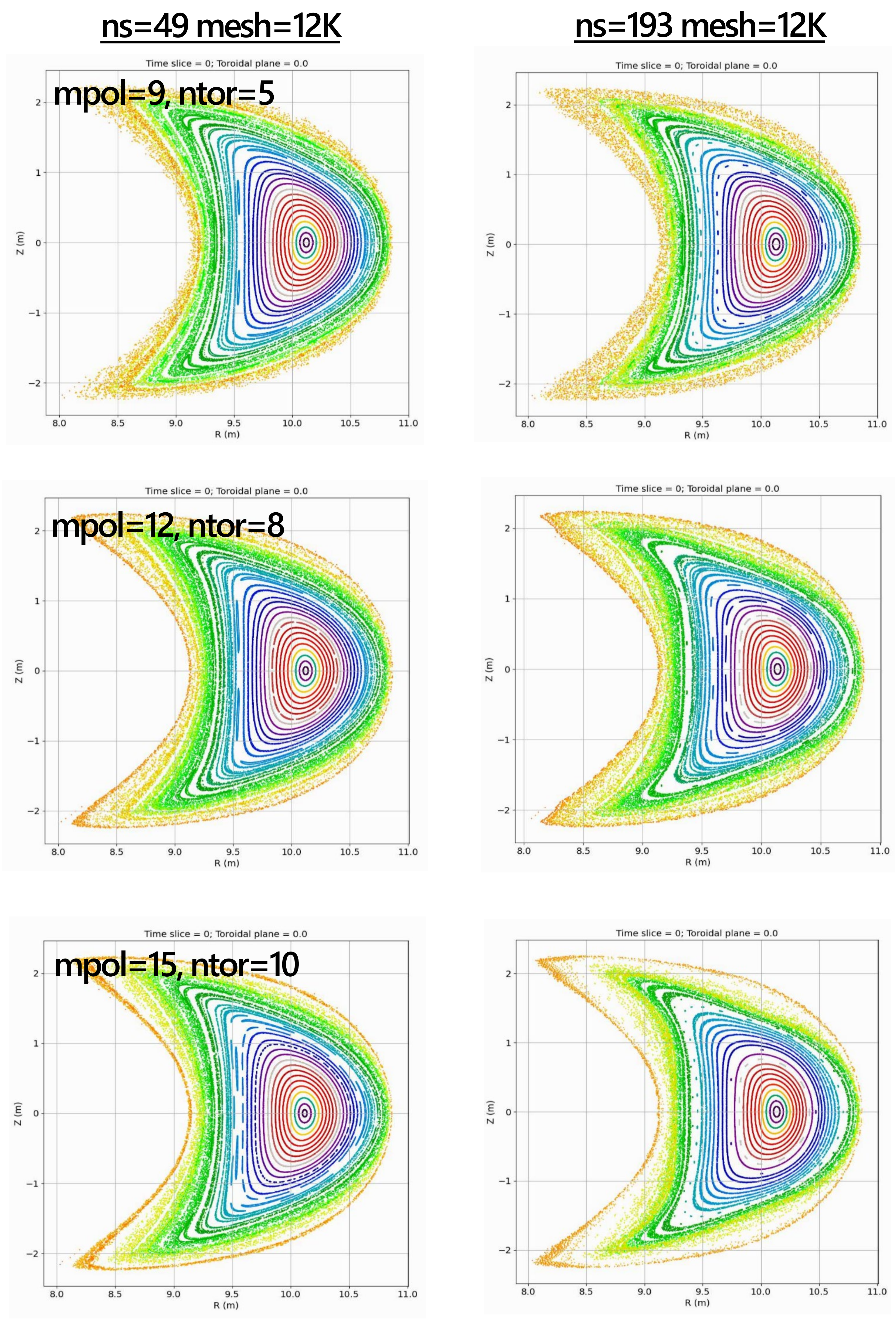
  Tokamak: orthogonality of (R, φ, Z) means derivatives up to 4^{th} order can be integrated (using integration-by-parts).

  Stellarator: (x, y, z) not orthogonal under mapping from (R, φ, Z), $\partial^{(4)}$ terms introduce mixed derivatives that are not C^1.

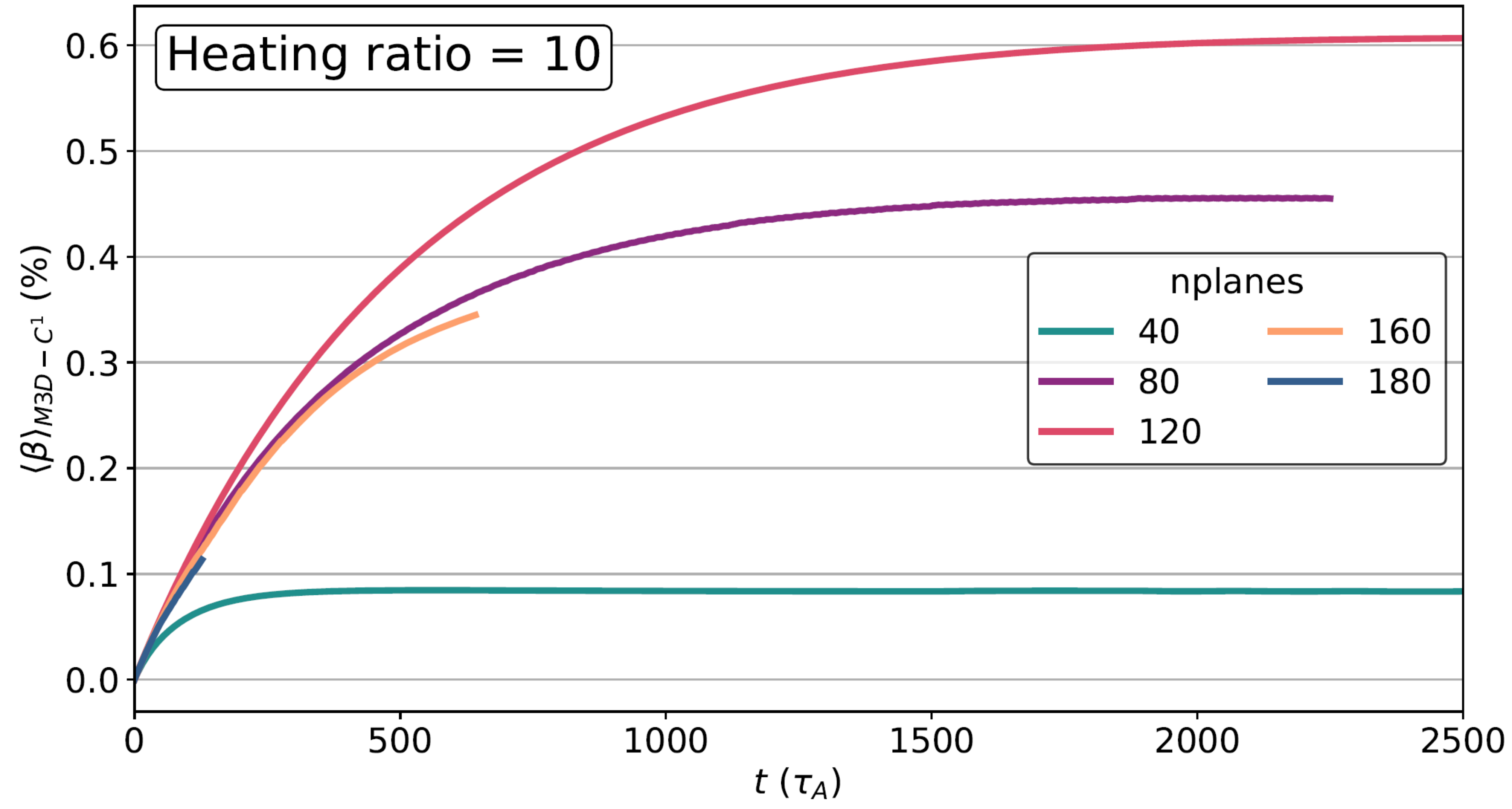


Impact of interpolation on initial magnetic field structure

- In both fixed- and free-boundary, the initial condition for the magnetic field is specified from an input (VMEC, FIELDLINES or MGRID).
- Interpolated onto M3D-C1 mesh with Zernike basis functions: `n_zer=2*mpol` (fixed boundary) or `1*mpol` (free boundary).
- The present scheme doesn't explicitly seek to preserve the topology of the input magnetic field.
- The topological structure of the magnetic field initial condition depends particularly on the resolution of input:
  E.g., VMEC: `ns`, `mpol`, `ntor`.
- Elements=307512, nplanes=24, cross-section=7.57m^2, typical poloidal linear dimension=2.43cm.
- In general, this is not an issue except when the objective is to study the physics interpretation of exactly integrable equilibria, which are not generally considered to be 'common'.

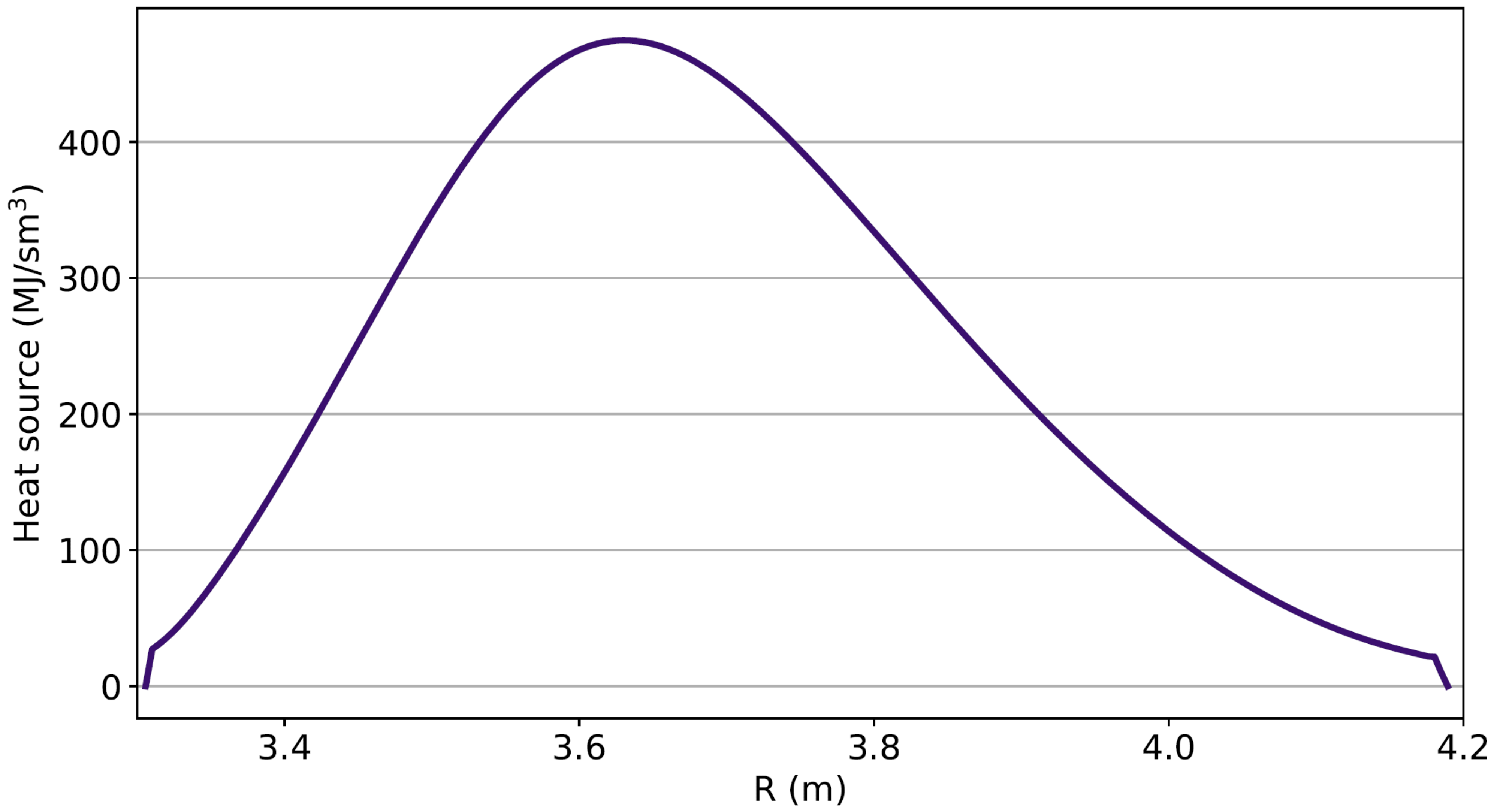


- For the LHD β-limit studies, large `nplanes` is needed for convergence of $\langle\beta\rangle_{M3D-C1}$.
- This leads to more realistic heating power needed to achieve same saturated $\langle\beta\rangle_{M3D-C1}$ (decreased by x5).
- Stability boundary $\langle\beta\rangle_{M3D-C1}{\sim}0.4\%$ and β_{limit} associated with low-n modes (n=1, n=2) so unchanged by lower `nplanes`.

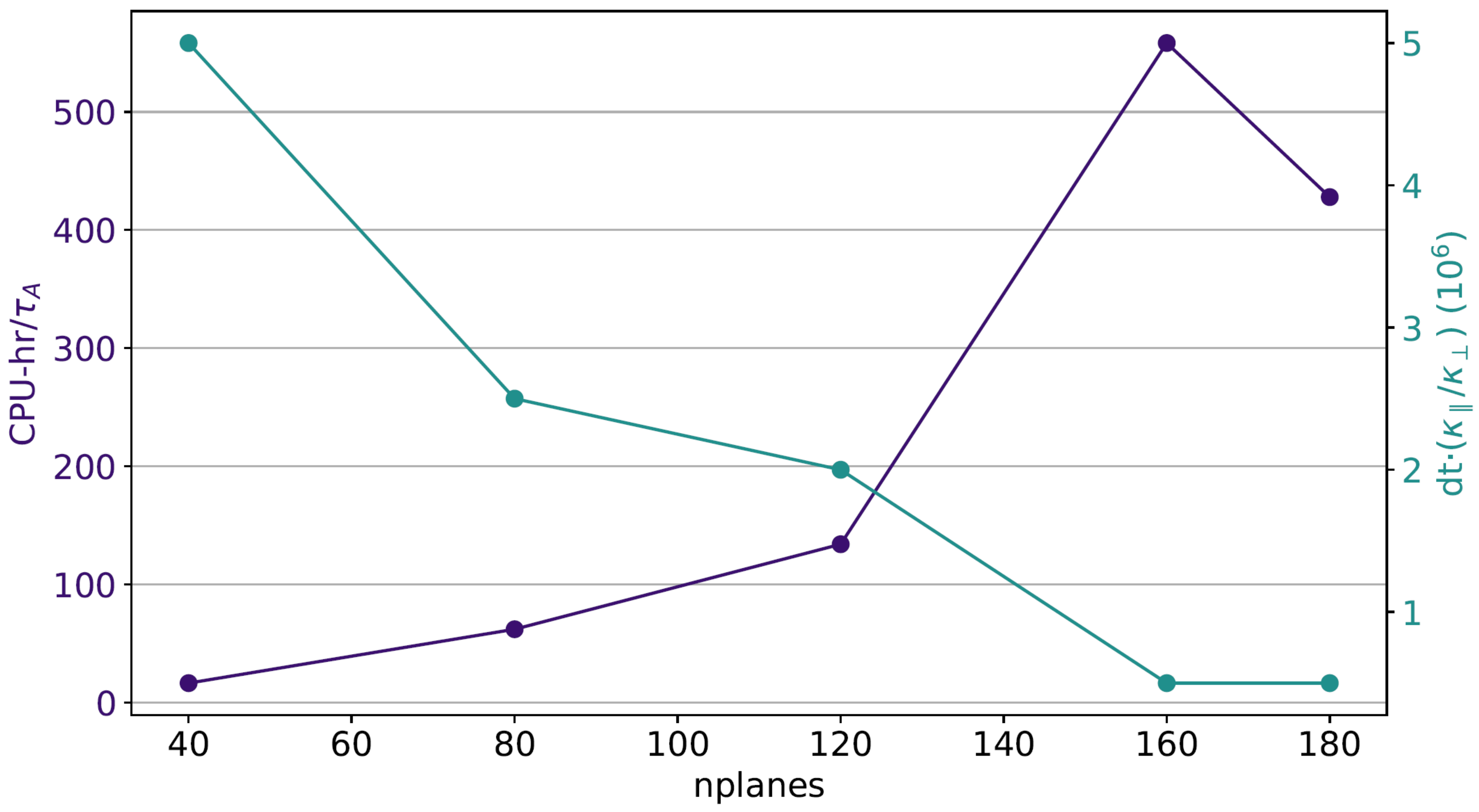


Why higher `nplanes` may be needed for stellarators:

- Discrete N-fold symmetry leads to toroidal mode coupling ("mode families") e.g., $n = 1 + c * N_{FP}$, $c \in \mathbb{Z}$
- Non-axisymmetric (strongly shaped) geometry -> inherently non-integrable magnetic fields -> existence of magnetic surfaces depends sensitively on resolving perturbations to magnetic field structure.
- Lower `nplanes` -> increased effective resistivity.



- CPU hours per τ_A scales strongly with `nplanes`.
- Numerical stability depends strongly on `dt*kappar/kappat`, which scales inversely with `nplanes`.



Comparison of approaches: Logical coordinates vs. adaptation of mesh to geometry

- A comparison [3] between two approaches to accommodating stellarator geometry found that mapping to logical coordinates was still **less expensive** than adapting the mesh to geometry.

Rotating ellipse in stellarator geometry:

Rotating ellipse in tokamak geometry:

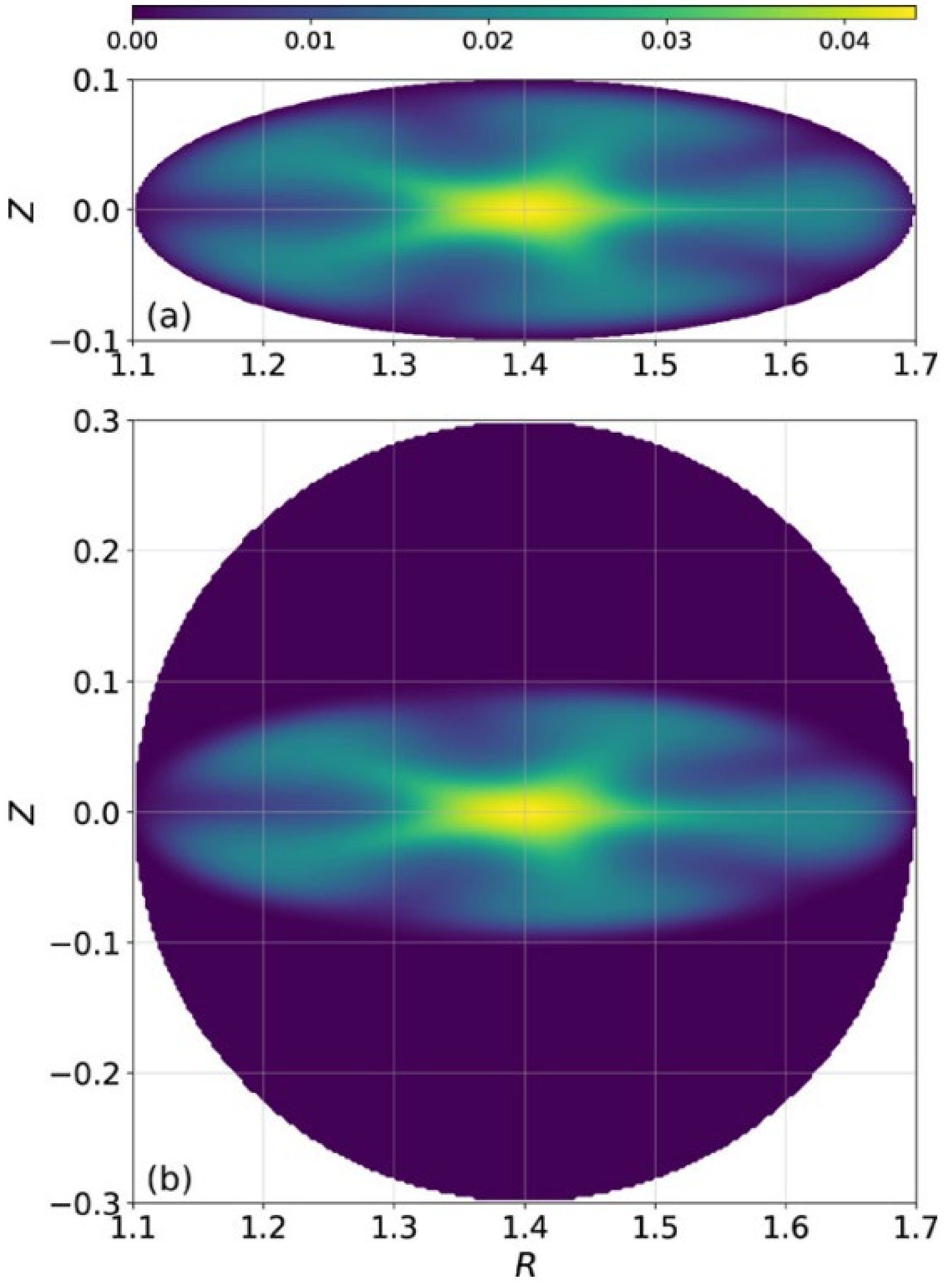


Figure 4. Final snapshot of pressure p at $\varphi = 0$ in the higher resolution simulations using (a) the rotating elliptical domain and (b) the cylindrical domain.

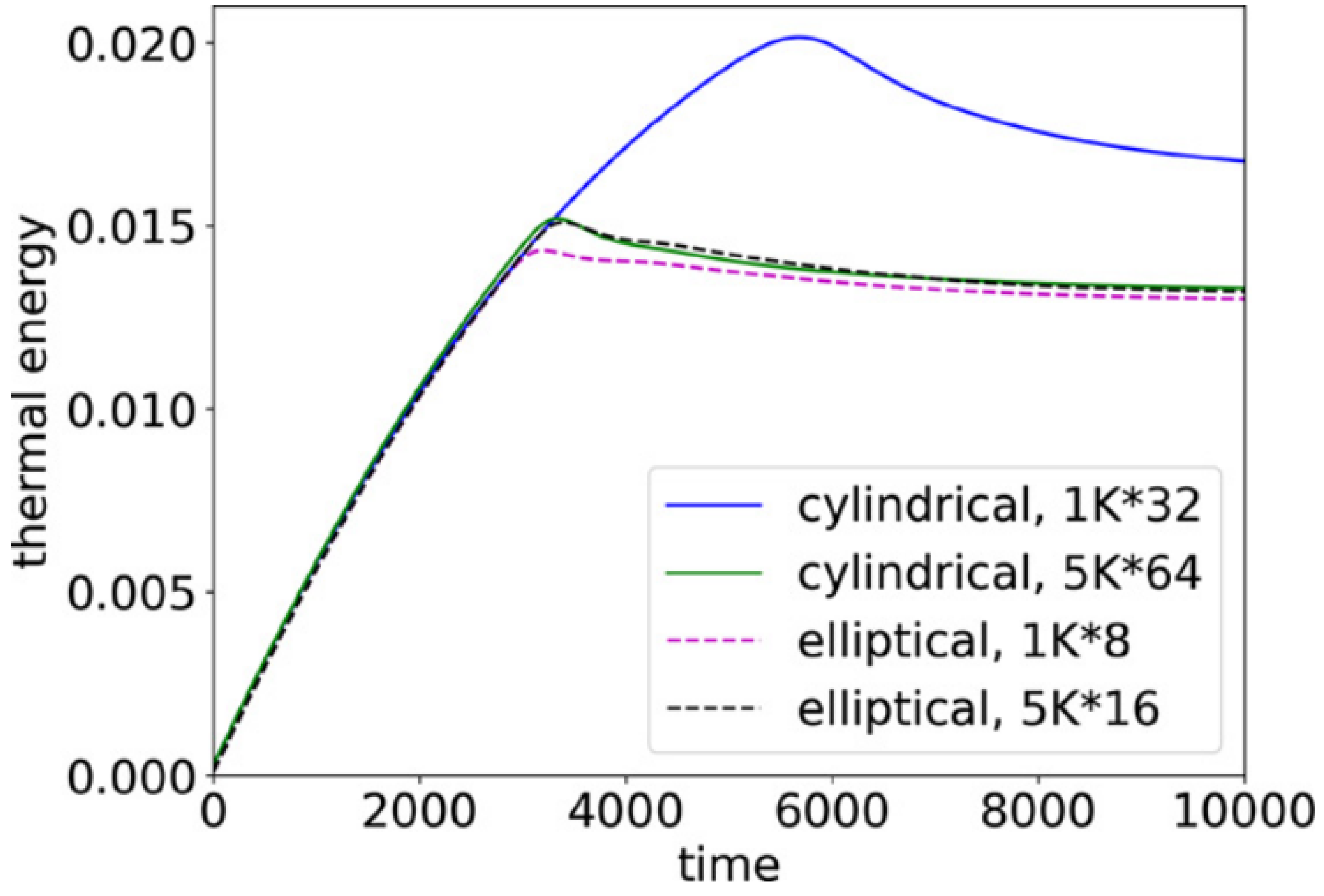


Figure 3. Total thermal energy versus time in simulations using a cylindrical (solid) or a rotating elliptical (dashed) domain. The number of elements at each toroidal plane is 1389 (1 K) in lower resolution runs and 5131 (5 K) in higher resolution runs. Simulations in the cylindrical domain require more toroidal planes (32 and 64) than the rotating elliptical domain (8 and 16) for comparable accuracy. The simulation parameters are $B_0 = 1$, $R_0 = 1$, $R_a = 1.4$, $Z_a = 0$, $\epsilon = 0.8$, $\sigma = 0.05$, $w = \kappa_\perp = \eta = 10^{-6}$, $\kappa_\parallel = 1$, and $\mu = \mu_c = 10^{-4}$.

[3] Zhou et al. Nuclear Fusion 61.8 (2021).

Contents

- *Motivation and existing approaches for MHD modeling in stellarators*
- *Examples*
- *M3D-C1 method and performance in stellarator geometry*
- **Outlook: Next steps**

Code improvements:
- Improve interface for initialising free-boundary equilibria.
- Expand post-processing utilities (incl. tools for analysing 3D field structure).
- Extend interface/options for prescribing (computational) boundary shape.

Physics models:
- Consider implementation of bootstrap current model.
- Multi-region mesh (resistive wall, coils)

On-going physics studies:
- Collaborations with LHD and W7-X (both verification and validation)

Opportunities:
- Lots of MHD experiments being proposed for upcoming W7-X campaign