A New Explanation of Sawtooth Phenomena in Tokamaks

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Acknowledgments:
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Motivation and Summary

- 45 years after it's discovery, there is still no widely-accepted theory for the sawtooth phenomena that is consistent with experimental observations

- The Kadomtsev[1] model is likely valid at low-temperatures & low pressures, but it cannot explain sawteeth in high-T_e, moderate to high β discharges

- There is now experimental and computational evidence that $q_0 \cong 1$ with low central shear in many high-performance discharges that exhibit sawteeth

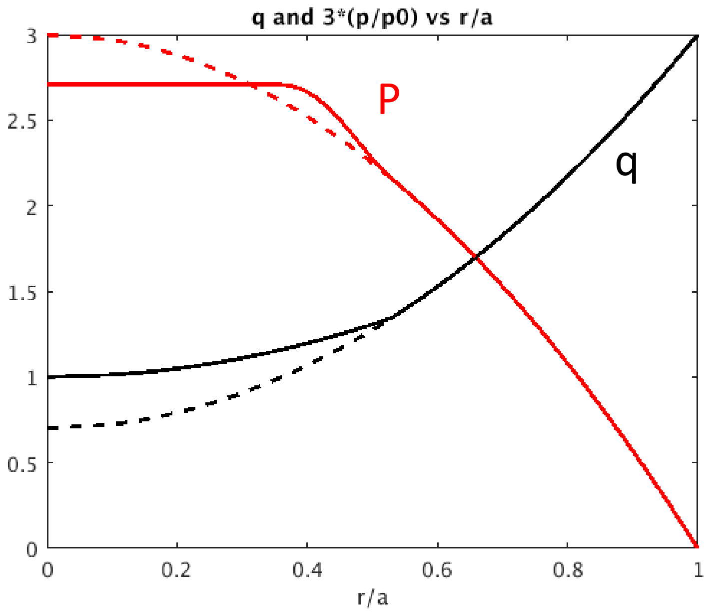
- This can be explained by a modified "interchange" model[2], with the addition of flux-pumping (dynamo) and higher order modes with n=m > 1

[1] Kadomtsev,, B. Fiz. Plazmy 1 710 (1975) [Sov. J. Plasma Phys. 1 389 (1976)

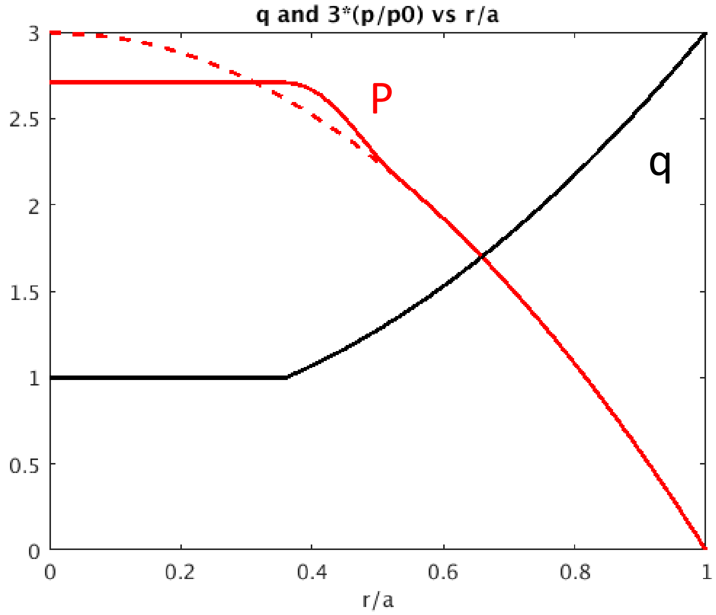
[2] J. A. Wesson, Plasma Physics and Controlled Fusion 28 243 (1986)

Two Dominant Competing Theories of Sawteeth

Kadomtsev (1975)



Wesson (1986)



Difference is in the evolution of the q-profile and the mechanism for the crash in p.

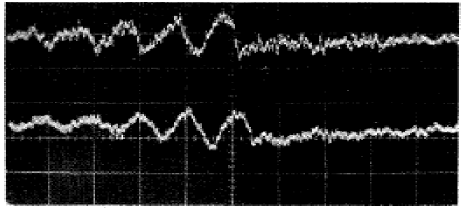
First measurement of sawtooth oscillations were in a low S, low β discharge

VOLUME 33, NUMBER 20 PHYSICAL REVIEW LETTERS 11 NOVEMBER 1974

Studies of Internal Disruptions and $m = 1$ Oscillations in Tokamak Discharges with Soft–X-Ray Techniques*

S. von Goeler, W. Stodiek, and N. Sauthoff
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540

(Received 11 July 1974)



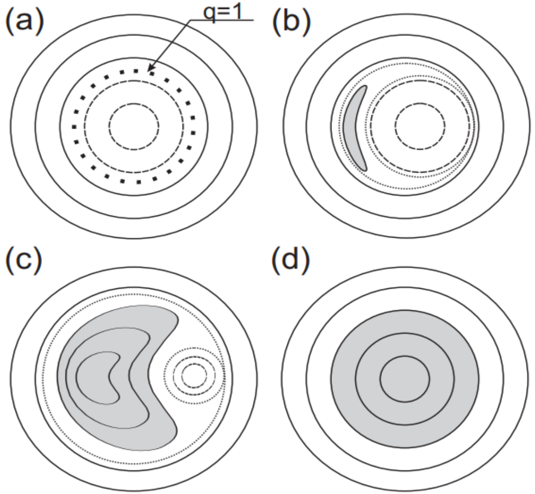
t (100 µs/cm) →

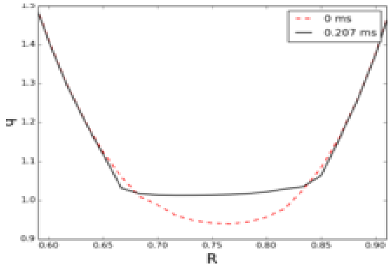
- ST Tokamak: T_{e0} = 800 eV , n_{e0} = .5 x 10^{14}
- Quasi-periodic (1,1) oscillations in central temperature (~ 100 μs)
- Low S ~ 10^5, low β < 1 %

$$S \equiv \frac{\tau_R}{\tau_A} = \frac{a^2 B_0}{\eta R}\left[\frac{\mu_0}{n_0 M_i}\right]^{1/2}$$

(Lundquist number)

Oscillations were explained shortly afterwards by Kadomtsev





- Current peaks and q_0 drops below 1 due to resistive diffusion with peaked temperature profile

$$\tau_R^{-1} \sim \eta \quad \sim S^{-1}$$

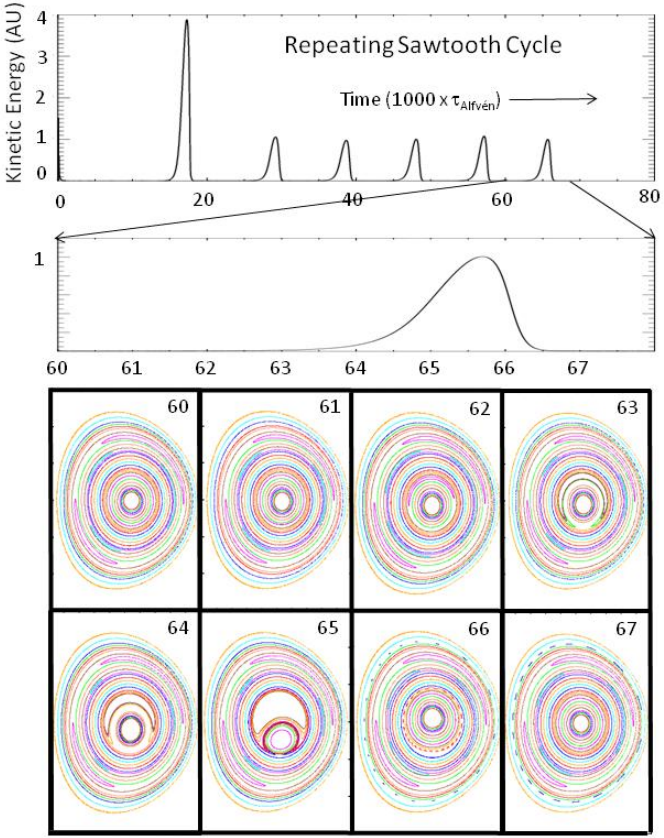
- When $q_0 < 1$, (1,1) resistive kink instability begins to grow.

$$\gamma \sim \eta^{1/3} \sim S^{-1/3}$$

- After several e-folding times, complete reconnection restores q_0 to 1 as the (1,1) island displaces the center surfaces.

Kadomtsev,, B. Fiz. Plazmy 1 710 (1975) [Sov. J. Plasma Phys. 1 389 (1976)

This "Kadomtsev reconnection" has been reproduced in many long-time nonlinear 3D resistive and 2F MHD simulations at low β & S



- Shown is a long-time M3D-*C*[1] simulation at $S \sim 10^5$, $\beta \sim .06$ %
- Repeated reconnection events occur. Well described by Kadomtsev model
- However, this model does not scale to high $S \gg 10^6$
- Since (1,1) growth rate is much faster than current diffusion rate, there will be negligible drop in q_0 before mode grows up.

$$\gamma \, \tau_R \sim \eta^{-2/3} \sim S^{2/3} \gg 1$$

High-T_e plasmas show much faster crash times than $\eta^{1/3}$

Investigation of magnetic reconnection during a sawtooth crash in a high-temperature tokamak plasma

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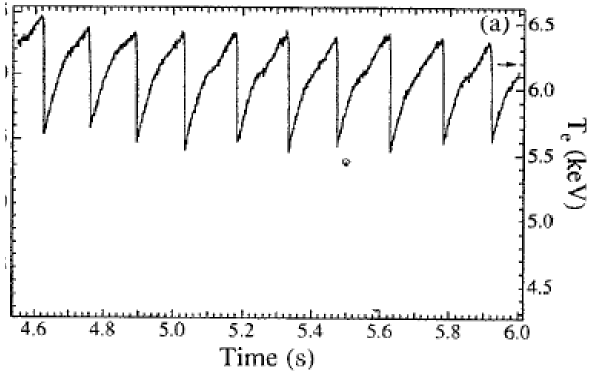
Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543

(Received 2 March 1994; accepted 9 June 1994)



- TFTR electron temperature crash times were very fast, ~ 100 μs. Even though T_e is over 10 times greater than T_e in the ST, crash times are comparable.

➢ Sawtooth fast crash times on TFTR and other large tokamaks not consistent with original Kadomtsev model

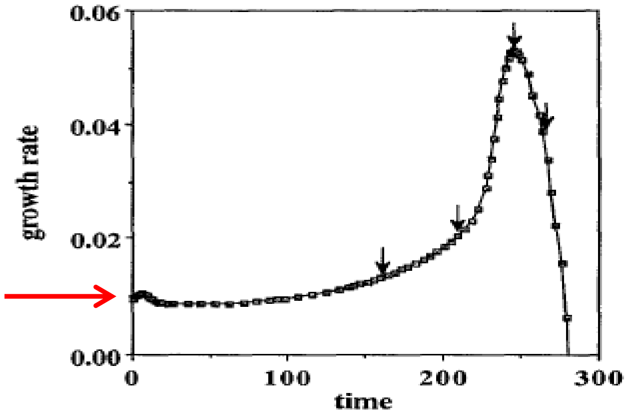
Many theory papers have offered explanations for fast crash times

With the Kadomtsev model in mind, many authors have "explained" fast crashes as being due to *fast magnetic reconnection*:

- Anomalous electron viscosity[1]
- Two-fluid effects [2-4]
- High-n ballooning modes [5]
- Plasmoids [6]
- Plasma compressibility [7]

[1] Aydemir, A. Y., Phys. Fluids B 2 2135 (1990)
[2] Aydemir, A. Y., Phys. Fluids B 4 3469 (1992)
[3] Yu, Q., Gunter, S., and Lackner, K., Nucl. Fusion 55 113008 (2015)
[4] Beidler, M., Cassak, P., Jardin, S., Ferraro, N., Plasma Phys. and Control. Fusion 59 025007 (2017)
[5] Nishimura, Y., Callen, J. D., Hegna, C., Phys. Plasma 64685 (1999)
[6] Gunter, S., Yu, Q., Lackner, K., et al. Plasma Phys. Control. Fusion 57 104017 (2015)
[7] Sugiyama, L. Phys. Plasmas 21, 022510 (2014)

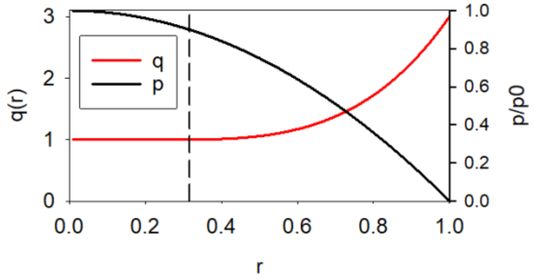
- However, all these numerical studies are initialized with an *unstable* plasma with $q_0 << 1$
- How did the plasma get into this initial unstable state ? Repeatable?
- Need to simulate *multiple* sawteeth to negate effect of initial contitions.



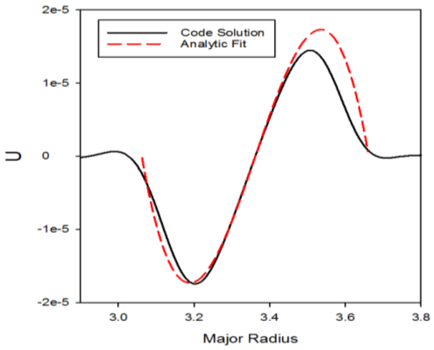
From Ref. [2]

An alternative to Kadomtsev model is the interchange model

- First introduced by Wesson [8] (coined the name quasi-interchange)
- A tokamak with q_0 slightly exceeding 1 and with very low central shear is unstable to a pressure-driven (1,1) interchange mode.

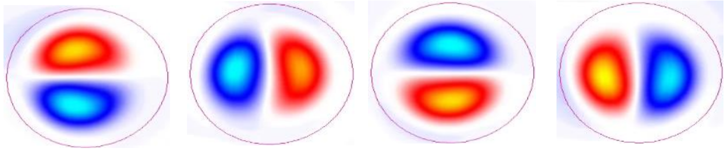


$$\mathbf{V}_{1,1} = R^2 \nabla U_{1,1} \times \nabla \varphi$$



- This (1,1) flow field found in M3D-C^1 simulations agrees with the linear eigenfunction found analytically [9,10]
- We now know that this (1,1) interchange mode saturates at a low amplitude thru dynamo effect (more later)

$U_{1,1}$



$\varphi = 0^o$ $\varphi = 90^o$ $\varphi = 180^o$ $\varphi = 270^o$

[8] J. Wesson, PPCF 28 243 (1986)
[9] J. Hastie and T. Hender, NF 28 585 (1988)
[10] F. Waelbroeck and R. Hazeline, PF 31 1217 (1988)

Both the Kadomtsev and Wesson Models are Incomplete

Kadomtsev (reconnection)

- How to explain fast crash times (ideal MHD time scale) in high-Te, high-β experiments
- What triggers the sudden crash?
- How to explain recent experimental measurements[1] that $q_0 \cong 1$ before and after the crash?

Wesson (Interchange)

- Why does q_0 stay at 1 in the center (with low shear)?
- What triggers the sudden crash?

[1]Nam, Y. B., Ko, J. S., Choe, G. H. et al Nucl. Fusion **58** 066009 (2018)

Both the Kadomtsev and Wesson Models are Incomplete

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Only at low T_e, low-β

Wesson (Interchange)

- Why does q_0 stay at 1 in the center (with low shear)?
  - **(1,1) interchange mode saturates and produces central loop voltage thru dynamo effect**
- What triggers the sudden crash?
  - **Ideal MHD stability boundary for modes with m=n>1 is crossed when central pressure increases sufficiently**

[1]Nam, Y. B., Ko, J. S., Choe, G. H. et al Nucl. Fusion **58** 066009 (2018)

(1,1) flow field produces a dynamo voltage that opposes drop in q_0

$$\underline{\nabla\Phi_{1,1} - \mathbf{V}_{1,1}\times\mathbf{B}} = -\eta\mathbf{J} + \frac{V_L}{2\pi}\nabla\varphi$$

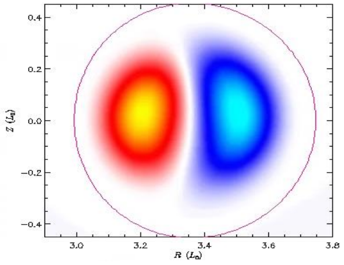
← Steady State Ohm's law

↑

These 2 large terms must almost cancel

- Perturbed electric potential $\Phi_{1,1}$ very similar in form to perturbed stream function $U_{1,1}$
- (1,1) velocity field also creates a $\mathbf{B}_{1,1}$ perturbed magnetic field:
- Perturbed electric potential and magnetic field produce a counter loop-voltage in center, keeping q_0 from dropping below 1:

potential $\Phi_{1,1}$ at one toroidal plane



$$\partial\mathbf{B}_{1,1} \sim \nabla\times\left(\mathbf{V}_{1,1}\times\mathbf{B} + \ldots\right)$$

$$V_{0,0} = \mathbf{B}_{1,1}\bullet\nabla\Phi_{1,1} + \cdots$$

[1]Jardin, Ferraro, Krebs, PRL , 21 215001 (2015)
[2]Krebs, Jardin, Guenter, et al, Phys. Plasmas 24 102511 (2017)

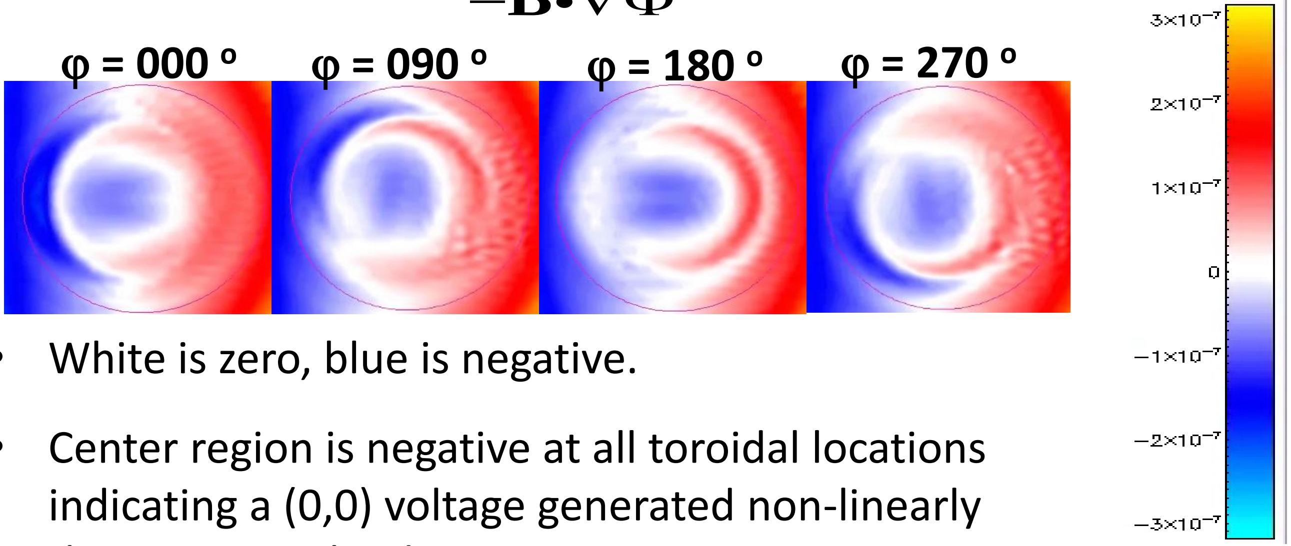
Consider the terms in the parallel Ohm's law

In 3D, the $\mathbf{B}_{1,1} \bullet \nabla\Phi_{1,1}$ term leads to an effective voltage along the field in center (dynamo effect)

$$\mathbf{B}\bullet\left[\nabla\Phi - \mathbf{V}\times\mathbf{B} + \eta\mathbf{J} = \frac{V_L}{2\pi}\nabla\varphi\right]$$

$$\Rightarrow \qquad \eta\mathbf{J}\bullet\mathbf{B} = -\mathbf{B}\bullet\nabla\Phi + \frac{V_L}{2\pi}\mathbf{B}\bullet\nabla\varphi$$

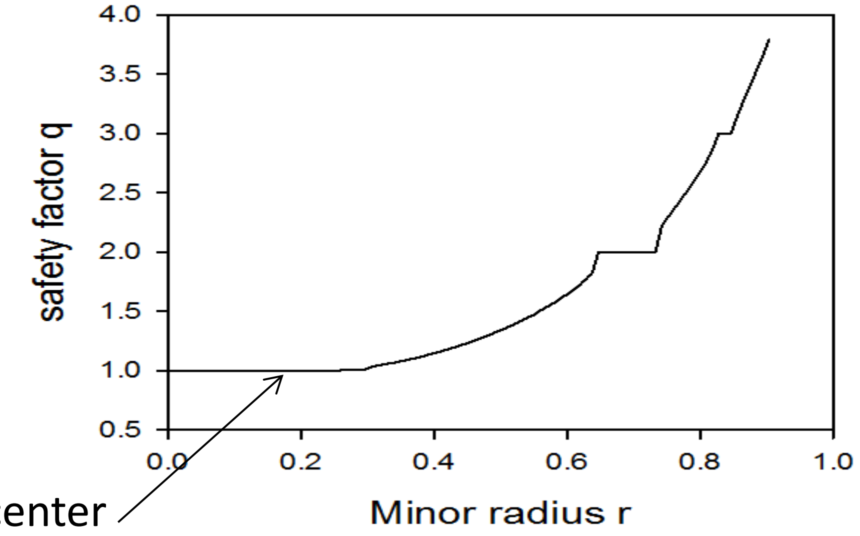
$$-\mathbf{B}\bullet\nabla\Phi$$



- White is zero, blue is negative.
- Center region is negative at all toroidal locations indicating a (0,0) voltage generated non-linearly that opposes the drop in q_0
- This mechanism keeps $q_0 = 1 + \varepsilon$ as shown on next slide

The $V_{0,0}$ voltage from $B_{1,1} \bullet \nabla \Phi_{1,1}$ keeps $q_0 \cong 1$

Results from long-time M3D-C^1 simulation

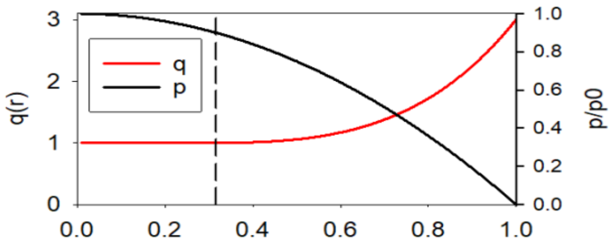


$$\beta = 2\%$$
$$S = 10^6$$

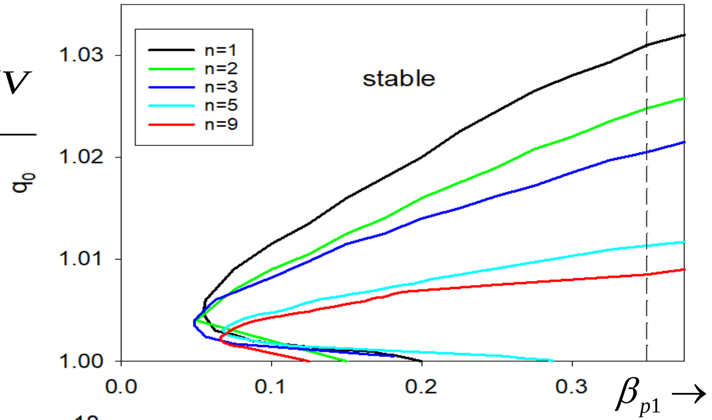
Large region in center with $q = 1+\varepsilon$

- Since the interchange instability drive and hence $\Phi_{1,1}$ is strongest at $q_0 = 1+\varepsilon$, this provides a natural feedback mechanism that keeps q_0 just above 1.0

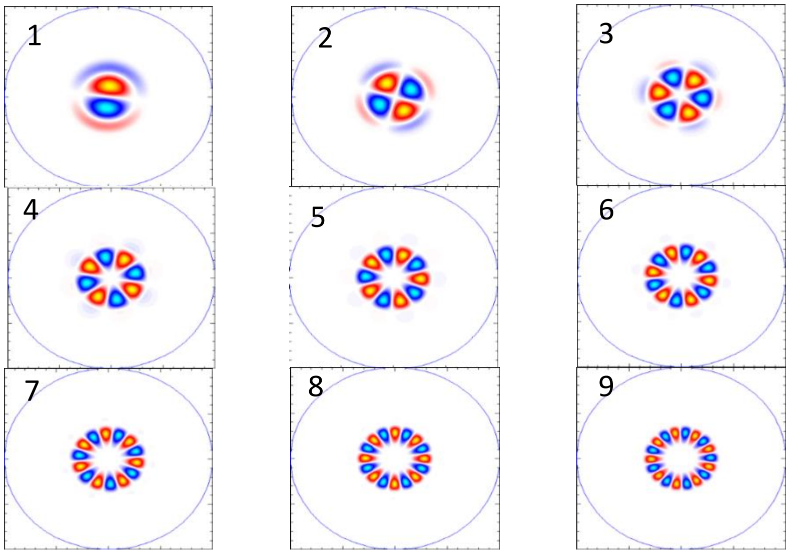
What causes T_e crash? Consider linear stability of modes with n=1-9 in circular cylinder geometry

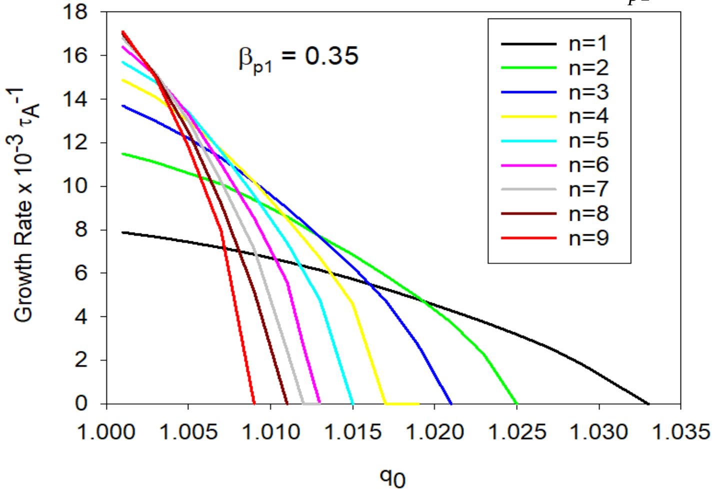


$$\beta_{p1} = \frac{\int_0^{r_1} [p - p_1] dV}{\int_0^{r_1} \frac{1}{2} B_p^2 dV}$$



Linear Eigenfunctions of m=n=1-9





3D Extended MHD Equations in M3D-C[1]

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\mathbf{V}) = \nabla \bullet D_n \nabla n + S_n$$

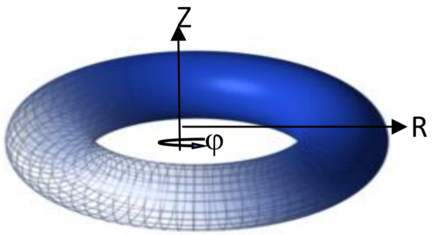
$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla\Phi, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla_\perp \bullet \frac{1}{R^2}\nabla\Phi = -\nabla_\perp \bullet \frac{1}{R^2}\mathbf{E}$$

$$nM_i\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\bullet\nabla\mathbf{V}\right) + \nabla p = \mathbf{J}\times\mathbf{B} - \nabla\bullet\mathbf{\Pi}_i + \mathbf{S}_m$$

$$\mathbf{E} + \mathbf{V}\times\mathbf{B} = \frac{1}{ne}\left(\mathbf{R_c} + \mathbf{J}\times\mathbf{B} - \nabla p_e - \nabla\bullet\mathbf{\Pi}_e\right) - \frac{m_e}{e}\left(\frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e\bullet\nabla\mathbf{V}_e\right) + \mathbf{S}_{CD}$$

$$\frac{3}{2}\left[\frac{\partial p_e}{\partial t} + \nabla\bullet\left(p_e\mathbf{V}\right)\right] = -p_e\nabla\bullet\mathbf{V} + \frac{\mathbf{J}}{ne}\bullet\left[\frac{3}{2}\nabla p_e - \frac{5}{2}\frac{p_e}{n}\nabla n + \mathbf{R}_c\right] + \nabla\left(\frac{\mathbf{J}}{ne}\right):\mathbf{\Pi}_e - \nabla\bullet\mathbf{q}_e + Q_\Delta + S_{eE}$$

$$\frac{3}{2}\left[\frac{\partial p_i}{\partial t} + \nabla\bullet\left(p_i\mathbf{V}\right)\right] = -p_i\nabla\bullet\mathbf{V} - \mathbf{\Pi}_i:\nabla\mathbf{V} - \nabla\bullet\mathbf{q}_i - Q_\Delta + S_{iE}$$



$$\mathbf{V}_e = \mathbf{V}_i - \mathbf{J}/ne$$

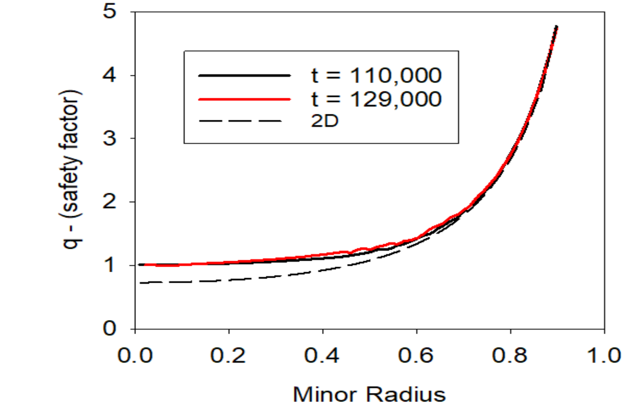
$$\mathbf{R}_c = \eta ne\mathbf{J}, \qquad \mathbf{\Pi}_i = -\mu\left[\nabla\mathbf{V} + \nabla\mathbf{V}^\dagger\right] - 2(\mu_c - \mu)(\nabla\bullet\mathbf{V})\mathbf{I} + \mathbf{\Pi}_i^{GV}$$

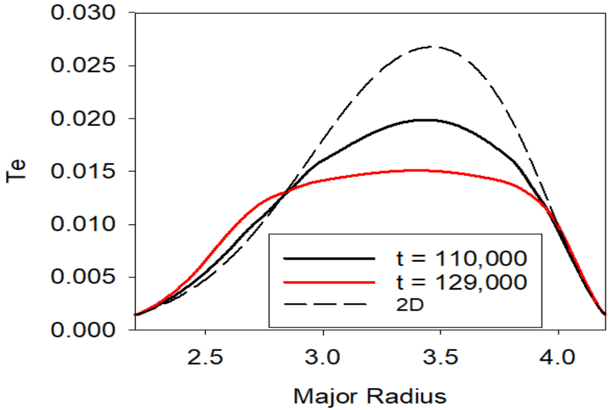
$$\mathbf{q}_{e,i} = -\kappa_{e,i}\nabla T_{e,i} - \kappa_\parallel \nabla_\parallel T_{e,i}$$

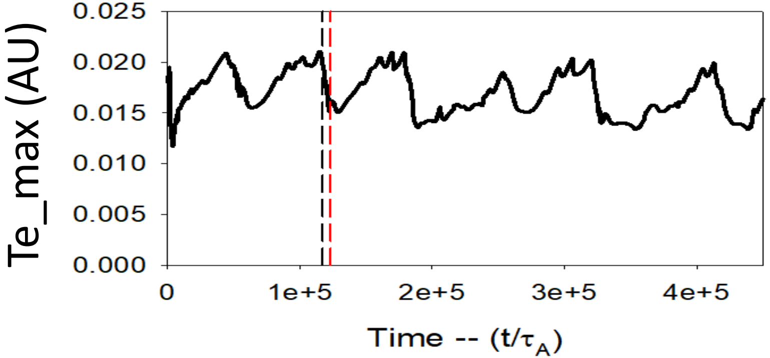
$$\mathbf{\Pi}_e = (\mathbf{B}/B^2)\nabla\bullet\left[\lambda_h\nabla\left(\mathbf{J}\bullet\mathbf{B}/B^2\right)\right], \qquad Q_\Delta = 3m_e(p_i - p_e)/\left(M_i\tau_e\right)$$

Blue terms are 2-fluid terms. Loop voltage at boundary, V_L, adjusted to keep I_P fixed. Energy and particle sources adjusted to keep β and <n> fixed.

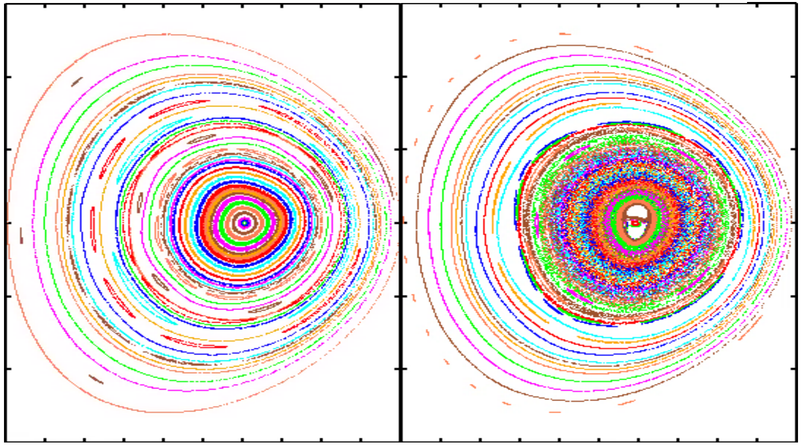
Central heating leads to periodic oscillations in Te(0)







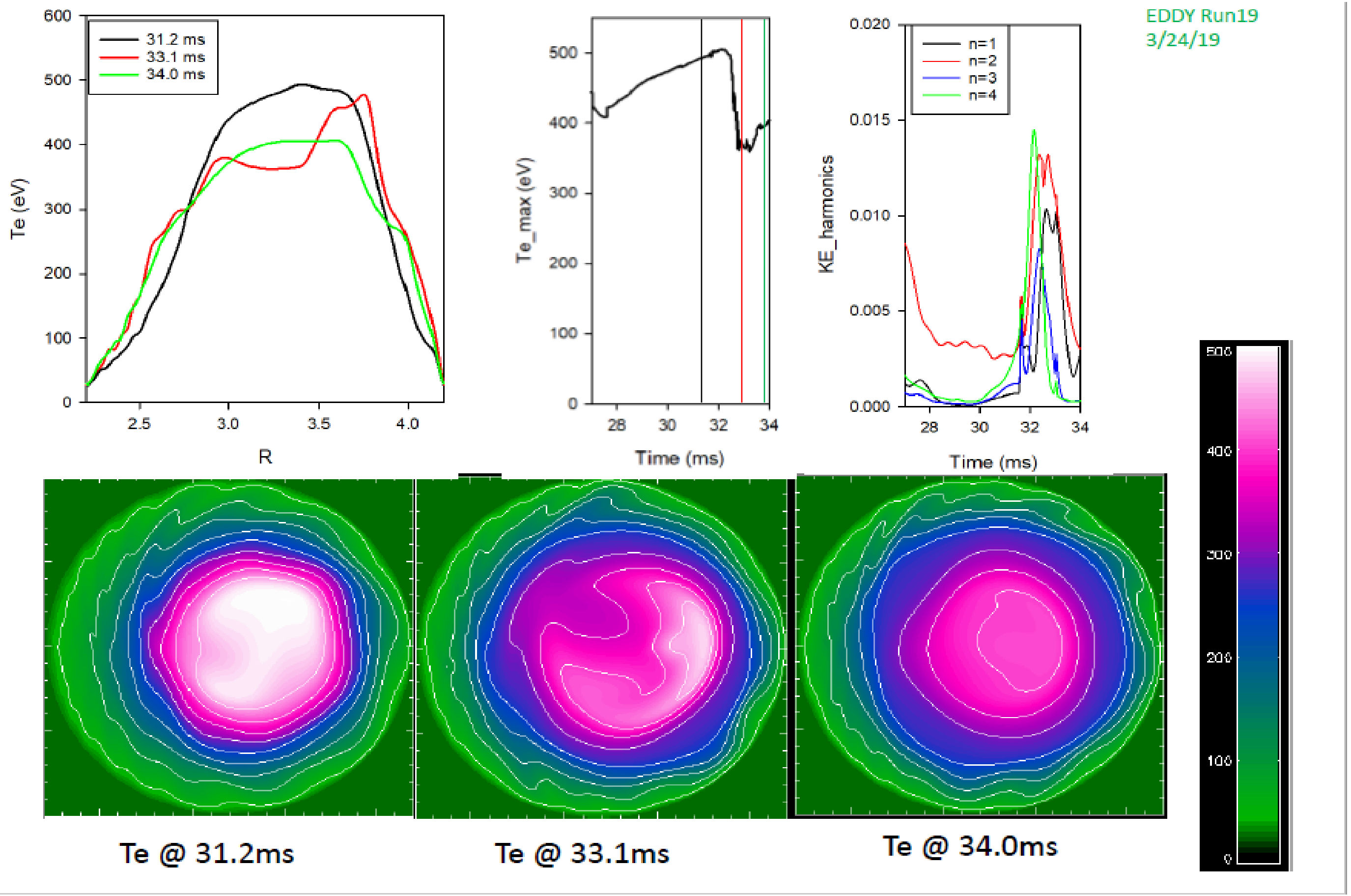
$B_T = 1$ T
$R_0 = 3.2$ m
$n_0 = 4x10^{19}$



t = 110,000 t = 129,000

Run CMOD-04

Clearly shows fast crash due to higher-n modes

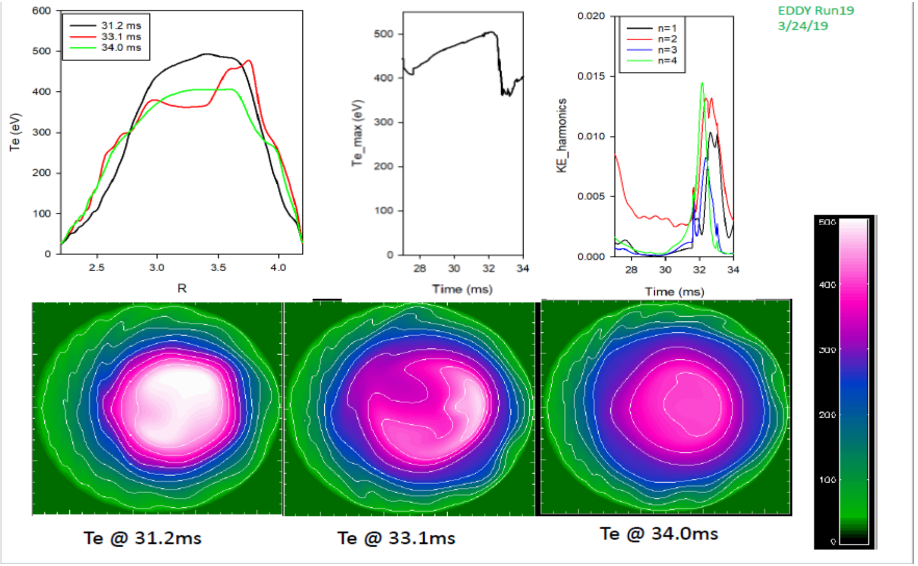


Te @ 31.2ms

Te @ 33.1ms

Te @ 34.0ms

Note similarities with published TFTR crash data



Investigation of magnetic reconnection during a sawtooth crash in a high-temperature tokamak plasma

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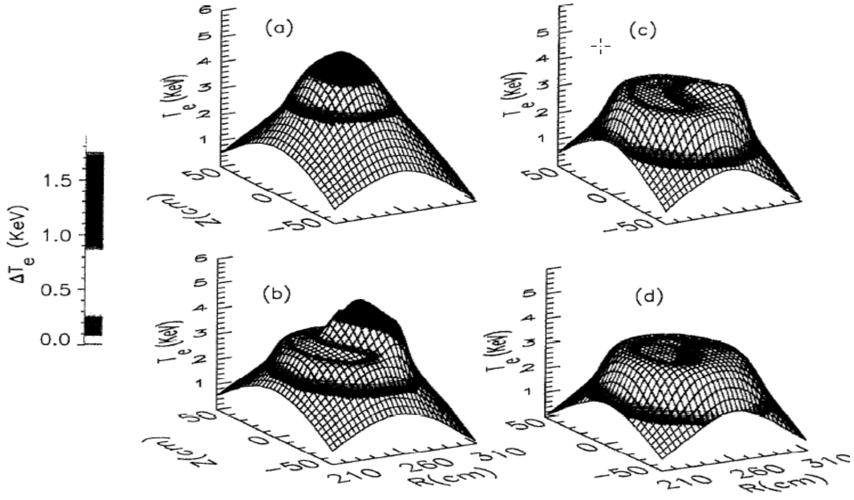
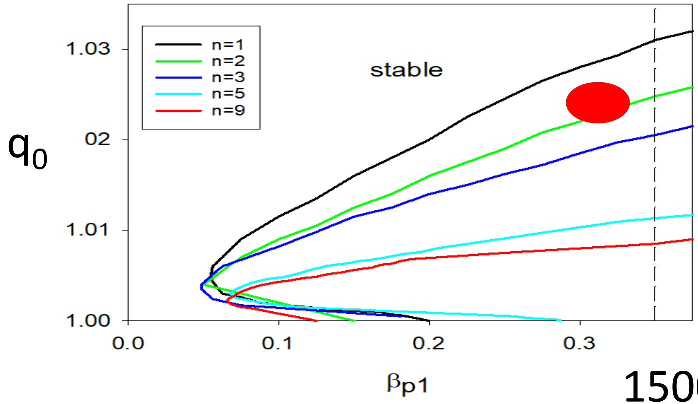
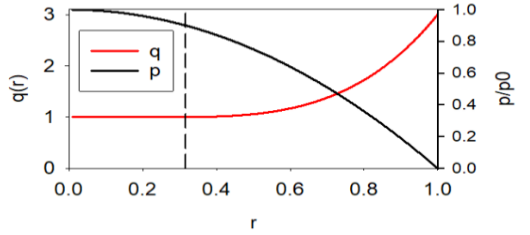
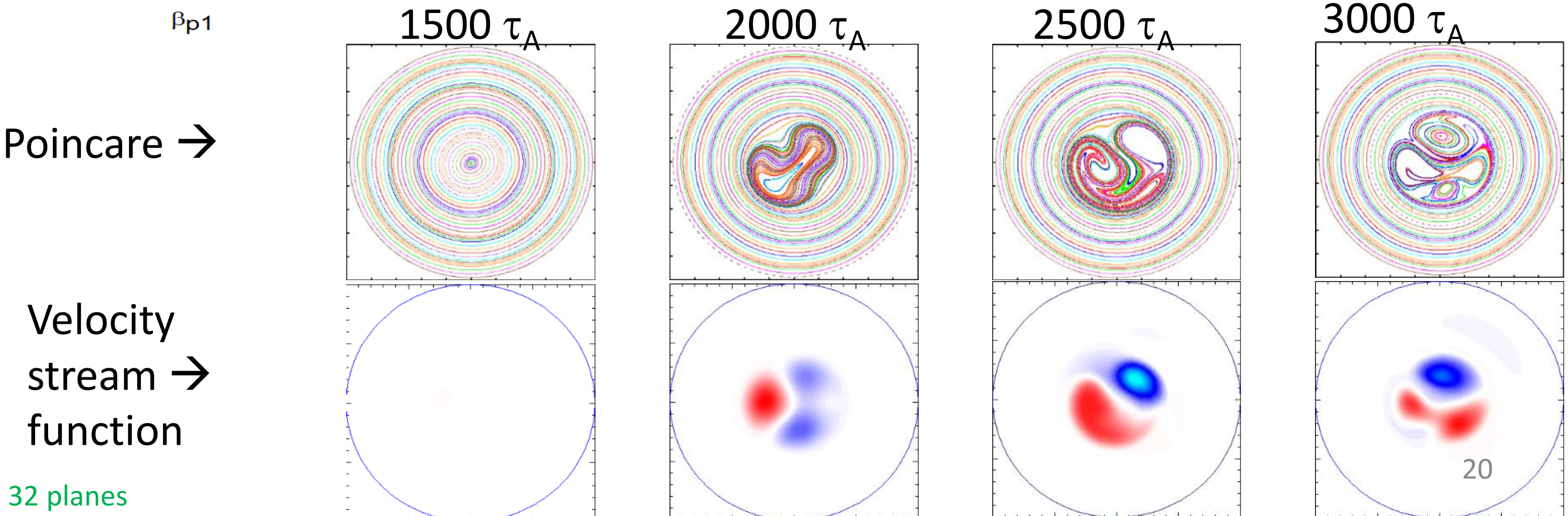


Initiate a NL M3D-C^1 run with one of these equilibria



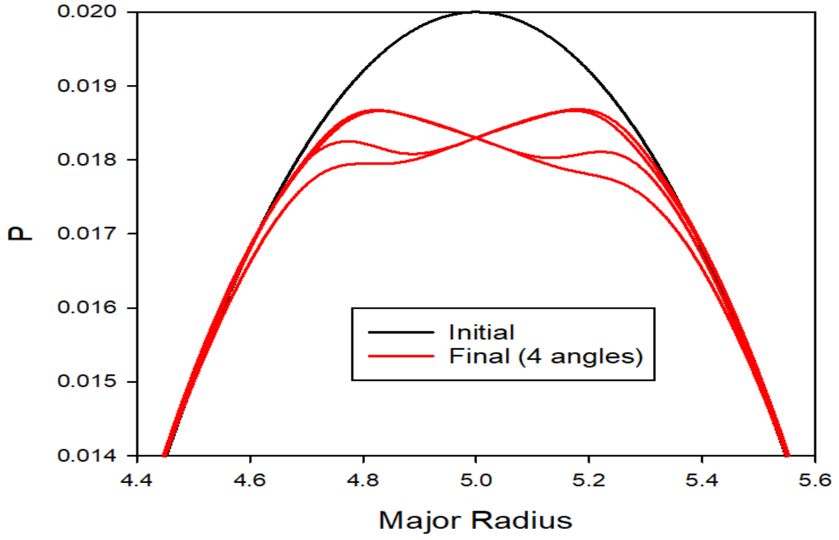


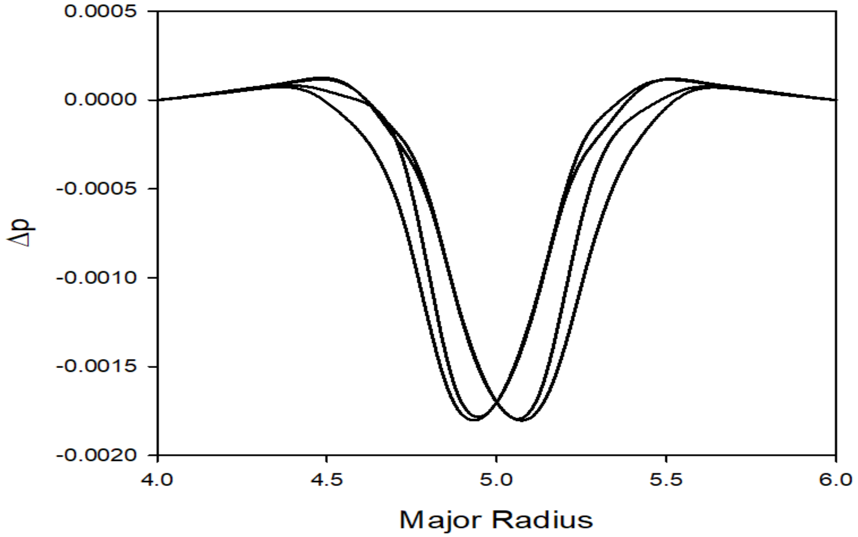
Assume there are sources to maintain equilibrium profiles. (eqsubtract = 1)



09a: 22,23,25,27: 32 planes

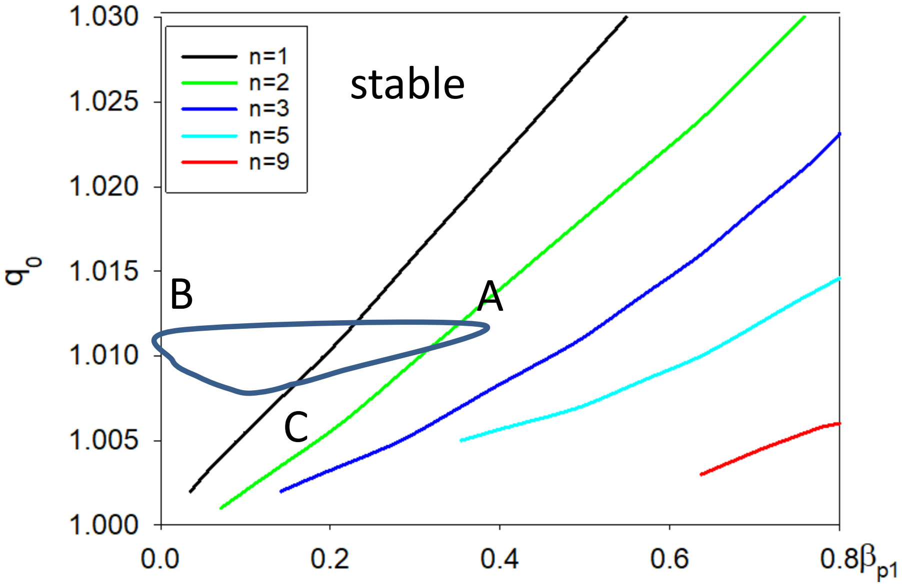
Central pressure flattens without affecting region with q > 1





Run09a: 31

The Sawtooth Cycle

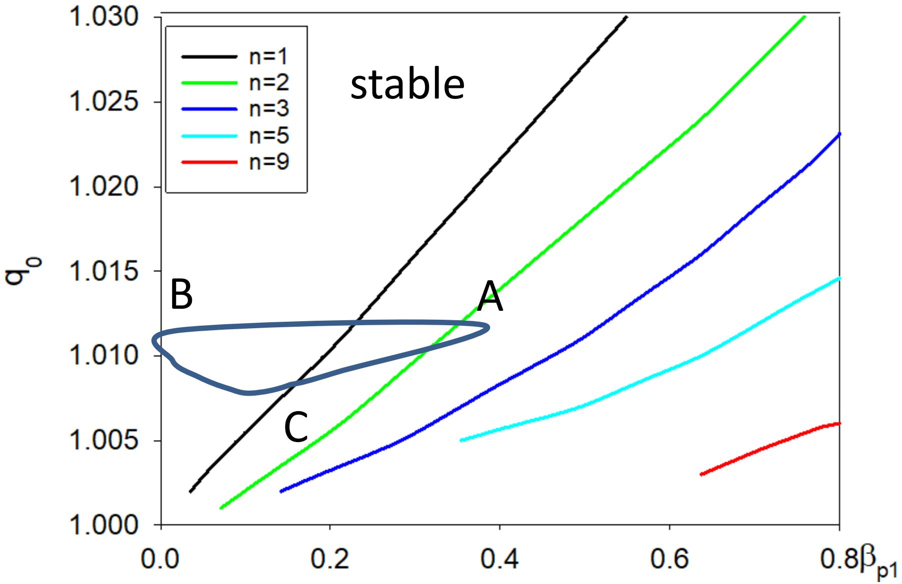


A. Fast crash when (2,2) ideal stability boundary is crossed. Other modes also excited by steep gradients that form in inner shear-free region

B. At low β_{p1}, plasma becomes axisymmetric, surfaces reform, β_{p1} begins to increase due to heating, and q_0 drops due to resistive diffusion

C. As (1,1) stability boundary is crossed, dynamo action works to increase q_0 as β_{p1} continues to increase due to heating.

The Sawtooth Cycle



No Reconnection!

A. Fast crash when (2,2) ideal stability boundary is crossed. Other modes also excited by steep gradients that form in inner shear-free region
B. At low β_{p1}, plasma becomes axisymmetric, surfaces reform, β_{p1} begins to increase due to heating, and q_0 drops due to resistive diffusion
C. As (1,1) stability boundary is crossed, dynamo action works to increase q_0 as β_{p1} continues to increase due to heating.

There is recent (and older) experimental evidence that q_0 stays near 1 during the entire sawtooth cycle.

- Wroblewski and Huang quote a value of q_0 very near unity in **TEXT** [1,2]
- Weisen used resonant Alfven waves to deduce that **TCA** had q_0 close to unity[3]
- Gill analyzed X-ray emission in **JET** when an injected pellet crosses the q=1 surface and found that the magnetic shear, dq/dr, interior to the q=1 surface was very low.[4]
- Wroblewski reports that q_0 in **DIII-D** is close to unity before and after sawtooth $\pm$ 0.05[5]
- Analysis of BAE modes during a sawtooth crash on **TORE SUPRE** imply that q_0 is normally slightly above unity after the sawtooth crash, and decreasing to unity[6]
- A recent study on **KSTAR** , supported by very high accuracy MSE measurements and supplemental MHD analysis concluded that q_0 was ~ 1 in sawtoothing discharges with relative accuracy +/- 0.03 and with compelling evidence that it is slightly above 1 after the crash.[7]

[1] Wroblewski, D., Huang, L, Moos, H. W. it et al Phys. Rev. Lett. **61**, 1724 (1988)
[2] Huang, L. K., Finkenthal, M.,Wroblewski, D., Phys. Fluids B. **2** 809 (1990
[3] Weisen, H., Borg, G., Joye, B., et al, Phys. Rev. Lett. **62**, 434 (1989)
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Summary and Future Directions

- Sawteeth in *low temperature, low-β* plasmas (like ST) can be explained by the Kadomtsev model
- Sawteeth in *high-temperature, high-β* tokamak discharges are likely caused by m=n > 1 *ideal MHD* modes causing turbulent convection in low shear region with $q \cong 1$
- The (1,1) interchange mode saturates at a low amplitude, and is responsible for keeping $q \cong 1$ in the center with very low shear via dynamo action … not for the crash.
- The rapid onset and fast crash time is caused by many ideal-MHD modes whose rapid growth rates are sensitive functions of q_0 and β_{p1}
- Since $q_0 \cong 1$ throughout the cycle, it is easy to see how (1,1) snakes can co-exist with sawteeth
- Next Step: Can this picture of sawteeth be used to explain “monster sawteeth” and RF sawtooth stabilization/destabilization? Other experimental tests?